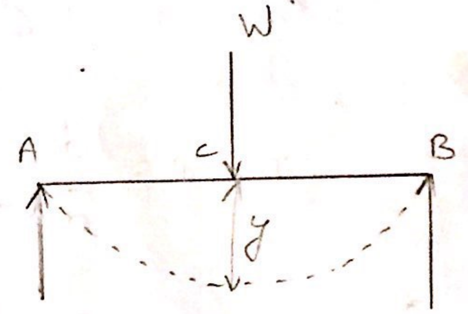
DEFLECTION OF BEAMS

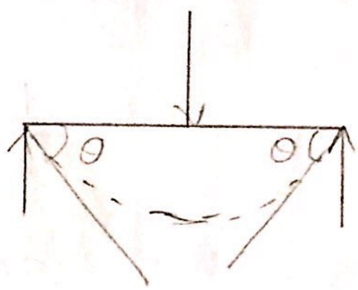
Deflection :-



It is the vertical distance of the beam measured before and after loading.

- It is denoted by 'y'
- Deflection at the supports is always zero.
- It is expressed in 'mm'.

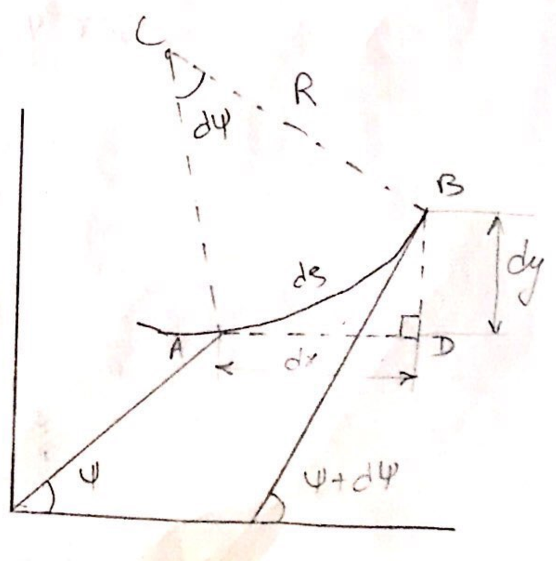
Slope :-



It is the angle measured between the tangent to the elastic curve & the original axis of the beam.

- It is denoted by 'θ' or $\frac{dy}{dx}$.
- It is expressed in radians.

Expression for Differential equation for Bending moment at any section.



we know from the diagram

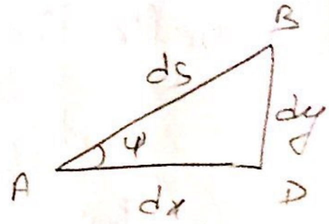
$\angle ACB = d\psi$

length of arc AB $= ds = d\psi \cdot R$

$\cancel{R = \frac{ds}{d\psi}} \Rightarrow \frac{ds}{R} = d\psi$

$\frac{1}{R} = \frac{d\psi}{ds}$ —— (1)

Consider Right Angled Δ^{le} ADB



$$\tan\psi = \frac{dy}{dx} \quad — \quad (2)$$

$$\cos\psi = \frac{dx}{ds} \quad — \quad (3)$$

$$\sec\psi = \frac{ds}{dx} \quad — \quad (4)$$

Differentiating eqn (2) w.r.t. x.

$$\frac{d}{dx}(\tan\psi) = \sec^2\psi$$

$$\frac{d}{dx}(\tan\psi) = \frac{d}{dx} \times \frac{dy}{dx}$$

$$\sec^2\psi \frac{d\psi}{dx} = \frac{d^2y}{dx^2}$$

Divide and multiply by ds

$$\sec^2\psi \frac{d\psi}{ds} \cdot \frac{ds}{dx} = \frac{d^2y}{dx^2}$$

$$\sec^2\psi \cdot \sec\psi \cdot \frac{d\psi}{ds} = \frac{d^2y}{dx^2}$$

$$\therefore \frac{d\psi}{ds} = \frac{1}{R})$$

$$\sec^3\psi \cdot \frac{1}{R} = \frac{d^2y}{dx^2}$$

$$\frac{1}{R} = \frac{\frac{d^2y}{dx^2}}{\sec^3\psi}$$

$$\frac{1}{R} = \frac{\frac{d^2y}{dx^2}}{(\sec^2\psi)^{\frac{3}{2}}}$$

$$(\sec^2\psi)^{\frac{3}{2}} = \sec\psi^{\cancel{2}\times\frac{3}{\cancel{2}}}$$
$$= \sec^3\psi$$

$$\frac{1}{R} = \frac{\frac{d^2y}{dx^2}}{(\tan^2\psi + 1)^{\frac{3}{2}}}$$

$(\sec^2\theta = \tan^2\theta + 1)$

$$\frac{1}{R} = \frac{\frac{d^2y}{dx^2}}{\left(\left(\frac{dy}{dx}\right)^2 + 1\right)^{3/2}}$$

$\tan\psi = \frac{dy}{dx}$

$\therefore$ denominator is very very small

neglecting the term $\left(\frac{dy}{dx}\right)^2$ we get

$$\frac{1}{R} = \frac{d^2y}{dx^2} \quad \text{--- (5)}$$

From bending equation.

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{E}{R}$$

$$\frac{M}{EI} = \frac{1}{R} \quad \text{--- (6)}$$

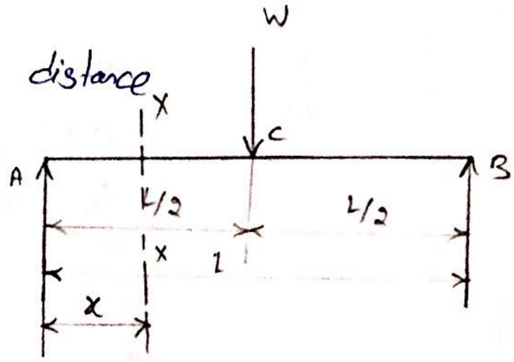
substitute eqn (6) in eqn (5)

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\boxed{M = EI \frac{d^2y}{dx^2}} \quad \text{--- (7)}$$

Deflection of a simply supported beam Carrying a point Load at the Centre.

Consider a section x-x at a distance x from A. The bending moment at this section is given by.



$$M_x = R_A \times x$$

$$\therefore R_A = \frac{W}{2} = R_B$$

$$= \frac{W}{2} \times x \quad \text{——} \quad (1)$$

But we know B.M at any section

$$M = EI \frac{d^2y}{dx^2} \quad \text{——} \quad (2)$$

Equate eqn (1) & (2)

$$EI \frac{d^2y}{dx^2} = \frac{W}{2} \times x. \quad \text{——} \quad (3)$$

Integrate the above equation (3) to get slope.

$$EI \int \frac{d^2y}{dx^2} = \int \frac{W}{2} \times x$$

$$EI \frac{dy}{dx} = \frac{W}{2}\left(\frac{x^2}{2}\right) + C_1 \quad \text{——} \quad (4)$$

For getting the C_1 value we have to apply boundary Conditions.

At $x = \frac{L}{2} \rightarrow \frac{dy}{dx} = 0$ $\qquad 0 = \frac{W}{4}\left(\frac{L^2}{2}\right) + C_1$

$$0 = \frac{WL^2}{4 \times 4} + C_1$$

$$\boxed{C_1 = -\frac{WL^2}{16}}$$

substitute C_1 value in eqn (4)

$$EI\frac{dy}{dx} = \frac{wx^2}{4} - \frac{wL^2}{16} \quad \text{—— (5)}$$

we know slope is maximum at A, $x = 0$.

$$EI\left(\frac{dy}{dx}\right)_{at\ A} = \frac{w(0)^2}{4} - \frac{wL^2}{16}$$

$$EI\,\theta_A = -\frac{wL^2}{16}$$

$$\theta_A = \frac{-wL^2}{16EI}$$

The slope at point will be equal to θ_A, since the load is symmetrically applied.

$$\boxed{\theta_B = \theta_A = \frac{-wL^2}{16EI}}$$

Deflection at any point.

Deflection at any point obtained by integrating the eqn (5)

$$\int EI\frac{dy}{dx} = \int\frac{wx^2}{4} - \int\frac{wL^2}{16}$$

$$EIy = \frac{w}{4}\times\frac{x^3}{3} - \frac{wL^2}{16}x + C_2 \quad \text{—— (6)}$$

where C_2 is another constant of integration.

At A $x = 0$, $y = 0$.

$$EI\times 0 = \frac{w(0)^3}{12} - \frac{wL^2}{16}(0) + C_2$$

$$\boxed{C_2 = 0}$$

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substitute the C_2 value in eqn 6

$$EIy = \frac{wx^3}{12} - \frac{wxL^2}{16} + 0$$

The above eqn is known as deflection equation.

we can find the deflection at any point by substituting the x value in the above eqn.

For maximum deflection $x = \frac{L}{2}$

$$EI\,y_c = \frac{w}{12}\left(\frac{L}{2}\right)^3 - \frac{wL^2}{16}\left(\frac{L}{2}\right)$$

$$y_c = \frac{w}{12}\frac{L^3}{8} - \frac{wL^3}{32}$$

$$= \frac{wL^3}{96} - \frac{wL^3}{32}$$

48 | 48 96,
2, | 1, 2

$$= \cancel{\frac{32wL^3 - 96wL^3}{}}$$

$$= \frac{wL^3 - 3wL^3}{96}$$

$$= \frac{-\cancel{2}wL^3}{\cancel{96}\ 48}$$

$$\boxed{y_c = \frac{-wL^3}{48EI}}$$

A beam 6m long, simply supported at its ends, is carrying a point load of 50KN at its centre. The M.O.I of the beam is $78 \times 10^6\ mm^4$. The young's modulus of the material is $2.1 \times 10^5\ N/mm^2$. Calculate (i) deflection at the centre.

ii) slope at the supports.

Given data:-

length of the beam $(l) = 6m$

point Load $(W) = 50KN$

M.O.I $(I) = 78 \times 10^6\ mm^4$

youngs modulus $(E) = 2.1 \times 10^5\ N/mm^2$.

we know the deflection at the centre in case of simply supported beam carries point load it is given by

$$y = \frac{WL^3}{48EI} = \frac{50 \times 10^3 \times (6000)^3}{48 \times 2.1 \times 10^5 \times 78 \times 10^6}$$

$$= \frac{1.08 \times 10^{16}}{7.8624 \times 10^{14}}$$

$$\boxed{y = 13.73mm}$$

$$\text{slope } (\theta) = \frac{Wl^2}{16EI} = \frac{50 \times 10^3 \times (6000)^2}{16 \times 2.1 \times 10^5 \times 78 \times 10^6}$$

$$= \frac{1.8 \times 10^{12}}{2.6208 \times 10^{14}} = 6.86 \times 10^{-3}\ rad \times \frac{180}{\pi}$$

$$\boxed{\theta = 0.3935^\circ}$$

A beam 3m long, simply supported at its ends is carrying a point load W at the centre. If the slope at the ends of the beam should not exceed 1°, find the deflection at the centre of the beam.

Given data:-

$$\text{Length } (L) = 3m$$

$$\text{slope } (\theta) = 1^\circ \times \frac{\pi}{180} = 0.0174 \text{ rad.}$$

we know

$$\text{deflection } (y) = \frac{Wl^3}{48EI} = \frac{L}{3} \times \frac{Wl^2}{16EI}$$

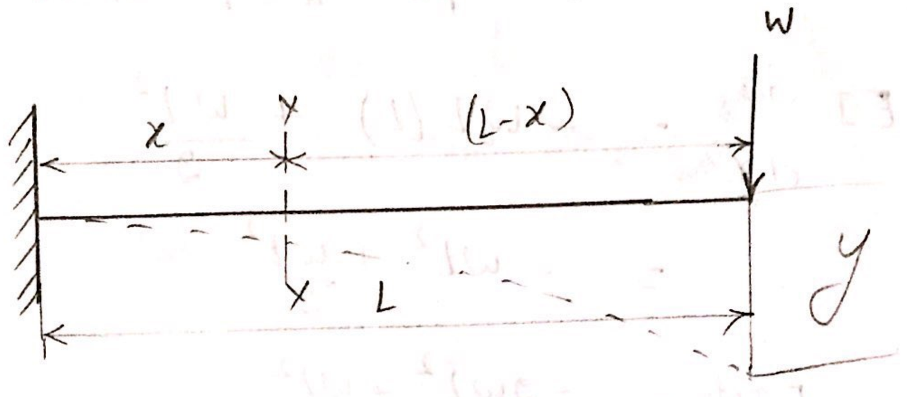
$$\theta = \frac{Wl^2}{16EI} = 0.0174 \text{ rad.}$$

$$y = \frac{\cancel{3000}^{1}}{\cancel{3}} \times 0.0174$$

$$= 1000 \times 0.0174$$

$$\boxed{y = 17.45 \text{ mm}}$$

DEFLECTION OF A CANTILEVER BEAM WITH A POINT LOAD AT THE FREE END :-



Bending moment at any section is given by

$$M = EI \frac{d^2y}{dx^2}$$

B.M at X-X section $= M_x = -W(L-x)$

$$EI \frac{d^2y}{dx^2} = -WL + Wx$$

For getting slope integrate the above eqn

$$\int EI \frac{d^2y}{dx^2} = -\int WL + \int Wx + C_1$$

$$EI \frac{dy}{dx} = -WLx + \frac{Wx^2}{2} + C_1$$

Apply boundary conditions

$$x = 0; \quad \frac{dy}{dx} = 0$$

$$EI(0) = -WL(0) + \frac{W(0)^2}{2} + C_1$$

$$\boxed{C_1 = 0}$$

$$EI \frac{dy}{dx} = -WLx + \frac{Wx^2}{2} \quad \text{--- (1)}$$

if $x = L$; then slope $\left(\frac{dy}{dx}\right)$ is maximum. For getting max. slope put $x = L$ in the above eqn.

$$EI\frac{dy}{dx} = -wL(L) + \frac{wL^2}{2}$$

$$= -wL^2 + \frac{wL^2}{2}$$

$$EI\frac{dy}{dx} = \frac{-2wL^2 + wL^2}{2}$$

$$\boxed{\frac{dy}{dx} = \frac{-wL^2}{2EI}}$$

Integrate the eqn ① to get max. deflection

$$\int EI\frac{dy}{dx} = \int -wLx + \int \frac{wx^2}{2} + C_2$$

$$EIy = -wL\frac{x^2}{2} + \frac{wx^3}{6} + C_2$$

Apply boundary Conditions

if $x = 0$; $y = 0$

$$EIy(0) = -\frac{wL(0)^2}{2} + \frac{w(0)^3}{6} + C_2$$

$$\boxed{C_2 = 0}$$

$$EIy = -\frac{wLx^2}{2} + \frac{wx^3}{6}$$

~~For~~ ~~Calculating~~ The maximum obtained at the free end i.e $x = L$

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$$EIy = -\frac{wL(L)^2}{2} + \frac{wL^3}{6}$$

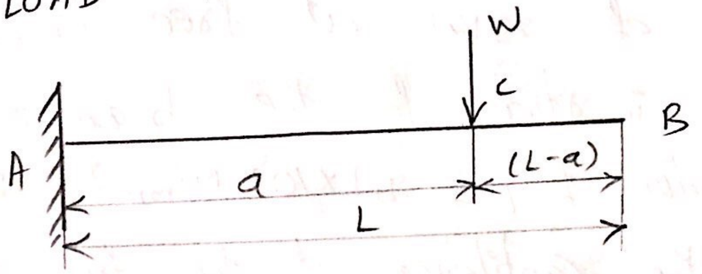
$$= -\frac{wL^3}{2} + \frac{wL^3}{6}$$

$$EIy = \frac{-6wL^3 + 2wL^3}{12}$$

$$y = -\frac{\cancel{4}^{1} wl^3}{\cancel{12}_{3} EI}$$

$$\boxed{y = -\frac{wl^3}{3EI}}$$

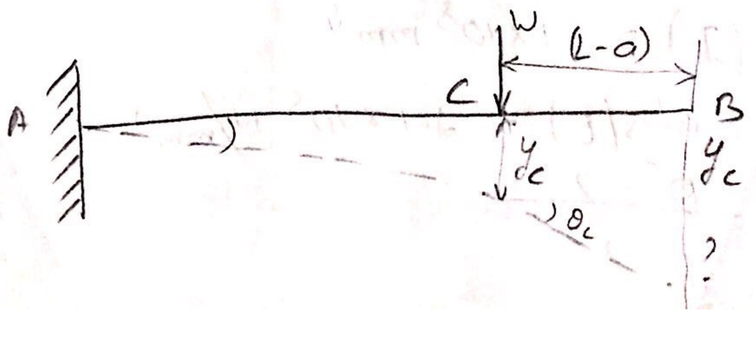
DEFLECTION OF CANTILEVER BEAM WITH A POINT LOAD AT A DISTANCE 'a' FROM FIXED END:-

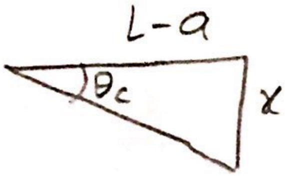


we know

$$y_c = \frac{wL^3}{3EI} = \frac{wa^3}{3EI}$$ $\boxed{\therefore L = a}$

$$\theta_c = \frac{wL^2}{2EI} = \frac{wa^2}{2EI}$$





$$\tan\theta_c = \frac{x}{L-a}$$

$$x = \tan\theta_c \times (L-a)$$

$$x = \theta_c (L-a)$$

$\boxed{\therefore \tan\theta = \theta}$

$$y_B = x + y_c$$

$$= \theta(L-a) + \left(\frac{Wa^3}{3EI}\right)$$

$$\boxed{y_B = \frac{Wa^2}{2EI}(L-a) + \left(\frac{Wa^3}{3EI}\right)}$$

Q) A Cantilever of length 3m is carrying a point load of 25KN at free end. If the moment of inertia of the beam = 1×10^8 mm4. and the value of $E = 2.1 \times 10^5$ N/mm2. Find.

i) slope of the Cantilever at the free end.

ii) Deflection at the free end.

Sol Given data :-

Length (L) = 3 m

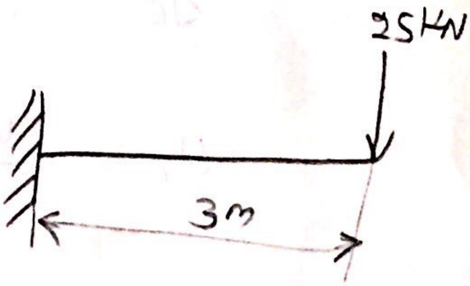
Load (W) = 25KN

M.O.I (I) = 1×10^8 mm4

youngs modulus (E) = 2.1×10^5 N/mm2

$\theta = ?$

$y = ?$



we know

$$\theta = \frac{WL^2}{2EI} = \frac{25\times10^3 \times (3000)^2}{2\times 2.1\times10^5 \times 1\times10^8}$$

$$\boxed{\theta = 0.005357\ rad}$$

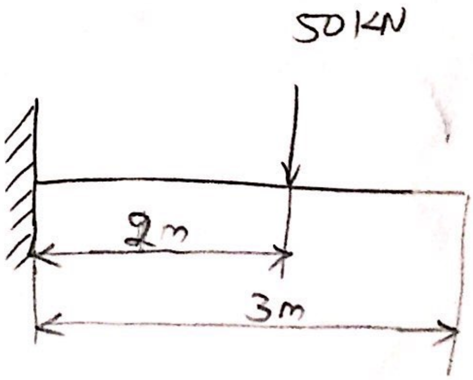
$$\text{Deflection } (y) = \frac{WL^3}{3EI} = \frac{25\times10^3 \times (3000)^3}{3\times 2.1\times10^5 \times 10^8}$$

$$\boxed{y = 10.71\,mm}$$

A Cantilever of length 3m is Carrying a point load of 50KN at a distance of 2m from the fixed end. of $I = 1\times10^8\ mm^4$. and $E = 2\times10^5\ N/mm^2$

find 1) Slope at free end.

2) Deflection at the free end.



Given data :-

moment of Inertia $(I) = 1\times10^8\ mm^4$

Youngs modulus $(E) = 2\times10^5\ N/mm^2$

we know

$$\text{Slope at free end} = \frac{Wa^2}{2EI}$$

$$= \frac{50000 \times (2000)^2}{2\times2\times10^5 \times 1\times10^8} = \frac{2\times10^{11}}{2\times10^{13}\times2}$$

$$\boxed{\theta = 5\times10^{-3}\ rad.}$$

Deflection at the free end

$$y = \frac{wa^3}{3EI} + \frac{wa^2}{2EI}(L-a)$$

$$= \frac{50\times10^3 \times (2000)^3}{3\times 2\times10^5 \times 10^8} + \frac{50\times10^3\times(2000)^2}{2\times2\times10^5\times10^8}(3000-2000)$$

$$= 6.67 + 5$$

$$\boxed{y = 11.67 \text{ mm}}$$

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Whenever a body is strained, the energy is absorbed in the body. The energy which is absorbed in the body due to straining effect is known as strain energy. The straining effect may be due to gradually applied load or suddenly applied or load with impact.

Resilience :-

The total strain energy stored in a body ~~in a body~~ is known as Resilience.

proof Resilience :-

The maximum strain energy stored in a body is known as proof Resilience.

modulus of Resilience :-

It is defined as the proof Resilience of a material per unit volume.

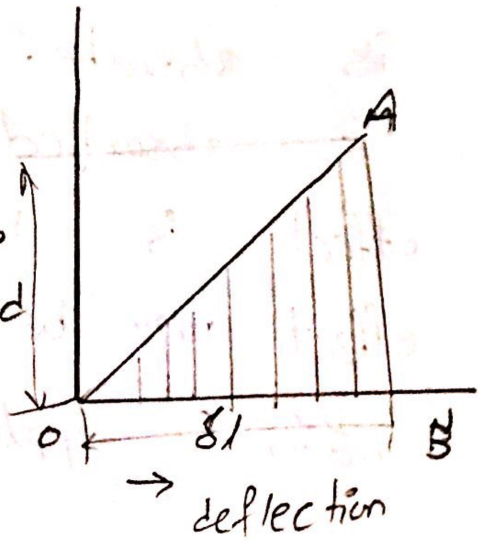
$$\text{modulus of Resilience} = \frac{\text{proof Resilience}}{\text{Volume of the body}}$$

Expression for strain energy stored in a body when the Load is applied gradually.

work done $= F \times S$

$= $ Area of Δ^{le} OAB ~~xxx~~

$= \frac{1}{2} \times P \times \delta l$



we know. $\boxed{\sigma = \frac{P}{A}}$

Load = Stress × Area

$P = \sigma \times A$

we also know $e = \frac{\delta l}{l}$

$\delta l = e \times l$

$= \frac{1}{2} \times \sigma \times A \times e \times l$

strain $= \frac{\text{stress}}{E}$

$= \frac{1}{2} \times \sigma \times A \times \frac{\sigma}{E} \times l$

work done by the load $= \frac{1}{2} \frac{\sigma^2}{E} \times Al$

But work done by the load in stretching the body is equal to the strain energy stored in the body

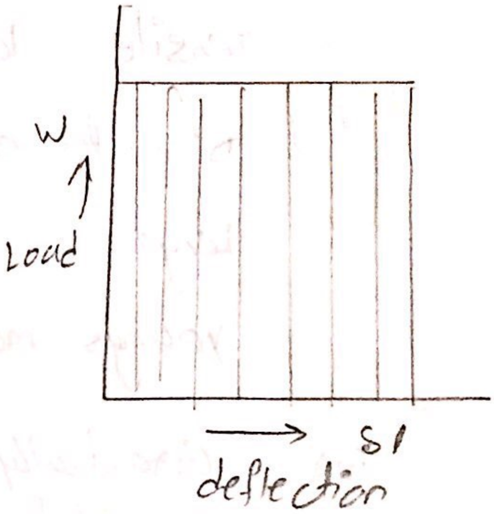
strain energy stored in the body $= \frac{\sigma^2}{2E} \times Al$

$\boxed{U = \frac{\sigma^2}{2E} \times \text{Volume.}}$ $A\cancel{l}$ $= \frac{1}{2} \times P \times \frac{\sigma}{E} \times A\cancel{l}$

Expression for strain energy stored in a body when the load is applied suddenly:-

when the Load is applied suddenly on a body, the load is constant throughout the process of the deformation of the body.

work done by the load $= P \times \delta l$



we know $\boxed{P = \sigma \times A}$ $\boxed{\delta l = e \times l}$

$\boxed{e = \frac{\sigma}{E}}$

$= P \times \frac{\sigma}{E} \times l$

$= P \times \frac{\sigma}{E} \times l$

max. strain energy stored in a body $= \frac{\sigma^2}{2E} \times A \times L$

strain energy stored = work done

$$\frac{\sigma^{\cancel{2}}}{2\cancel{E}} \times A \times \cancel{l} = P \times \cancel{\frac{\sigma}{E}} \times \cancel{l}$$

cancelling $\frac{\sigma l}{E}$ from both sides

$$\frac{\sigma A}{2} = P$$

$$\boxed{\sigma = \frac{2P}{A}}$$

A tensile load of 60KN is gradually applied to a circular bar of 4cm diameter and 5m long. of youngs modulus is 2×10^5 N/mm². Determine

1) stretch in the rod.
2) stress in the rod.
3) strain energy absorbed by the rod.

Given data :-

Tensile load (P) = 60KN = 60000 N

Diameter of bar (d) = 4cm = 40mm

Length of bar (L) = 5m = 5000mm

youngs modulus (E) = 2×10^5 N/mm².

For Gradually applied Load

$$\boxed{\text{stress } (\sigma) = \frac{P}{A}}$$

$$\text{Area} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(40)^2 = 1256.63 \text{ mm}^2.$$

$\boxed{e = \frac{\delta l}{l}}$

$\boxed{\frac{\sigma}{e} = E}$

1) $$\sigma = \frac{60000}{1256.63} = 47.74 \text{ N/mm}^2.$$

2) $$\delta l = \frac{\sigma}{E} \times l$$

$$= \frac{47.74}{2\times10^5} \times 5000$$

$$\boxed{\delta l = 1.19 \text{ mm}}$$

3) $U = \dfrac{\sigma^2}{2E} \times V$

$= \dfrac{(47.74)^2}{2 \times 2 \times 10^5} \times (1256.63 \times 5000)$

$= \dfrac{1.43 \times 10^{10}}{4 \times 10^5}$

$= 35799.93 \text{ N-mm}$

$\boxed{U = 35.79 \text{ N-m}}$

A Tensile load of 60KN is applied suddenly on a circular rod of 4cm diameter & 5m long. Determine elongation, stress and strain energy absorbed in a rod. Take $E = 2 \times 10^5 \text{ N/mm}^2$.

$P = 60 \text{KN} = 60000 \text{ N}$

$d = 4 \text{ cm} = 40 \text{ mm}$

$l = 5 \text{ m} = 5000 \text{ mm}$

$E = 2 \times 10^5 \text{ N/mm}^2$.

we know for suddenly applied load

$\sigma = \dfrac{2P}{A} = \dfrac{2 \times 60000}{\frac{\pi}{4}(40)^2} = \dfrac{12 \times 10^4}{1256.63}$

$\boxed{\sigma = 95.49 \text{ N/mm}^2}$

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$$\delta l = \frac{\sigma}{E} \times l$$

$$= \frac{95.49 \times 5000}{2 \times 10^5} = \frac{477467.51}{200000}$$

$$\boxed{\delta l = 2.38\ mm}$$

3) strain energy

$$U = \frac{\sigma^2}{2E} \times V \qquad \boxed{\therefore V = A \times l}$$

$$= \frac{(95.49)^2}{2 \times 2 \times 10^5} \times (1256.63) \times (5000)$$

$$= 143238\ N\text{-}mm$$

$$\boxed{U = 143.23\ N\text{-}m}$$

A steel rod is 2m long and 50mm in dia. An Axial pull of 100KN is suddenly applied to the rod. Calculate the instantaneous stress induced and also elongation produced in the rod. Take $E = 200\ GN/m^2$.

length of rod $(l) = 2m = 2000mm$

Dia of rod $(d) = 50mm$

Axial pull $(P) = 100KN$

youngs modulus $(E) = 200 GN/m^2 = 200 \times 10^9 N/m^2$

$$= \frac{200 \times \cancel{10^9}^{3}}{\cancel{10^6}} = 2 \times 10^5\ N/mm^2$$

For suddenly applied load

$$\sigma = \frac{2P}{A} = \frac{2 \times 100 \times 10^3}{\frac{\pi}{4}(50)^2} = \frac{2 \times 10^5}{1963.49}$$

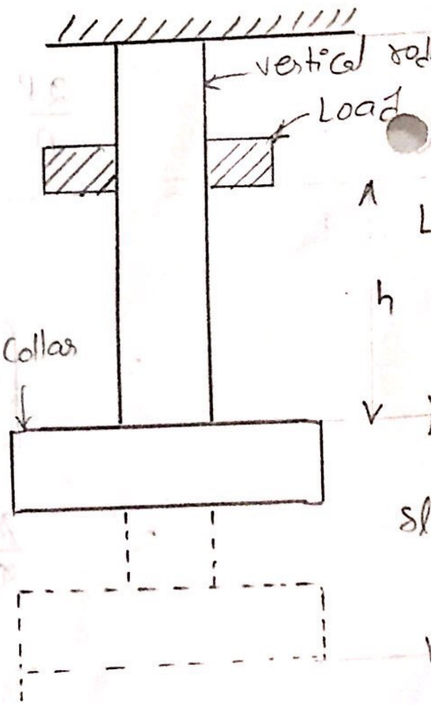
$$\boxed{\sigma = 101.86 \ N/mm^2}$$

Elongation $\delta l = \frac{\sigma}{E} \times L$

$$= \frac{101.86}{2 \times 10^5} \times 2000$$

$$\boxed{\delta l = 1.01 \ mm}$$

EXPRESSION FOR STRAIN ENERGY STORED IN A BODY WHEN THE LOAD IS APPLIED WITH IMPACT :-



work done = Force x distance

$$W.D = P(h + \delta l)$$

we know

strain energy $(U) = \frac{\sigma^2}{2E} \times V$

$$W.D = U$$

$$Ph + P\delta l = \frac{\sigma^2}{2E} \times V$$

$$Ph + P\frac{\sigma}{E} \times l = \frac{\sigma^2}{2E} \times A \times l$$

$$\frac{\sigma^2}{2E} \times Al - \frac{P\sigma}{E} \times l - Ph = 0.$$

$$\boxed{\therefore e = \frac{\delta l}{l}}$$

$$\boxed{\frac{\sigma}{e} = E}$$

~~100~~ 10⁹ = 200×10⁹ N/

= 2.

multiply the above equation by $\frac{2E}{Al}$ to get in the form of quadratic equation.

$$\frac{\sigma^2}{\cancel{2E}} \times \frac{\cancel{2E}}{\cancel{Al}} \times \cancel{Al} - \frac{P\sigma \cancel{l}}{\cancel{E}} \times \frac{2\cancel{E}}{A\cancel{l}} - Ph \times \frac{2E}{Al} = 0$$

$$\sigma^2 - \frac{2P\sigma}{A} - \frac{2EPh}{Al} = 0$$

The above equation in the form of $ax^2+bx+c=0$

we know $$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\frac{-\left(-\frac{2P}{A}\right) \pm \sqrt{\left(-\frac{2P}{A}\right)^2 - 4 \times 1 \times \left(-\frac{2EPh}{Al}\right)}}{2 \times 1}$$

$$= \frac{\frac{2P}{A} \pm \sqrt{\frac{4P^2}{A^2} + \frac{8EPh}{Al}}}{2}$$

$$= \left[\frac{2P}{A} \pm \sqrt{\frac{4P^2}{A^2} + \frac{8EPh}{Al}}\right] \times \frac{1}{2}$$

$$= \frac{\cancel{2}P}{\cancel{2}A} \pm \sqrt{\frac{\cancel{4}P^2}{\cancel{4}A^2} + \frac{\cancel{8}^{2}EPh}{\cancel{4}Al}}$$

$$= \frac{P}{A} \pm \sqrt{\frac{P^2}{A^2} + \frac{2EPh}{Al}}$$

neglect -ve root.

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$$P(h+\delta l) = \frac{\sigma^2}{2E} \times Al$$

$$10\times10^3(30+\delta l) = \frac{(187.76)^2}{2\times210\times10^3} \times 1000 \times 4000$$

$$30+\delta l = \frac{1.4101\times10^{11}}{42\times10^8}$$

$$\delta l = 33.57 - 30$$

$$\boxed{\delta l = 3.57mm}$$

a)

Sol

A Load of 100N falls to a height of 2cm Collar rigidity attached to the Lower end of the vertical rod 1.5m long and of $1.5cm^2$ cross-sectional area. The upper end of the vertical bar is fixed. Determine
1) stress 2) Elongation.
3) strain energy stored in the vertical rod.
Take $E = 2\times10^5$ N/mm².

Given data :-

Load (P) = 100 N

Height (h) = 2Cm = (20 mm)

length (l) = 1.5m = 1500 mm

Area (A) = $1.5cm^2$ = 1.5×10^2 mm^2

youngs modulus (E) = 2×10^5 N/mm² we know for impact Load

$$\sigma = \frac{P}{A}\left[1 + \sqrt{1 + \frac{2EAh}{Pl}}\right]$$

$$= \frac{100}{1.5\times10^2}\left[1 + \sqrt{1 + \frac{2\times2\times10^5\times1.5\times10^2\times20}{100\times1500}}\right]$$

$$\frac{100}{150}\left[1 + \sqrt{1 + \frac{12\times10^8}{15\times10^4}}\right]$$

$$0.66\left[1 + \sqrt{1 + 8000}\right]$$

$$= 0.66\left[1 + \sqrt{8001}\right]$$

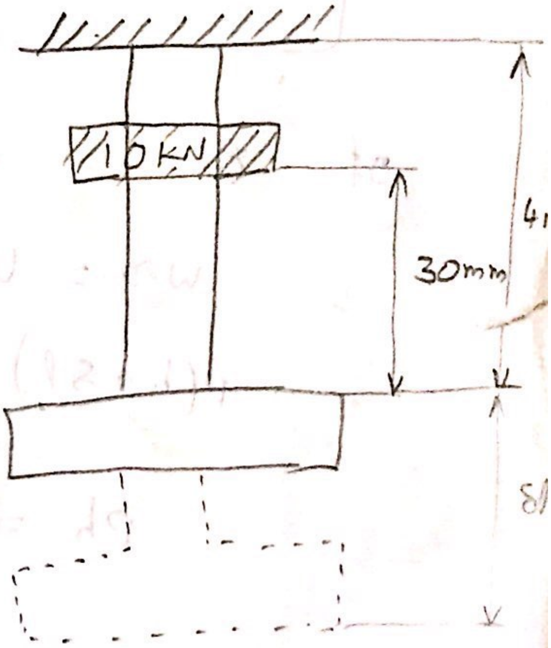
$$\boxed{\sigma = 59.69 \ N/mm^2}$$

A weight of 10KN falls by 30mm on a Collar rigidly attached to a vertical bar 4m long and 1000 mm^2 in cross-section. Find the expansion of the bar. Take E = 210GPa.

Given data :-

weight (P) = 10KN

$= 10 \times 10^3 N$



Height (h) = 30mm

Length (l) = 4m

$= 4 \times 10^3 mm$

Area (A) = 1000 mm^2

youngs modulus (E) = 210 GPa

$= 210 \times 10^9 N/m^2$

$= \frac{210 \times \cancel{10^9}^{3}}{\cancel{10^6}} N/mm^2$

$= 210 \times 10^3 N/mm^2$

$$\sigma = \frac{P}{A}\left[1 + \sqrt{1 + \frac{2EAh}{Pl}}\right]$$

$$= \frac{10 \times 10^3}{1000}\left[1 + \sqrt{1 + \frac{2 \times 210 \times 10^3 \times 1000 \times 30}{10 \times 10^3 \times 4000}}\right]$$

$$\boxed{\sigma = 187.76 \ N/mm^2}$$

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$$= \frac{P}{A} + \sqrt{\frac{P^2}{A^2} + \frac{2EPh}{Al}}$$

$$\therefore \frac{P}{A} + \sqrt{\frac{P^2}{A^2} + \frac{2EAh}{Pl} \times \frac{P}{A}}$$

$$= \frac{P}{A} + \frac{P}{A}\left[\sqrt{1 + \frac{2EAh}{Pl}}\right]$$

$$\boxed{\sigma = \frac{P}{A}\left[1 + \sqrt{1 + \frac{2EAh}{Pl}}\right]}$$

If δl is very small

$$W.D = U$$

$$P(h + \delta l) = \frac{\sigma^2}{2E} \times Al$$

$$Ph = \frac{\sigma^2}{2E} \times Al$$

$$\sigma^2 = \frac{2EPh}{Al}$$

$$\boxed{\sigma = \sqrt{\frac{2EPh}{Al}}}$$

if $h = 0$

$$= \frac{P}{A}\left[1 + \sqrt{1 + \frac{2EA(0)}{Pl}}\right]$$

$$= \frac{P}{A}\left[1 + \sqrt{1}\right]$$

$$\boxed{\sigma = \frac{2P}{A}}$$