# Unit -1 Bearings

1) <u>Hydrodynamic Lubricated Bearings:</u>

In Hydrodynamic lubricated Bearings, there is a thick film of lubricant between the journal & bearing. when the bearing is suppried with sufficient lubricant, a pressure is build up in the clearance space when the journal is rotating about an axis that is escentric with the bearing axis. The load can be supported by this fluid pressure without any actual contact between the journal and bearing.

The load carrying ability of hydrodynamic bearing arises simply because a viscous fluid resists bein pushed around.

Under proper conditions, this resistance to motion will develop a pressure distribution in the lubicant film that can support a useful load.

The load supporting pressure in hydrodynamic bearing arises from either

1. The flow of a viscous fluid in a converging channel (known as wedge film lubrication) or
2. The resistance of a viscous fluid to being squeezed out from betn approaching surfaces (known as squeeze film lubrication).

2. <u>Bearing characteristic Number:</u>

The co-efficient of friction in design of bearings is of great importance, because it affords a means for detemining the loss of power due to bearing friction.

Co-efft of friction for a full lubricated journal bearing is a function of three variables, ie

(i) $\frac{ZN}{P}$ (ii) $\frac{d}{c}$ (iii) $\frac{l}{d}$

$\therefore$ co-efficient of friction may be expressed as

$$\mu = \phi\left(\frac{ZN}{P}, \frac{d}{c}, \frac{l}{d}\right)$$

where $\mu$ = co-efft of friction

$\phi$ - A functional relationship

Z - Absolute viscosity of the lubricant in kg/m-s.

N - Speed of the journal in r/m.

P - Bearing Pressure on the Projected bearing area in $\frac{N}{mm^2}$.

= load on journal $\div$ l×d.

d - diameter of the jourat

l - length of the bearing

c - diametral clearance.

## Assumptions in Hydrodynamic Lubricated Bearings:

1. The lubricant obeys Newton's law of viscous flow.
2. The pressure is assumed to be constant throughout the film thickness.
3. The lubricant is assumed to be incompressible.
4. The viscosity is assumed to be constant throughout the film.
5. The flow is one dimensional ie side leakage is neglect

## Important Factors for the formation of Thick oil film in Hydrodynamic lubricated Bearings:

According to Reynolds, the following factors are essential for the formation of a thick film of oil in hydrodynamic lubricated Bearings:

1. A continuous supply of oil,
2. A relative motion betn the two surfaces in a direction approximately tangential to the surfaces.
3. The ability of one of the surfaces to take up a small inclination to the other surface in the direction of the relative motion.

The factor $\frac{ZN}{P}$ is termed as bearing characteristic number and is a dimensionless number.

The variation of co-efft of friction with the operating values of bearing characteristic number $\left(\frac{ZN}{P}\right)$ as obtained by Mc. Kee brothers in an actual test of friction is shown in fig.

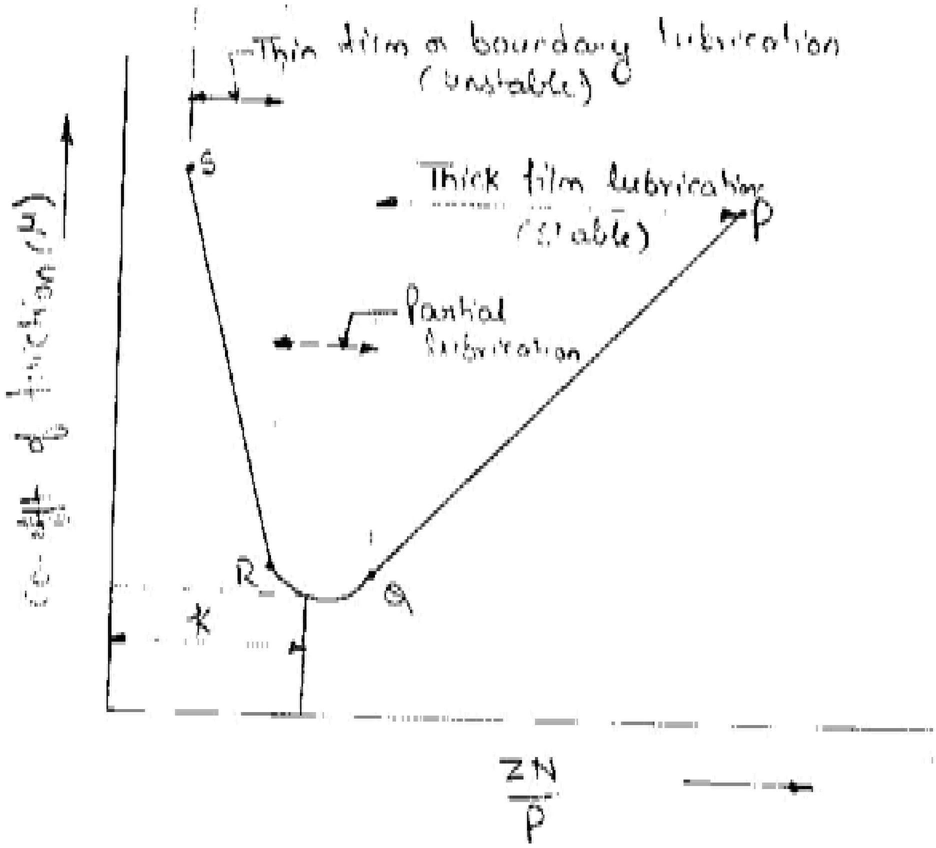
The factor $\frac{ZN}{P}$ helps to predict the performance of a bearing.

The part of the curve PQ represents the region of thick film lubrication. Between Q & R, the viscosity (Z) or the speed (N) are so low, or the pressure (P) is so great that their combination $\frac{ZN}{P}$ will reduce the film thickness so that partial metal to metal contact will result.

The thin film or boundary lubrication or imperfect lubrication exists between R & S on the curve.

This is the region where the viscosity of the lubricant ceases to be a measure of friction characteristics but the oiliness of the lubricant is effective in preventi complete metal to metal contact and seizure of the parts.

It may be noted that the part PQ of the curve represents stable operating conditions, since from any point of stability, a decrease in viscosity (Z) will reduce $\frac{ZN}{P}$. This will result in a decrease in co-efft of friction ($\mu$) followed by a lowering of bearing temperature that will raise the viscosity.




3. Select a single row deep groove ball bearing for a radial load of 4000N and an axial load of 5000N, operating at a speed of 1600 rpm for an average life of 5 yrs at 10 hours per day. Assume uniform and steady load.

Sol: Given:

$W_R = 4000$ N

$W_A = 5000$ N

$N = 1600$ rpm

The distance from the top of the piston to the first ring groove i.e. width of the top land,

$b_1 = t_H$ to $1.2t_H$

$= 16$ to $1.2 \times 16 = 16$ to $19.2$ mm.

width of other ring lands,

$b_2 = 0.75t_2$ to $t_2$

$= 0.75 \times 3$ to $3$ mm $= 2.25$ to $3$ mm.

Let us adopt $\underline{\underline{b_1 = 18\text{mm}}}$ & $\underline{\underline{b_2 = 2.5\text{ mm}}}$.

5. Design a journal bearing to carry a radial load of 3000 N. The journal having 50mm diameter rotates at 1500 rpm. The viscosity of oil at the operating temp is 25cp. if the allowable bearing pressure is $1.6 \frac{N}{mm^2}$.

<u>Sol.</u>

Given data:

$W = 3000N$, $d = 50mm$

$N = 1500$ rpm

Let $t_0 = 15.5°c$.

$Z = 25cp = \frac{25}{100}$ Poise $= 0.25$ Poise $= \frac{0.25}{10} \frac{Ns}{m^2}$

$= 0.025 \frac{Ns}{m^2}$

The journal bearing is designed in the following steps.

1. Let us find the length of the journal ($l$).

from the table, considering $\frac{l}{d}$ for centrifugal pump.

$\frac{l}{d}$ varies from 1-2.

Let us adopt, $\frac{l}{d} = 1.6$

$$l = 1.6d$$
$$= 1.6 \times 50$$
$$= 80 \text{ mm}.$$

2. We know that bearing pressure,

$$P = \frac{W}{l \times d}$$

$$= \frac{3000}{80 \times 50} = 0.75 \frac{N}{mm^2}.$$

Since the given bearing pressure is $1.6 \frac{N}{mm^2}$

$\therefore$ the above value of P is safe & hence the dimensions of l & d are safe.

3. $\frac{ZN}{p} = \frac{0.025 \times 1500}{0.75}$

$= 50.$

The minimum value of the bearing modulus at which the oil film will break is given by

$$3K = \frac{ZN}{p}$$

$$\Rightarrow K = \frac{1}{3}\frac{ZN}{p}$$

$$= \frac{1}{3} \times 28 = 9.33.$$

$\because$ the operating value of $\frac{ZN'}{p} = 28$, from table.

Since the calculated value of bearing characteris-tic number $\left(\frac{ZN}{p} = 50\right)$ is more than 9.33,

$\therefore$ the bearing will operate under hydrodynamic conditions.

4. We find that for centrifugal pumps, the clearance ratio $\left(\frac{c}{d}\right) = 0.0013$.

5. we know that co-efficient of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{P}\right)\left(\frac{d}{c}\right) + K$$

$$= \frac{33}{10^8} \times 50 \times \frac{1}{0.0013} + 0.002$$

$$= 0.0126 + 0.002$$

$$= \underline{\underline{0.0146}}$$

6. Heat generated,

$$Q_g = \mu W V$$

$$= \mu \cdot W \left(\frac{\pi d N}{60}\right) W$$

$$= 0.0146 \times 3000 \left(\frac{\pi \times 50 \times 1500}{60}\right) W$$

$$= \frac{173001.92}{1000} \text{ W}$$

( d is in meters)

$$= \underline{\underline{173 \text{ W}}}$$

$50\text{mm} = \frac{50}{1000}$

7. Heat dissipated,

$$Q_d = C \cdot A (t_b - t_a)$$

$$= C \cdot l \cdot d \times \frac{(t_o - t_a)}{2}$$

Let $\epsilon = 1232 \ W/m^2/°c$

$\therefore \quad \phi_d = 1232 \times 80 \times 50 \times 20'$

$= 98.56 W.$

($l$ & $d$ are in meters.)

We see that the heat generated is greater than heat dissipated which indicates that the bearing is warming up.

Therefore, the bearing should be cooled artificially.

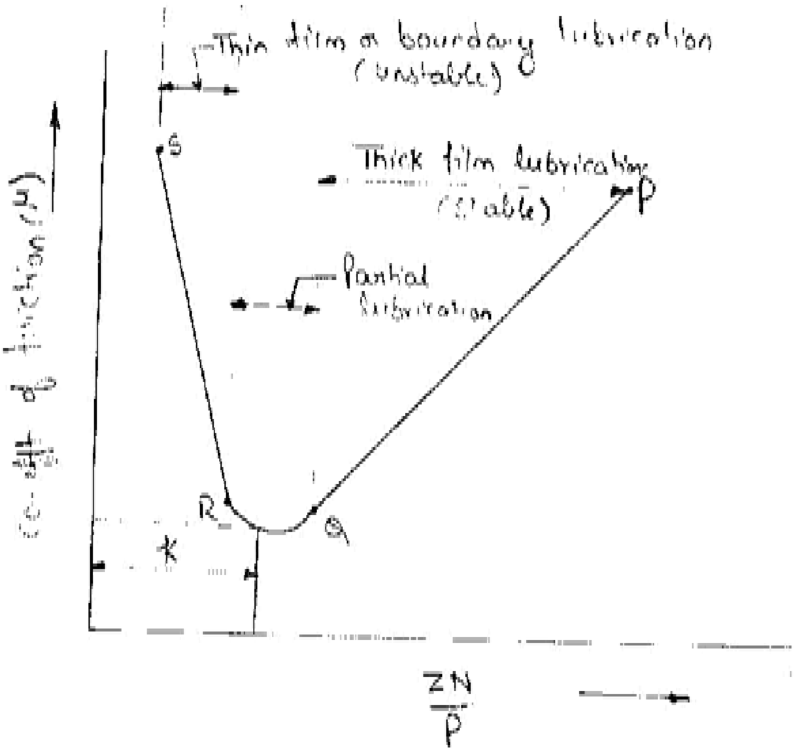
Amount of artificial cooling required,

$=$ Heat generated $-$ Heat dissipated

$= 173 - 98.56$

$= 74.44 \ W.$

reduce $\frac{ZN}{p}$. This will result in a decrease in co-efft of friction ($\mu$) followed by a lowering of bearing temperature that will raise the viscosity.




3. Select a single row deep groove ball bearing for a radial load of 4000N and an axial load of 5000N, operating at a speed of 1600 rpm for an average life of 5 yrs at 10 hours per day. Assume uniform and steady load.

Sol: Given:

$W_R = 4000$ N

$W_A = 5000$ N

$N = 1600$ rpm

Since the average life of the bearing is 5 years at 10 hours per day, therefore life of the bearing in hours,

$L_H = 5 \times 300 \times 10 = 15{,}000$ hours.

(assuming 300 working days per year)

and life of the bearing in revolutions,

$L = 60N \times L_H = 60 \times 1600 \times 15{,}000$

$= 1440 \times 10^6$ rev.

Basic dynamic equivalent radial load,

$W = XV \cdot W_R + Y \cdot W_A$

In order to determine the radial load factor (X) and axial load factor (Y), we require $\frac{W_A}{W_R}$ and $\frac{W_A}{C_o}$. since the value of basic static load capacity ($C_o$) is not known,

therefore let us take $\frac{W_A}{C_o} = 0.5$.

From table, we find that the values of X & Y corresponding to $\frac{W_A}{C_o} = 0.5$ & $\frac{W_A}{W_R} = \frac{5000}{4000}$

$\frac{W_A}{W_R} = 1.25$. are

$X = 0.56$ and $Y = 1$

Since the rotational factor (V) for most of the bearings is 1,

$\therefore$ Basic dynamic equivalent radial load,

$W = 0.56 \times 1 \times 4000 + 1 \times 5000 = 7240\ N.$

From table, we find that for uniform & steady load, the service factor ($K_S$) for ball bearings is 1.

$\therefore$ Bearing would be selected for $W = 7240\ N.$

Basic dynamic load rating,

$$C = W\left(\frac{L}{10^6}\right)^{1/K} = 7240\left(\frac{1440 \times 10^6}{10^6}\right)^{1/3}$$

$$= 81760\ N.$$

($\because$ $K = 3$, for ball bearings)

From table, select the bearing No. 315 which has the following basic capacities,

$C_0 = 72\,kN = 72000\ N$ and $C = 90\,kN$
$= 90{,}000\ N.$

$$\frac{W_A}{C_0} = \frac{5000}{72000} = 0.07$$

From tables, the values of X & Y are

$X = 0.56$ & $Y = 1.6$

Substituting these values in eq (i), we have dynamic equivalent load,

$$W = 0.56 \times 1 \times 4000 + 1.6 \times 5000$$
$$= 10240N$$

∴ Basic dynamic load rating,

$$C = 10240 \left( \frac{1440 \times 10^6}{10^6} \right)^{1/3}$$

$$= 115635N = 115.635KN$$

Thus, from table, the bearing number 319 having $C = 120KN$ may be selected.

1. Design a cast iron piston for a single acting four four stroke engine for the following data:

Cylinder bore = 100mm

Stroke = 125mm

Max gas pressure = $5 \frac{N}{mm^2}$

Indicated mean effective pressure = $0.75 \frac{N}{mm^2}$

Mechanical efficiency = 80%

Fuel consumption = 0.15 Kg per brake power per hour

Higher calorific value of fuel = $42 \times 10^3$ KJ/Kg;

Speed = 2000 rpm.

Any other data required for the design may be assumed.

Sol: Given

$D = 100mm$

$L = 125mm = 0.125m$

$P = 5 \frac{N}{mm^2}$

$P_m = 0.75 \frac{N}{mm^2}$

$\eta_m = 80\% = 0.8$

$m = 0.15\ Kg/B.P/h = 41.7 \times 10^{-6}\ Kg/B.P/s$

$HCV = 42 \times 10^3$ KJ/Kg;

$N = 2000$ rpm.

The dimensions for various components of the piston are determined as follows;

1. Piston Head or crown:

The thickness of the piston head or crown is determined on the basis of strength as well as on the basis of heat dissipation and the larger of the two values is adopted.

Thickness of piston head on the basis of strength,

$$t_H = \sqrt{\frac{3PD^2}{16\sigma_t}} = \sqrt{\frac{3 \times 5 \times 100^2}{16 \times 38}}$$

$$= 15.7 \text{ or } 16 \text{ mm.}$$

$\therefore$ taking $\sigma_t$ for cast iron $= 38$ MPa $= 38 \frac{N}{mm^2}$

since the engine is a four stroke engine, therefore the number of working strokes per minute,

$$n = \frac{N}{2} = \frac{2000}{2} = 1000.$$

Cross sectional area of cylinder,

$$A = \frac{\pi D^2}{4} = \frac{\pi (100)^2}{4} = 7855 \, mm^2$$

Indicated Power,

$$I.P = \frac{P_m . L . A . n}{60}$$

$$= \frac{0.75 \times 0.125 \times 7855 \times 100}{60} = 12270 W$$

$$= 12.27 \, KW.$$

$\therefore$ Brake Power, $BP = I.P \times \eta_m$

$$= 12.27 \times 0.8$$

$$= 9.8 \, KW$$

Heat flowing through the piston head,

$$H = C \times HCV \times m \times BP$$

$$= 0.05 \times 42 \times 10^3 \times 41.7 \times 10^{-6} \times 9.8$$

$$= 0.86 \, kW = 860 W.$$

taking $c = 0.05$.

$\therefore$ Thickness of Piston Head on the basis of Heat dissipation,

$$t_H = \frac{H}{12.56K(T_C - T_E)}$$

$$= \frac{860}{12.56 \times 46.6 \times 22.0} = 0.0067m$$

$$= 6.7mm.$$

$\because$ for cast iron, $K = 46.6 \; W/m/^{\circ}C$ &

$T_C - T_E = 220^{\circ}C$.

Taking the larger of the two values,

$t_H = \underline{16 \; mm}$.

Since the ratio of $\frac{L}{D}$ is 1.25, therefore a cup in the top of the piston head with a radius equal to $0.7D$ (ie 70mm) is provided.

2. <u>Radial ribs</u>

The radial ribs may be four in number. The thickness of the ribs varies from $t_H/3$ to $\frac{t_H}{2}$.

$\therefore$ Thickness of ribs, $t_R = \frac{16}{3}$ to $\frac{16}{2} = 5.33$ to $8mm$.

Let us adopt $t_R = \underline{\underline{7mm}}$.

3. Piston rings

Let us assume that there are total four rings (i.e. $n_R = 4$) out of which three are compression rings & one is an oil ring.

Radial thickness of the piston rings,

$$t_1 = D\sqrt{\frac{3p_w}{\sigma_t}}$$

$$= 100\sqrt{\frac{3\times 0.035}{90}} = 3.4 \text{ mm}.$$

taking $p_w = 0.035 \frac{N}{mm^2}$ & $\sigma_t = 90$ MPa.

Axial thickness of the piston rings.

$t_2 = 0.7t_1$ to $t_1$

$= 0.7\times 3.4$ to $3.4 = 2.38$ to $3.4$ mm.

Let $t_2 = 3$mm.

minimum axial thickness of the piston ring,

$$t_2 = \frac{D}{10n_R} = \frac{100}{10\times 4} = 2.5 \text{ mm}.$$

Thus axial thickness of the piston ring as already calculated (i.e $t_2 = 3$ mm) is satisfactory.

UNIT-IV. (II) B.T (1)

DMM-II

III MECH.

## Flat Belt drives.

### Ratio of Driving Tensions for Flat Belt drive:

The distance from the top of the piston to the first ring groove, width of the top land,

Consider a driven pulley rotating in the clockwise direction to 0.8 $t_H$ shown in fig.

= 16 to 1.2 x 16 = 16 to 19.2 mm.

Let $T_1$ = Tension in the belt on the width of other ring lands, tight side,

$b_2$ = 0.75 $t_2$ to $t_2$

$T_2$ = Tension in the belt on the = 0.75 x 3 to 3 mm = 2.25 to 3 mm.

slack side,

Let us adopt $b_1$ = 18 mm & $b_2$ = 2.5 mm.

$\theta$ = Angle of contact in radians.

5. Design a journal bearing for a small radial load of 3000 N. The journal having 50 mm

Now consider a small portion of the belt PQ subtending diameter angle rotates at 1500 rpm centre viscosity the of oil at the operating temp is 25cp. pulley as shown in the fig allowable bearing pressure is 1.6 N/mm².

The belt sol. PQ is in equilibrium under the following forces: Given data

W = 3000 N, d = 50 mm

1) Tension N = 1500 rpm the belt at P.

let $t_o$ = 15.5°c.

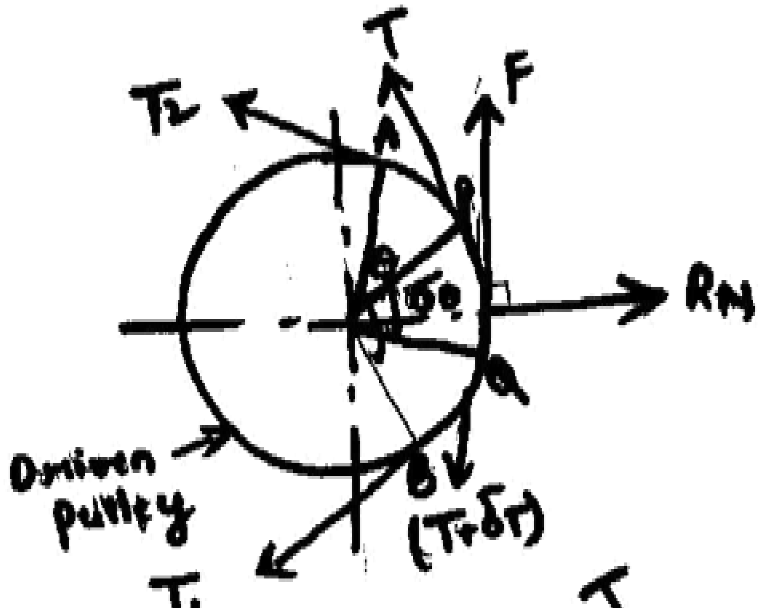
2) Tension = (25 cp) T $\frac{25}{100}$ Poise at Q = 0.25 Poise = $\frac{0.25}{10}$ $\frac{Ns}{m^2}$

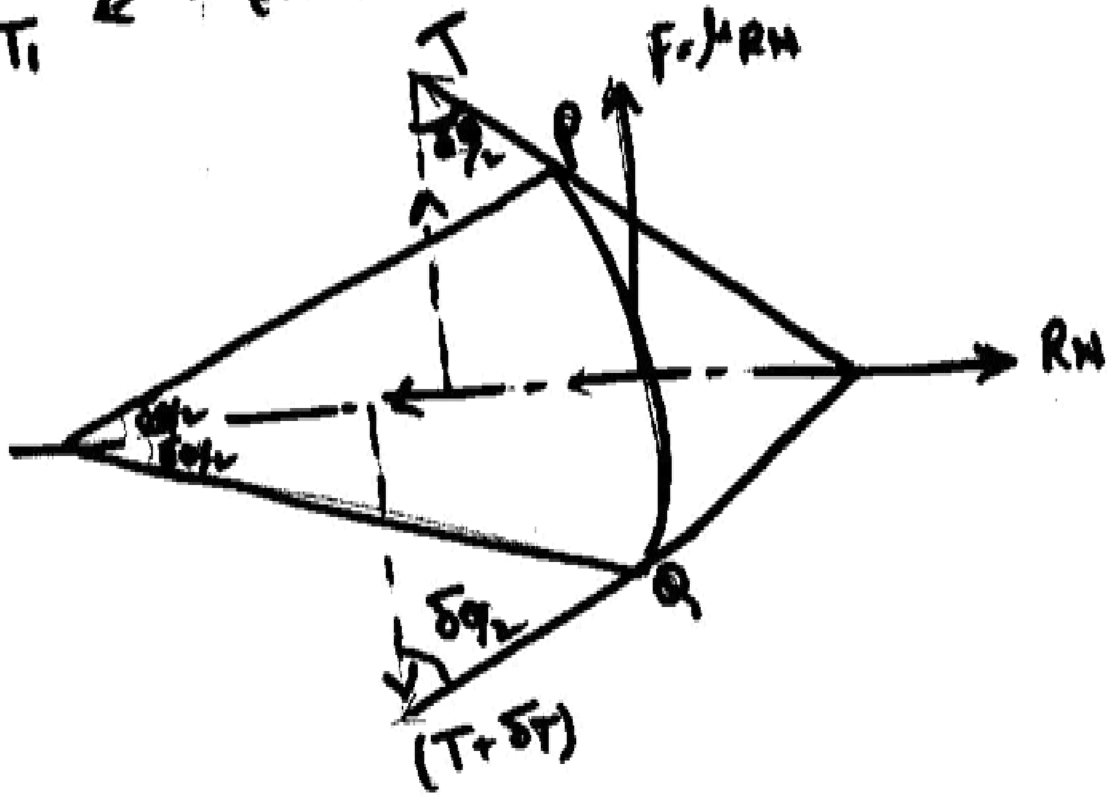
= 0.025 $\frac{Ns}{m^2}$

3) Normal reaction $R_N$ &

4) Frictional force $F = \mu \cdot R_N$

where $\mu$ is the co-efft of friction betn the belt and pulley.




Ratio of driving tensions for flat belt

Resolving all the forces horizontally, we have

$$R_N = (T+\delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2} \quad -(1)$$

Since the angle $\delta\theta$ is very small, therefore putting $\sin\frac{\delta\theta}{2} = \frac{\delta\theta}{2}$ in eq (1) we have

$$R_N = (T+\delta T)\frac{\delta\theta}{2} + T\cdot\frac{\delta\theta}{2}$$

$$= T\frac{\delta\theta}{2} + \delta T\cdot\frac{\delta\theta}{2} + T\cdot\frac{\delta\theta}{2}$$

$$= T\delta\theta \quad \left(\because \text{neglecting } \frac{\delta T\,\delta\theta}{2}\right) \quad -(3)$$

Now resolving the forces vertically, we have

$$\mu \times R_N = (T+\delta T)\cos\frac{\delta\theta}{2} - T\cdot\cos\frac{\delta\theta}{2} \quad -(2)$$

Since the angle $\delta\theta$ is very small, therefore putting $\cos\frac{\delta\theta}{2} = 1$ in eq (2) we have

$$\mu \times R_N = T + \delta T - T = \delta T$$

$$\therefore \quad R_N = \frac{\delta T}{\mu}.$$

Equating the values of RN from eqns -(4)

$$T.\delta\theta = \frac{\delta T}{\mu} \quad \text{or} \quad \frac{\delta T}{T} = \mu.\delta\theta.$$

Integrating the above equation between the limits $T_2$ & $T_1$ and from 0 to $\theta$, we have

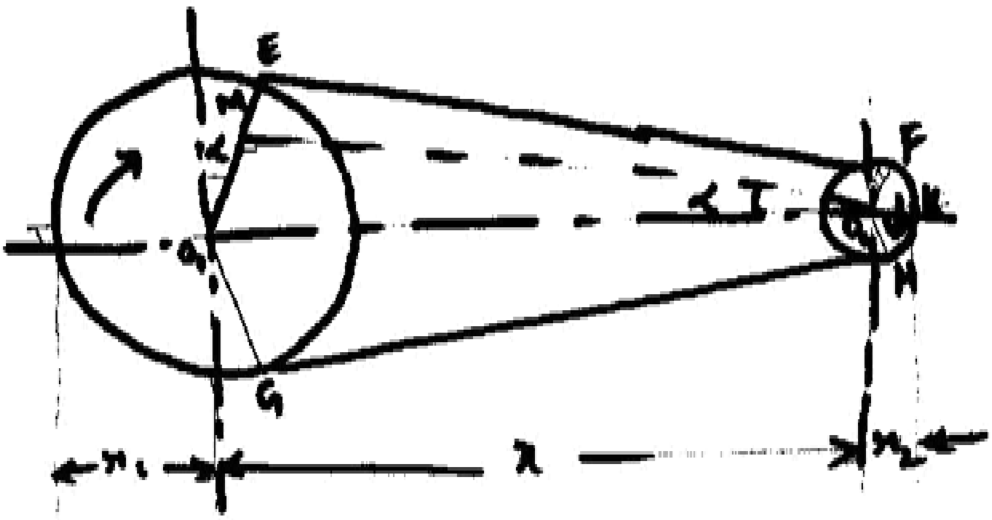
$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^{\theta} \delta\theta$$

$$\log_e\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

$$\Rightarrow \boxed{\frac{T_1}{T_2} = e^{\mu\theta}}.$$

(2) <u>Length of</u> an <u>open Belt drive.</u>
(Derivation).

In open belt drive, both the pulleys rotate in the same direction as shown in fig.




Open belt drive.

Let $r_1$ & $r_2$ = Radii of the larger & smaller pulleys.

$x$ = Distance between the centres of two pulleys (ie $O_1O_2$) &

$L$ = Total length of the belt.

Let the belt leaves the larger pulley at E & G & the smaller pulley at F & H as shown in fig. Through $O_2$ draw $O_2M$ $\parallel^{le}$ to FE.

From the geometry of the fig, we find that $O_2M$ will be perpendicular to $O_1E$.

Let the angle $MO_2O_1 = \alpha$ radians.

We know that length of the belt,

$$L = \text{Arc}\,(GJE + EF + \text{Arc}\,FKH + HG)$$

$$= 2(\text{Arc}\,JE + EF + \text{Arc}\,FK)$$

from the geometry of the fig, we find that, $\Delta^{le}\ O_2O_1M$

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E - EM}{O_1O_2} = \frac{r_1 - r_2}{x}.$$

Since the angle $\alpha$ is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radians)}$$

$$= \frac{r_1 - r_2}{x}.$$

$\because l = R\theta$

$$\text{Arc}\ JE = r_1\left(\frac{\pi}{2} + \alpha\right)$$

$$\text{arc}.\ FK = r_2\left(\frac{\pi}{2} - \alpha\right)$$

$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} \qquad (\text{from } \triangle^{le} O_1O_2M)$$

$$= \sqrt{x^2 - (r_1 - r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem,

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2 + \ldots\right]$$

$$= x - \frac{(r_1 - r_2)^2}{2x}.$$

Substituting the values of arc JE, arc FK & EF, we get

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 - r_2)^2}{2x} + r_2\left(\frac{\pi}{2} - \alpha\right)\right]$$

$$= 2\left[r_1\frac{\pi}{2} + r_1\alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2\frac{\pi}{2} - r_2\alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}\right]$$

$$= \pi(r_1+r_2) + 2\alpha(r_1-r_2) + 2x - \frac{(r_1-r_2)^2}{x}$$

Substituting the value of $\alpha = \left(\frac{r_1-r_2}{x}\right)$

we get –

$$L = \pi(r_1+r_2) + 2x\left(\frac{r_1-r_2}{x}\right)(r_1-r_2) + 2x - \frac{(r_1-r_2)^2}{x}$$

$$= \pi(r_1+r_2) + \frac{2(r_1-r_2)^2}{x} + 2x - \frac{(r_1-r_2)^2}{x}$$

$$= \pi(r_1+r_2) + \frac{(r_1-r_2)^2}{x} + 2x.$$

(in terms of pulley radii)

$$= \frac{\pi}{2}(d_1+d_2) + 2x + \frac{(d_1-d_2)^2}{4x}$$

(in terms of pulley diameter).

## Problems:

Two pulleys, one 450mm diameter and the other 200mm diameter, on parallel shafts 1.95m apart are connected by a crossed belt. Find the length of the belt required and the angle of contact between the belt & each pulley.

What power can be transmitted by belt when the larger pulley rotates at 200 rev/min, if the maximum permissible tension in the belt is 1KN, and the co-efft of friction betn the belt and pulley is 0.25.

**Sol.**

Given:

$d_1 = 450\ mm = 0.45m$

$r_1 = 0.225\ m$

$d_2 = 200\ mm = 0.2m$

$r_2 = 0.1\ m$

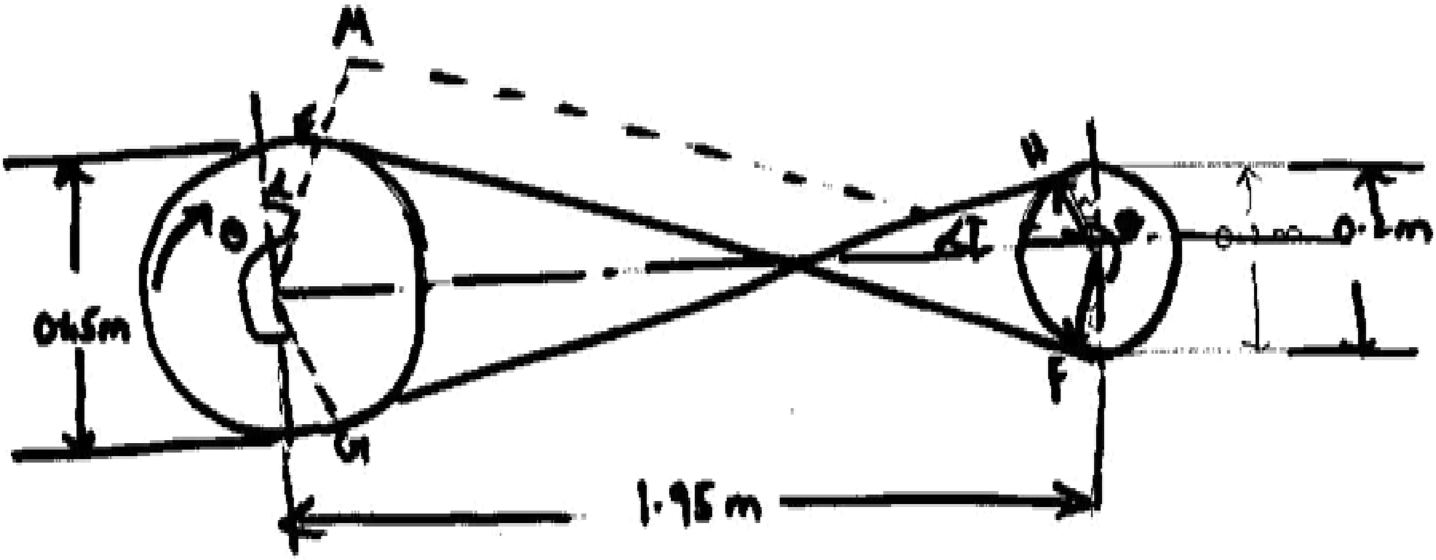
$x = 1.95m$

$N_1 = 200\ rpm$

$T_1 = 1KN = 1000N$

$\mu = 0.25$

The arrangement of crossed belt drive is shown in fig.




<u>Length of the belt:</u>

we know that length of the belt,

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(0.225 + 0.1) + 2 \times 1.95 \left(\frac{0.225 + 0.1}{1.95}\right)^2$$

$$= \underline{\underline{4.974}}\ m.$$

<u>Angle of contact betn the belt and each pulley:</u>

Let $\theta$ = Angle of contact betn the belt and each pulley.

For crossed belt drive,

$$\sin\alpha = \frac{r_1 + r_2}{x}$$

$$= \frac{0.225 + 0.1}{1.95} = 0.1667.$$

$$\Rightarrow \quad \alpha = 9.6^\circ$$

and $\theta = (180 + 2\alpha)\frac{\pi}{180}$ rad. for crossed belt drive.

$$= (180 + 2 \times 9.6)\frac{\pi}{180}$$

$$= 3.477 \text{ rad.}$$

Power Transmitted,

Let $T_1$ = Tension in the tight side of belt &

$T_2$ = Tension in slack side of belt.

we know that,

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu \cdot \theta$$

$$= 0.25 \times 3.477$$

$$= 0.8693$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.8693}{2.3} = 0.378$$

or $$\frac{T_1}{T_2} = e^{0.378}$$

$$\Rightarrow \frac{T_1}{T_2} = 2.387.$$

$$T_2 = \frac{T_1}{2.387} = \frac{1000}{2.387} = 419\,N.$$

we know that the velocity of belt,

$$V = \frac{\pi d_1 N_1}{60}$$

$$= \frac{\pi \times 0.45 \times 200}{60} = \underline{\underline{4.713\ m/s}}.$$

Power transmitted,

$$P = (T_1 - T_2)v$$

$$= (1000 - 419) \times 4.713$$

$$= \underline{\underline{2738\,W}}.$$

UNIT- IV

## Initial tension in the belt :-

When a belt is wound round the two pulleys (i.e. driver and follower), its two ends are joined together, so that the belt may continuously move over the pulleys, since the motion of the belt from the driver and the follower from the belt is governed by a firm grip due to friction betn the belt and the pulleys.

In order to increase this grip, the belt is tightened up. At this stage, even when the pulleys are stationary, the belt is subjected to some tension, called initial tension.

Let $T_0$ = Initial tension in the belt,

$T_1$ = Tension in the tight side of the belt,

$T_2$ = Tension in the slack side of the belt &

$\alpha$ = co-efft of increase of the belt length per unit force.

A little consideration will show that the increase of tension in the tight side

$= T_1 - T_0$.

and increase in the length of the belt on the tight side $= \alpha(T_1 - T_0)$.

IIIly decrease in tension in the slack side

$= T_0 - T_2$.

decrease in length of the belt on the slack side

$= \alpha (T_0 - T_2)$

Assuming that the belt material is perfectly elastic such that length of the belt remains constant, when it is at rest or in motion, therefore increase in length on the tight side is equal to decrease in the length on the slack side. Thus,

$$\alpha(T_1 - T_0) = \alpha(T_0 - T_2)$$

$$T_1 - T_0 = T_0 - T_2$$

$$T_0 = \frac{T_1 + T_2}{2}$$

$$\boxed{T_0 = \frac{T_1 + T_2 + 2T_c}{2}}$$

## Chain Drives:

Design a chain drive to actuate a compressor from 15KW electric motor running at 1000 rpm., the compressor speed being 350 rpm. The minimum centre distance is 500mm. The compressor operates 16 hours per day. The chain tension may be adjusted by shifting the motor on slides.

**Sol:** Given data:

Rated power = 15KW

$N_1 = 1000 rpm$

$N_2 = 350 rpm.$

velocity of ratio of chain drive,

$$\frac{N_1}{N_2} = \frac{1000}{350} = 2.86 \text{ say } 3.$$

From table, we find that for the roller chain, the number of teeth on the smaller sprocket or pinion ($T_1$) for a velocity ratio of 3 are 25.

$\therefore$ Number of teeth on the larger sprocket or gear, $T_2 = T_1 \times \frac{N_1}{N_2}$

$$= \frac{25 \times 1000}{350} = 71.5 \text{ say } 72.$$

Design Power = Rated Power × Service factor ($K_s$)

Load Factor ($K_1$) for variable load with heavy shock = 1.5

$K_2$ for drop lubrication = 1.

Rating factor $K_3$ for 16 hours per day = 1.25

$\therefore$ Service factor, $K_s = K_1 \cdot K_2 \cdot K_3$

$= 1.5 \times 1 \times 1.25$

$= 1.875$.

design power $= 15 \times 1.875 = 28.125$ kw.

From table, we find that corresponding to a pinion speed of 1000 rpm, the power transmitted for chain No. 12 is 15.65 kw per strand.

From table, we find that,

Pitch, $p = 19.05$ mm.

Roller diameter, $d = 12.07$ mm.

Minimum width of roller,

$w = 11.68$ mm.

Breaking load, $W_B = 59 \text{ kN}$
$= 59 \times 10^3 \text{ N}.$

Pitch circle diameter of the smaller sprocket or pinion,

$$d_1 = p \operatorname{cosec}\left(\frac{180}{T_1}\right) = 19.05 \operatorname{cosec}\left(\frac{185}{25}\right) \text{mm}$$

$$= 19.05 \times 7.98 = 152 \text{ mm}.$$

Pitch circle diameter of the larger gear

$$d_2 = p \operatorname{cosec}\left(\frac{180}{T_2}\right) = 19.05 \operatorname{cosec}\left(\frac{180}{72}\right)$$

$$= 19.05 \times 22.9 = 436 \text{ mm}.$$

Pitch line velocity of the smaller sprocket,

$$v_1 = \frac{\pi d_1 N_1}{60}$$

$$= \frac{\pi \times 0.152 \times 1000}{60}$$

$$= 7.96 \text{ m/s}.$$

$\therefore$ load on the chain,

$$W = \frac{\text{Rated Power}}{\text{Pitch line velocity}}$$

$$= \frac{15}{7.96} = 1.844 \text{ kN}.$$

$$\text{Factor of safety} = \frac{W_B}{W} = \frac{59\times10^3}{1844} = 32.$$

This value is more than the value given in table ie 11.

The minimum centre distance betn the smaller and larger sprockets should be 30 to 50 times the pitch. Let us take it as 30 times the pitch.

∴ Centre distance between the sprockets,

$= 30p$

$= 30\times19.05 = 572 \text{ mm}.$

In order to accomodate initial sag in the chain, the value of centre distance is reduced by 2 to 5 mm.

∴ Correct centre distance

$x = 572 - 4 = 568 \text{ mm}$

Number of chain links

$$K = \frac{T_1+T_2}{2} + \frac{2x}{p} + \left[\left(\frac{T_2-T_1}{2\pi}\right)\right]^2 \frac{p}{x} = 110.$$

∴ length of chain, $L = k\cdot p = 110\times19.05 = 2.096 \text{m}.$

UNIT- VII

# Power Screws

**Qn:** A differential screw jack is to be made as shown in fig. Neither screw rotatates. The outside screw dia is 50mm. The screw threads are of square form single start and the co-efft of thread friction is 0.15.

Determine:

(1) Efficiency of screw jack

(2) load than can be lifted if the shear stress in the body of the screw is limited to 28 MPa.

**Sol:** Given:

$d_o = 50mm$

$\mu = \tan\phi = 0.15;$

$P_1 = 16mm$

$P_2 = 12mm$

$\tau_{max} = 28 MPa = 28 \frac{N}{mm^2}.$

1) Efficiency of the screw jack

Mean diameter of the upper screw,

$d_1 = d_o - \frac{P_1}{2} = 50 - \frac{16}{2} = 42 mm.$

Mean dia of the lower screw,

$d_2 = d_o - \frac{P_2}{2} = 50 - \frac{12}{2} = 44 mm.$

$$\tan\alpha_1 = \frac{p_1}{\pi d_1} = \frac{16}{\pi \times 42} = 0.1212.$$

$$\tan\alpha_2 = \frac{p_2}{\pi d_2} = \frac{12}{\pi \times 44} = 0.0868.$$

Let $W$ = load than can be lifted in N.

we know that torque required to overcome friction at the upper screw,

$$T_1 = W\tan(\alpha_1+\phi)\frac{d_1}{2}$$

$$= W\left[\frac{\tan\alpha_1 + \tan\phi}{1-\tan\alpha_1\tan\phi}\right]\frac{d_1}{2}$$

$$= W\left[\frac{0.1212+0.15}{1-0.1212\times 0.15}\right]\frac{42}{2} = 5.8\,W\,\text{N-mm}.$$

Similarly, torque required to overcome friction at the lower screw,

$$T_2 = W\tan(\alpha_2-\phi)\frac{d_2}{2}$$

$$= -1.37\,W\,\text{N-mm}.$$

$\therefore$ Total torque required to overcome friction,

$$T = T_1 - T_2 = 5.8\,W - (-1.37\,W)$$
$$= 7.17\,W\,\text{N-mm}.$$

We know that the torque required when there is no friction,

$$T_0 = \frac{W}{2\pi}(P_1 - P_2) = \frac{W}{2\pi}(16-12)$$

$$= 0.636\,W \text{ N-mm.}$$

$\therefore$ Efficiency of the Screw jack,

$$\eta = \frac{T_0}{T} = \frac{0.636W}{7.17W} = 8.87\%.$$

2) <u>load that can be lifted:</u>

Since the upper screw is subjected to a larger torque,

$\therefore$ the load to be lifted (W) will be calculated on the basis of large torque ($T_1$)

Core diameter of the upper screw,

$$d_{c1} = d_0 - P_1 = 50 - 16 = 34\text{mm}.$$

Since the screw is subjected to direct compressive stress due to load W and shear stress due to torque $T_1$ therefore

Direct Compressive Stress,

$$\sigma_c = \frac{W}{A_{c1}}$$

$$= \frac{W}{\frac{\pi}{4}(d_{c1})^2} = \frac{W}{\frac{\pi}{4}(34)^2}$$

$$= \frac{W}{908} \frac{N}{mm^2}$$

and shear stress,

$$\tau = \frac{16T_1}{\pi(d_{c1})^3} = \frac{16 \times 5.8W}{\pi(34)^3}$$

$$= \frac{W}{1331} \frac{N}{mm^2}.$$

max shear stress ($\tau_{max}$),

$$28 = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{\left(\frac{W}{908}\right)^2 + 4\left(\frac{W}{1331}\right)^2}$$

$$= \frac{1}{2}\sqrt{1.213 \times 10^{-6} W^2 + 2.258 \times 10^{-6} W}$$

$$= \frac{1}{2} 1.863 \times 10^{-3} W$$

$$W = \frac{28 \times 2}{1.863 \times 10^{-3}} = \underline{\underline{30060 \text{ N}}}$$

$$= \underline{\underline{30.06 \text{ kN.}}}$$

Qn:12 The mean diameter of the square threaded screw having pitch of 10mm is 50mm. A load of 20kN is lifted through a distance of 170mm. Find the work done in lifting the load and the efficiency of the screw, when

(1) The load rotates with the screw, &

(2.) The load rests on the loose head which does not rotate with the screw.

The external & internal diameter of the bearing surface of the loose head are 60mm and 10mm respbly. $\mu = 0.08$.

Sol: Given:

$p = 10mm$

$d = 50mm$

$W = 20kN = 20 \times 10^3 N.$

$D_1 = 60mm$ or $R_1 = 30mm$

$D_2 = 10mm \Rightarrow R_2 = 5mm.$

$\mu = \tan\phi = \mu_1 = 0.08.$

we know that,

$$\tan\alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 50}$$

$$= 0.0637.$$

$\therefore$ Force required at the circumferance of the screw to lift the load,

$$P = W\tan(\alpha+\phi) = W\left[\frac{\tan\alpha+\tan\phi}{1-\tan\alpha\tan\phi}\right]$$

$$= 20\times10^3\left[\frac{0.0637+0.08}{1-0.0673\times0.08}\right]$$

$$= 2890N.$$

And torque required to overcome friction at the screw,

$$T = P\times\frac{d}{2} = 2890\times\frac{50}{2} = 72250\,N\text{-}mm.$$

Since the load is lifted through a vertical distance of 170mm and the distance moved by the screw in one rotation is 10mm (equal to pitch).

$$\therefore N = \frac{170}{10} = 17.$$

1. When the load rotates with the screw

work done in lifting the load

$$= T\times2\pi N = 72.25\times2\pi\times17$$
$$= 7718\,Nm.$$

Efficiency of the screw,

$$\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)}$$

$$= \tan\alpha\left(\frac{1-\tan\alpha\tan\phi}{\tan\alpha+\tan\phi}\right)$$

$$= \frac{0.0637\,(1-0.0637\times0.08)}{0.0637+0.08}$$

$$= 0.441 = 44.1\%.$$

2. When the load does not rotate with the screw

mean radius of the bearing surface

$$R = \frac{R_1+R_2}{2} = \frac{30+5}{2} = 17.5\,mm.$$

Torque required to overcome friction at the screw & collar,

$$T = P\times\frac{d}{2} + \mu_1 W R$$

$$= 2890\times\frac{50}{2} + 0.08\times20\times10^3\times17.5$$

$$= 100.25\ Nm.$$

$\therefore$ workdone by the torque in lifting the load

$= T \times 2\pi N$

$= 100.25 \times 2\pi \times 17 = 10710\ Nm.$

Torque required to lift the load, neglecting friction,

$T_0 = P_0 \times \frac{d}{2} = W \tan\alpha \times \frac{d}{2}$ $\quad \because P_0 = W\tan\alpha$

$= 20 \times 10^3 \times 0.0637 \times \frac{50}{2}$

$= 318.50\ N\text{-}mm.$

$\therefore$ Efficiency of the screw,

$\eta = \frac{T_0}{T} = \frac{31.85}{100.25} = 31.8\%.$

Qn:3 Short note on Acme or Trapezoidal Threads.

we know that the normal reaction in case of a square threaded screw is

$R_N = W\cos\alpha;$

$\alpha$ - Helix angle.

But in case of Acme or trapezoidal thread, the normal reaction between the screw and nut is increased because the axial component of this normal reaction must be equal to the axial load (W).

Consider an Acme or trapezoidal thread as shown in fig.

Let $2\beta$ = Angle of Acme thread and
$\beta$ = semi-angle of the thread.

$$R_N = \frac{W}{\cos\beta}$$

i.e. frictional force,

$$F = \mu \cdot R_N = \mu \cdot \frac{W}{\cos\beta} = \mu_1 \cdot W$$

where $\frac{\mu}{\cos\beta} = \mu_1$ known as virtual co-efft of friction.