TORSION OF SHAFTS AND SPRINGS

TORSION

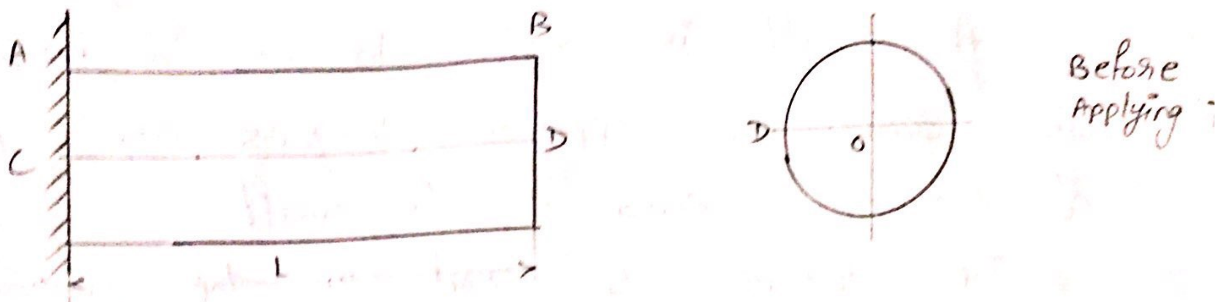
A shaft is said to be in torsion, when equal and opposite torques are applied at the two ends of the shaft.

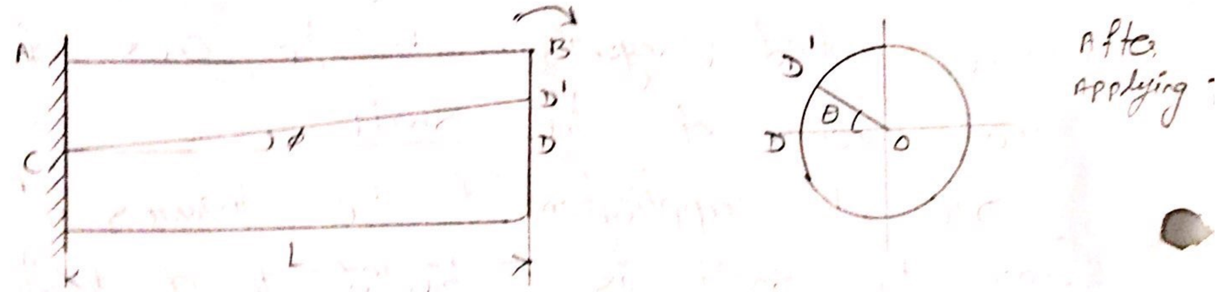
The torque is equal to the product of Force applied (Tangentially to the ends of a shaft) and radius of the shaft.

Due to application of the torques at the two ends the shaft is subjected to a twisting moment.

DERIVATION OF TORSION EQUATION :-

Consider a shaft fixed at one end AA and free at the end BB as shown in below Let CD is any line on the outer surface of the shaft. Now let the shaft is subjected to a torque T at the end BB as shown in fig (2). As a result of this torque T, the shaft at the end BB will rotate clockwise and every cross- section of the shaft will be subjected to shear stresses. The point D will shift to D' and hence line CD will be deflected to CD' as shown in fig (2) The line OD will be shifted to OD' as shown in fig 2b.





we know $\tan \phi = \dfrac{DD'}{CD}$ $\quad \boxed{\therefore \tan \phi = \phi}$

$$\boxed{\phi = \frac{DD'}{CD}}$$

we Also know

$$\text{shear strain} = \frac{\text{Distorsion at outer surface}}{\text{Length of shaft}}$$

$$\phi = \frac{DD'}{L}$$

Here $DD' = R\theta$

$$\phi = \frac{R\theta}{L}$$

Now the modulus of Rigidity(G) is given by

$$G = \frac{\text{shear stress}}{\text{shear strain}} = \frac{\tau}{\phi}$$

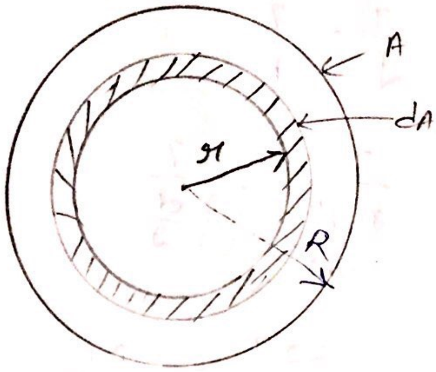
$$G = \frac{\tau}{\frac{R\theta}{L}} \Rightarrow G = \frac{L\tau}{R\theta}$$

Scanned with CamScanner

$$\frac{\tau}{R} = \frac{G\theta}{L}$$

For outer Area

$$\tau = \frac{F}{A}$$



<u>For small elemental strip</u>

Force $(dF) = \tau \cdot dA$

$$\boxed{\text{Torque } (T) = \text{Force } (F) \times \text{Radius}}$$

$$dT = dF \times r$$

$$dT = \tau \, dA \cdot r$$

To get Total Torque we have to integrate above eqn

$$\int dT = \int \frac{G\theta}{L} \times r \times dA \cdot r$$

At radius r shear stress

$$\tau = \frac{G\theta}{L} \times r$$

$$T = \frac{G\theta}{L} \int r^2 \, dA$$

$$\boxed{\therefore \int r^2 \, dA = J}$$

$$= \frac{G\theta}{L} \times J$$

$$\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{R}$$

T = Torque

J = Polar MoI

G = modulus of Rigidity

θ = Angle of twist in radians

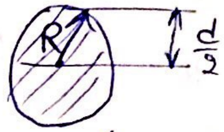
L = length of shaft

τ = Shear stress

R = Distance from the centre of shaft.

maximum torque transmitted by solid circular shaft :-

we know



$$\frac{T}{J} = \frac{\tau}{R}$$

$$\frac{T}{\frac{\pi d^4}{32}} = \frac{\tau}{\frac{d}{2}}$$

$$\frac{32T}{\pi d^4} = \frac{2\tau}{d}$$

$$T = \frac{\cancel{2}^{1} \pi d^{\cancel{4}^{3}} \cdot \tau}{\cancel{32}_{16} \cdot \cancel{d}}$$

$$\boxed{T = \frac{\pi d^3}{16} \cdot \tau}$$

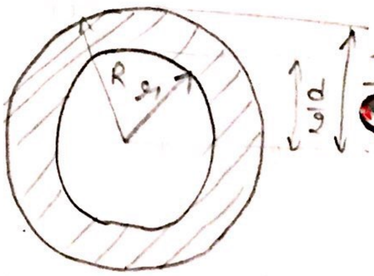
For solid or shaft
polar moment of inertia =

$$= I_{xx} + I_{yy}$$

$$= \frac{\pi d^4}{64} + \frac{\pi d^4}{64} \Rightarrow \frac{\cancel{2}^{1} \pi d^4}{\cancel{64}_{32}}$$

$$= \frac{\pi d^4}{32}$$

maximum torque transmitted by Hollow circular shaft



$$\frac{T}{J} = \frac{\tau}{R}$$

$$T = \frac{\tau}{\frac{D}{\cancel{2}_{1}}} \times \frac{\pi (D^4 - d^4)}{\cancel{32}_{16}}$$

$$\boxed{T = \frac{\pi (D^4 - d^4)}{16D} \cdot \tau}$$

$$\boxed{J = \frac{\pi (D^4 - d^4)}{32}}$$

power Transmitted by shafts:-

$$P = \omega \times T$$

$$\omega = \frac{2\pi N}{60}$$

N = speed of the shaft in r.p.m

ω = Angular speed of the shaft.

T = Average (or) mean torque. (N-m)

P = power (watt)

$$\boxed{P = \frac{2\pi N T}{60}}$$

Q1) A solid shaft of 150mm diameter is used to transmit torque. Find the maximum torque transmitted by the shaft if the max shear stress induced to the shaft is 45 N/mm².

Sol Given data:-

Diameter of shaft (d) = 150mm

max. shear stress (τ) = 45 N/mm²

we know

$$T_{max} = \frac{\pi d^3}{16} \cdot \tau$$

$$= \frac{\pi (150)^3}{16} \times 45 = \frac{477129384.3}{16}$$

$$\boxed{T = 29820.58 \text{ N-m}}$$

The shearing stress of a solid shaft is not to exceed 40N/mm² when the torque transmitted is 20000 N-m. Determine the minimum diameter of the shaft.

Given data :-

Shear stress (τ) = 40 N/mm².

Torque (T) = 20000 N-m.

= 20000×10^3 N-mm

We know

$$T = \frac{\pi d^3}{16} \times \tau$$

$$d^3 = \frac{16T}{\pi \tau} = \frac{16 \times 2 \times 10^7}{\pi \times 40} = \frac{32 \times 10^7}{125.66}$$

$$d = \sqrt[3]{2546479.089}$$

$$\boxed{d = 136.55 \text{ mm}}$$

A hollow shaft of external diameter 120mm transmits 300KW power at 200rpm. Determine the maximum internal diameter if the maximum stress in the shaft is not to exceed 60 N/mm².

Given data :-

Exeternal diameter (D) = 120mm

Power (P) = 200r.p.m

maximum shear stress $(\tau) = 60\ N/mm^2$

we know for hollow circular shaft

$$T = \frac{\pi(D^4 - d^4)}{16D} \cdot \tau$$

But we also know

$$P = \frac{2\pi NT}{60}$$

$$\frac{300 \times 10^3 \times 60}{2\pi \times 200} = T$$

$$T = \frac{18 \times 10^6}{400\pi} = \cancel{141371.66\ N\text{-}mm}\ 14323.94\ N\text{-}m$$

$$= 14323.94 \times 10^3\ N\text{-}mm$$

$\boxed{\cancel{P = 141.371 N-m}}$

$$\overset{14323.94 \times 10^3}{\cancel{141.371 \times 10^3}} = \frac{\pi(120^4 - d^4)}{16 \times 120} \times 60$$

$$\frac{\overset{14323.94 \times 10^3}{\cancel{141.371 \times 10^3}} \times 16 \times 120}{60\pi} = 120^4 - d^4$$

$$\frac{27501974.17 \times 10^3\ \cancel{27143360.5.3}}{188.49} = 207360000 - d^4$$

$$d^4 = 207360000 - \overset{145906.8096 \times 10^3}{\cancel{144004247.1}}$$

$d^4 = (61453190.4)$ $= \cancel{\sqrt[4]{208919995755}}$ $\cancel{d = \sqrt[4]{2072214093}}$

$d = \sqrt[4]{61453190.4}$ $\cancel{d = 119.79\ mm}$ $\cancel{d =}$

$\boxed{d = 88.53\ mm}$

A hollow shaft is to transmit 300KW power at 80r.p.m. If the shear stress is not to exceed 60 N/mm^2 and the internal diameter is 0.6 of the external diameter. find the external and internal diameters. Assuming that the maximum torque is 1.4 times the mean.

Given data:

Power $(P) = 300\,kW = 300 \times 10^3\,W$

speed $(N) = 80\,r.p.m.$

shear stress $(\tau) = 60\,N/mm^2$

Internal diameter $(d) = 0.6\,D$

maximum Torque $(T_{max}) = 1.4\,T_{mean}$

we know mean Torque $(T_{mean}) = \dfrac{60P}{2\pi N}$ $\boxed{\therefore P = \dfrac{2\pi NT}{60}}$

$$T_{mean} = \frac{60 \times 300 \times 10^3}{2 \times \pi \times 80} = \frac{18 \times 10^6}{502.65}$$

$$T_{mean} = 35809.86\ N\text{-}m.$$

$$T_{mean} = 35809.86 \times 10^3\ N\text{-}mm$$

$$T_{max} = 1.4\,T_{mean}$$

$$= 1.4 \times 35809.86 \times 10^3$$

$$\boxed{T_{max} = 50133.80 \times 10^3\ N\text{-}mm}$$

we Also know for hollow shaft

$$T = \frac{\pi (D^4 - d^4)}{16D} \times \tau$$

$$50133.80 \times 10^3 = \frac{\pi \left[D^4 - (0.6D)^4 \right]}{16 D} \times 60$$

$$\frac{802140 \times 10^3 D}{60\pi} = D^4 - 0.1296 D^4$$

$$= D^4 (1 - 0.1296)$$

$$\cancel{4199952.19} \; 4889114.049 \, \cancel{D} = D^{\cancel{4}\,3} (0.8704)$$

$$D^3 = \cancel{\frac{4199952.19}{0.8704}} = \frac{\cancel{4889114.049}\; 4255484.868}{0.8704}$$

$$D = \cancel{\sqrt[3]{4825362.54}} = \sqrt[3]{4889114.049}$$

$$\boxed{D = 169.72 \text{ mm}}$$

$$d = 0.6 \times D$$

$$d = 0.6 \times 169.72$$

$$\boxed{d = 102 \text{ mm}}$$

Q) Determine the diameter of a solid steel shaft which will transmit 90kW at 160 r.p.m. Also determine the length of the shaft if the twist must not exceed 1° over the entire length. The maximum shear stress is limited to 60 N/mm². Take the value of modulus of Rigidity = 8×10^4 N/mm²

Given data :-

Power $(P) = 90 KW = 90 \times 10^3 W$

speed $(N) = 160 r.p.m$

Angle of twist $(\theta) = 1^\circ \Rightarrow 1 \times \frac{\pi}{180} = 0.0174$ radian.

shear stress $(\tau) = 60 N/mm^2$.

modulus of Rigidity $(G) = 8 \times 10^4 N/mm^2$.

we know

$$T = \frac{60P}{2\pi N} = \frac{60 \times 90000}{2 \times \pi \times 160}$$

$$\boxed{\therefore P = \frac{2\pi NT}{60}}$$

$$= 14323.94 \, N\text{-}m$$

$$= 14323.94 \times 10^3 \, N\text{-}mm.$$

we Also know for solid circular shaft

$$T = \frac{\pi d^3}{16} \times \tau$$

from the above eqn we can calculate the diameter

$$d^3 = \frac{16T}{\pi \times \tau} = \frac{16 \times 14323.94 \times 10^3}{\pi \times 60} = \frac{229183118.1}{60\pi}$$

$$d = \sqrt[3]{1215854.204}$$

$$\boxed{d = 106.73 \ mm}$$

For calculating the length of the shaft we have

$$\frac{T}{J} = \frac{G\theta}{L}$$

For solid circular shaft polar m.o.I (J) is equal to

$$J = \frac{\pi d^4}{32}$$

$$\frac{14323.94 \times 10^3}{\frac{\pi (106.73)^4}{32}} = \frac{8 \times 10^4 \times 0.0174}{L} \Rightarrow \frac{32 \times 14323.94 \times 10^3}{\pi (106.73)^4} = \frac{1392}{L}$$

$$L = \frac{1392 \times 407657944.6}{458366080}$$

$$= \frac{5.67 \times 10^{11}}{458366080}$$

$$\boxed{L = 1238\,mm}$$

Q) A hollow shaft of diameter ratio 3:8 (internal dia to external dia) is to transmit 375 KW power at 100 r.p.m. The maximum torque being 20% greater than the mean. The shear stress not to exceed 60 N/mm² and twist in a length of 4m not to exceed 2°. Calculate its external and internal diameters which would satisfy both the above conditions. Assume modulus of rigidity, $G = 0.85 \times 10^5$ N/mm².

Sol Given data :-

Diameter ratio $\left(\frac{d}{D}\right) = \frac{3}{8} \Rightarrow d = \frac{3}{8}D = 0.375D$

Power $(P) = 375\,KW = 375 \times 10^3\,W$

speed $(N) = 100\,r.p.m$

maximum Torque $(T_{max}) = 1.2\,T_{mean}$

Shear stress $(\tau) = 60\,N/mm^2$

Angle of twist $(\theta) = 2^\circ \times \frac{\pi}{180} = 0.0349$

length $(L) = 4m = 4000mm$

modulus of Rigidity $(G) = 0.85 \times 10^5 \ N/mm^2$.

we know for ~~sol~~ hollow circular shaft

$$T_{max} = \frac{\pi (D^4 - d^4)}{16D} \times \tau$$

For calculating Tmean (mean Torque)

$$T_{mean} = \frac{60P}{2\pi N} = \frac{60 \times 375 \times 10^3}{2 \times \pi \times 100} = \frac{225 \times 10^5}{628.31}$$

$$T_{mean} = 35810.34 \ N\text{-}m$$

$$= 35810.34 \times 10^3 \ N\text{-}mm$$

$$T_{max} = 1.2 \ T_{mean}$$

$$= 1.2 \times 35810.34 \times 10^3$$

$$= 42972.41 \times 10^3 \ N\text{-}mm$$

$$T_{max} = \frac{\pi (D^4 - d^4)}{16D} \times \tau$$

$$42972.41 \times 10^3 = \frac{\pi \left[D^4 - (0.375D)^4\right]}{16D} \times 60$$

$$\frac{42972.41 \times 10^3 \times 16}{60 \times \pi} = \frac{D^4 - 0.01977 D^4}{D}$$

$$\frac{687558.68 \times 10^3}{60\pi} = \frac{\cancel{D^4}^{\,3} (1 - 0.01977)}{\cancel{D}}$$

$$\cancel{360004883} = D^3 (0.9802) = 3647611.45$$

$$\boxed{D = 154.96 \ mm}$$

from given data

$$d = \frac{3}{8} D \Rightarrow \frac{3}{8} \times 154.96$$

$$\boxed{d = 58.11 \text{ mm}}$$

Condition 2 :-

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\frac{42972.41 \times 10^3}{\frac{\pi \left[D^4 - (0.375 D)^4\right]}{32}} = \frac{0.85 \times 10^5 \times 0.0349}{4000}$$

$$\frac{32 \times 42972.41 \times 10^3 \times 4000}{\pi \times 0.85 \times 10^5 \times 0.0349} = D^4 - 0.01977 D^4$$

$$\frac{5.50 \times 10^{12}}{9319.53} = D^4 (1 - 0.01977)$$

$$D^4 = \frac{590158224.8}{0.9802}$$

$$D = \sqrt[4]{602079336.9}$$

$$\boxed{D = 156.64 \text{ mm}}$$

$$d = \frac{3}{8} \times D \Rightarrow \frac{3}{8} \times 156.64$$

$$\boxed{d = 58.74 \text{ mm}}$$

The diameter of the shaft greater of the two values

COMBINED BENDING AND TORSION :-

when a shaft is transmitting Torque or power, it is subjected to shear stresses. At the same time the shaft is also subjected to bending moments due to gravity or inertia loads. Due to the bending moment, bending stresses are setup in the shaft. Hence each particle in a shaft is subjected to shear stress and bending stress.

For design purpose it is necessary to find the principal stresses and maximum shear stress.

major principal stress $= \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$

$$= \frac{\frac{\cancel{32}^{16} M}{\pi d^3}}{\cancel{2}_1} + \sqrt{\left(\frac{\frac{\cancel{32}^{16} M}{\pi d^3}}{\cancel{2}_1}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{16M}{\pi d^3} + \sqrt{\left(\frac{16M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{16}{\pi d^3}\left(M + \sqrt{M^2 + T^2}\right)$$

minor principal stress

$$= \frac{\sigma}{2} - \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{M}{I} \times y$$

$$= \frac{M}{\frac{\pi d^4}{64}} \times \frac{d}{2}$$

$$= \frac{\cancel{64}^{32} M}{\pi \cancel{d^4}_3} \times \frac{\cancel{d}^1}{\cancel{2}}$$

$$\boxed{\sigma = \frac{32M}{\pi d^3}}$$

similarly

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\tau = \frac{T}{\frac{\pi \cancel{d^4}_3}{\cancel{32}_{16}}} \times \frac{\cancel{d}^1}{\cancel{2}_1}$$

$$\boxed{\tau = \frac{16T}{\pi d^3}}$$

minor pricipal stress

$$= \frac{\frac{\overset{16}{\cancel{32}}m}{\pi d^3}}{\cancel{2}} - \sqrt{\left(\frac{\frac{\overset{16}{\cancel{32}}m}{\pi d^3}}{\cancel{2}}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{16m}{\pi d^3} - \sqrt{\left(\frac{16m}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{16}{\pi d^3}\left(m - \sqrt{m^2 + T^2}\right)$$

maximum shear stress $= \dfrac{\text{major principal stress} - \text{minor principal stress}}{2}$

$$= \frac{\frac{16}{\pi d^3}\left(m + \sqrt{m^2+T^2}\right) - \frac{16}{\pi d^3}\left(m - \sqrt{m^2+T^2}\right)}{2}$$

$$= \frac{\cancel{\frac{16}{\pi d^3}m} + \frac{16}{\pi d^3}\sqrt{m^2+T^2} - \cancel{\frac{16m}{\pi d^3}} + \frac{16}{\pi d^3}\sqrt{m^2+T^2}}{2}$$

$$= \frac{\frac{16}{\pi d^3}\sqrt{m^2+T^2}\,(1+1)}{2} = \frac{16}{\pi d^3}\sqrt{m^2+T^2} \times \frac{\cancel{2}}{\cancel{2}}$$

$$= \frac{16}{\pi d^3}\sqrt{m^2+T^2}.$$

position of max. shear with normal cross-section

$$\boxed{\tan 2\theta = \frac{T}{m}}$$

Similarly for Hollow shaft

major principal stress $= \dfrac{16D}{\pi(D^4-d^4)}\left(m + \sqrt{m^2+T^2}\right)$

minor principal stress $= \dfrac{16D}{\pi(D^4-d^4)}\left(m - \sqrt{m^2+T^2}\right)$

max. shear stress $= \dfrac{16D}{\pi(D^4-d^4)}\left(\sqrt{m^2+T^2}\right)$

Q) A solid shaft of diameter 80mm is subjected to a twisting moment of 8MN-mm and bending moment of 5MN-mm at a point. Determine (i) principal stresses (ii) position of the plane on which they act.

Sol Given data :-

Diameter of shaft $(d) = 80mm$

Twisting moment $(T) = 8 MN-mm$

$= 8 \times 10^6 \ N-mm$

Bending moment $(M) = 5 MN-mm$

$= 5 \times 10^6 \ N-mm$

we know for solid shaft

$$\text{major principal stress} = \frac{16}{\pi D^3}\left(M + \sqrt{M^2+T^2}\right)$$

$$= \frac{16}{\pi(80)^3}\left(5\times10^6 + \sqrt{(5\times10^6)^2 + (8\times10^6)^2}\right)$$

$$\frac{16}{(512000)\pi}\left(5\times10^6 + \sqrt{(2.5\times10^{13}) + (6.4\times10^{13})}\right)$$

$$= 9.9471\times10^{-6}\left[5\times10^6 + \sqrt{8.9\times10^{13}}\right)$$

$$\cancel{= 9.9471\times10^{-6} \times (4996394.45 \cdot (5003605.55))}$$

~~major principal stress = 49.69 N/mm²~~

$$= 9.9471\times10^{-6}\left[5\times10^6 + 9433981.132\right]$$

$$= 9.9471\times10^{-6}\left[14433981.13\right]$$

$$= 143.57 \ N/mm^2$$

Minor principal stress $= \frac{16}{\pi D^3}\left[M - \sqrt{M^2 + T^2}\right]$

$= \frac{16}{\pi (80)^3}\left[5\times10^6 - \sqrt{(5\times10^6)^2 + (8\times10^6)^2}\right]$

$= 9.9471\times10^{-6}\left[5\times10^6 - \sqrt{8.9\times10^{13}}\right]$

$= 9.9471\times10^{-6}\left[5\times10^6 - 9433981.13\right]$

$= 9.9471\times10^{-6}\left[-4433981.13\right]$

$= -44.10\ N/mm^2$. (Compressive)

ii) position of plane on which they act

$\tan 2\theta = \frac{T}{M} = \frac{8\times10^6}{5\times10^6}$

$\tan 2\theta = \frac{8}{5}$

$2\theta = \tan^{-1}(1.6)$

$\theta = \frac{57.99}{2} = 28.99^\circ$

9) The maximum allowable shear stress in a hollow shaft of external diameter equal to twice the internal diameter is $80\ N/mm^2$. Determine the diameter of the shaft if it is subjected to a torque of 4×10^6 N-mm and bending moment of 3×10^6 N-mm.

Sol Given data :-

External diameter (D) = 2 × Internal diameter.

$D = 2d$

shear stress (τ) := $80\ N/mm^2$.

Torque (T) = 4×10^6 N-mm

Bending moment (M) = 3×10^6 N-mm.

For Calculating maximum shear stress we know

$$\text{max. shear stress} = \frac{16D}{\pi(D^4 - d^4)} \left(\sqrt{M^2 + T^2}\right]$$

$$80 = \frac{16 \times 2d}{\pi\left[(2d)^4 - d^4\right]} \times \sqrt{(3\times10^6)^2 + (4\times10^6)^2}$$

$$80 = \frac{32d}{\pi\left[16d^4 - d^4\right]} \times \sqrt{9\times10^{12} + 1.6\times10^{13}}$$

$$= \frac{32d}{\pi(15d^4)} \times \sqrt{2.5\times10^{13}}$$

$$\frac{d^{\cancel{4}\,3}}{\cancel{d}} = \frac{32\times(5\times10^6)}{\pi\times15\times80} = \frac{16\times10^7}{3769.911}$$

$$d^3 = 42441.32$$

$$d = \sqrt[3]{42441.32}$$

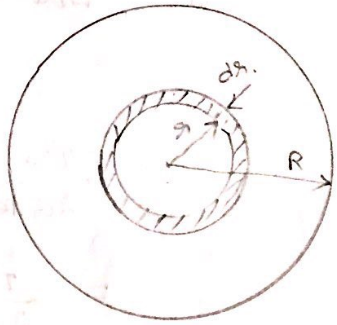
$$\boxed{d = 34.88\ mm}$$

$$D = 2d \Rightarrow 2\times34.88$$

$$\boxed{D = 69.76\ mm}$$

EXPRESSION FOR STRAIN ENERGY STORED IN A BODY DUE TO TORSION :-

Consider a solid shaft which is subjected to torsion. Take an elementary ring of width dr at a radius r as shown in fig.



we know shear stress due to torsion at a radius r is equal to

$$\frac{\tau_r}{r} = \frac{\tau}{R}$$

shear strain energy is given by

$$= \frac{(\text{Shear stress})^2}{2G} \times \text{volume}.$$

$$\boxed{U = \frac{\tau^2}{2G} \times \text{Volume}}$$

volume of ring $=$ Area $\times$ length

$$= 2\pi r \times dr \times l$$

shear strain energy in the ~~..~~ ring at a radius r

$$= \frac{\left(\frac{\tau}{R} \times r\right)^2}{2G} \times 2\pi r\, dr \times l \Rightarrow \frac{\tau^2}{R^2} \times \frac{r^2}{2G} \times 2\pi r\, dr.\, l$$

$$= \frac{\tau^2 . l}{2GR^2}\left[r^2\, 2\pi r.\, dr\right] \Rightarrow \frac{\tau^2 . l}{2GR^2}\left[r^2 . dA\right]$$

Total strain energy $= \dfrac{\tau^2 . l}{2GR^2}\displaystyle\int r^2 dA = \dfrac{\tau^2 l}{2GR^2} \times J$

$$= \frac{\tau^2 . l}{2GR^2} \times \frac{\pi d^4}{32} \Rightarrow \cancel{\frac{\tau^2 . l \times 4}{2 . G \times d^4} \times \frac{\pi d^4}{32}}$$

$$= \frac{\tau^2 l}{2GR^2} \times \frac{\pi (2R)^4}{32} \Rightarrow \frac{\tau^2 . l}{2GR^2} \times \frac{\pi}{32} \times 16R^4$$

$$= \frac{\tau^2 \cdot l}{2GR^2} \times \frac{\pi}{32} \times 16 R^2 \cdot R^2 \Rightarrow \frac{\tau^2}{4G} \times \pi R^2 \cdot l$$

$$\boxed{U = \frac{\tau^2}{4G} \times V}$$

Total strain energy in a hollow shaft due to torsion

we know

$$\cancel{U = \frac{\tau^2}{4G} \times l} \Rightarrow \cancel{\frac{\tau^2}{4G} \times \pi}$$

$$= \frac{\tau^2 \times l}{2G R^2} \times J \Rightarrow \frac{\tau^2 \times l}{2GR^2} \times \frac{\pi}{32}\left[D^4 - d^4\right]$$

$$= \frac{\tau^2 \times l}{2G\left(\frac{D}{2}\right)^2} \times \frac{\pi}{32}\left[D^2 + d^2\right]\left[D^2 - d^2\right]$$

$$= \frac{4\tau^2 \times l}{2G \times (D^2)} \times \frac{\pi}{32_8}(D^2 + d^2)(D^2 - d^2)$$

$$= \frac{\tau^2}{4G \cdot D^2} \times \frac{\pi}{4}(D^2 - d^2) \times l \times (D^2 + d^2)$$

$$\boxed{U = \frac{\tau^2}{4G \cdot D^2} \times V (D^2 + d^2)}$$

Q) Determine the maximum strain energy stored in a solid shaft of diameter 10 cm and length of 1.25 m, if the maximum allowable shear stress is 50 N/mm². Take $G = 8 \times 10^4$ N/mm².

Sol Given data

Dia. of shaft (d) = 10 cm

Area of shaf $= \frac{\pi}{4} d^2 = \frac{\pi}{4}(10 \times 10)^2$

$= 7853.98 \text{mm}^2$

Length $(L) = 1.25\text{m} = 1250\text{mm}$

Shear stress $(\tau) = 50 \text{ N/mm}^2$

modulus of Rigidity $(G) = 8 \times 10^4 \text{ N/mm}^2$.

$$U = \frac{\tau^2}{4G} \times V$$

$$= \frac{(50)^2}{4 \times 8 \times 10^4} \times 7853.98 \times 1250$$

$$= \frac{2.45 \times 10^{10}}{32 \times 10^4}$$

$$\boxed{U = 76699.02 \text{ N-mm.}}$$

9) The external and internal diameters of a hollow shaft are 40cm and 20cm. Determine the maximum strain energy stored in the hollow shaft if the maximum allowable shear stress is 50 N/mm^2 and length of the shaft is 5m. Take $G = 8 \times 10^4 \text{ N/mm}^2$.

Sol Given data :-

External diameter $(D) = 40\text{cm} = 400\text{mm}$

Internal diameter $(d) = 20\text{cm} = 200\text{mm}$

Shear stress $(\tau) = 50 \text{ N/mm}^2$.

Length of the shaft $(L) = 5\text{ m} = 5000\text{ mm}$

modulus of Rigidity (G) = 8×10^4 N/mm².

we know strain energy for hollow shaft

$$U = \frac{\tau^2}{4GD^2}(D^2+d^2) \times V$$

$$= \frac{(50)^2}{4 \times 8 \times 10^4 \times (400)^2}(400^2+200^2) \times \frac{\pi}{4}(400^2-200^2) \times 5 \times 10^3$$

$$= \frac{2500 \times (200000) \times \pi(120000)}{2.048 \times 10^{11}} \times (5000)$$

$$= \frac{1.8849 \times 10^{14}}{2.048 \times 10^{11}} \times 5000 \Rightarrow \frac{9.4245 \times 10^{17}}{2.048 \times 10^{11}}$$

$$= 4601806.64 \text{ N-mm}$$

$$U = 4601.806 \text{ N-m}$$

9) A solid circular shaft of 10cm diameter of length 4m is transmitting 112.5 KW power at 150 r.p.m. Determine (i) maximum shear stress induced in the shaft and (ii) strain energy stored in the shaft. Take $G = 8 \times 10^4$ N/mm².

Sol Given data:-

Diameter of shaft (d) = 10 cm = 100mm.

length of shaft (L) = 4 m

Power (P) = 112.5 kW ⇒ 112500 W

Speed (N) = 150 r.p.m

modulus of Rigidity (G) = 8×10^4 N/mm².

Sol Dia. of shaft (d) = 10 cm

we Know

$$\text{power} = \frac{2\pi N T}{60}$$

$$T = \frac{112500 \times 60}{2 \times \pi \times 150} = \frac{6750000}{942.47} = 7161.97 \text{ N-m}$$

$$T = 7161972.43 \text{ N-mm}.$$

we also Know for solid shaft

$$T = \frac{\pi d^3}{16} \times \tau$$

$$7161972.43 = \frac{\pi \times (100)^3}{16} \times \tau$$

$$\tau = \frac{114591559}{3141592.654} = 36.47 \text{ N/mm}^2.$$

$$\text{strain energy } (U) = \frac{\tau^2}{4G} \times V$$

$$= \frac{(36.47)^2}{4 \times 8 \times 10^4} \times \frac{\pi}{4}(100)^2 \times 4000$$

$$= \frac{1330.47 \times \pi \times 10000 \times 400}{4 \times 8 \times 10^4 \times 4}$$

$$= \frac{1.67191 \times 10^{10}}{1280000} =$$

$$\boxed{U = 13061.7968 \text{ N-mm}}$$

SPRINGS

Springs are elastic members which distort under load and regain their original shape when Load is removed. They are used in scooters, motor Cars, railway Carriages, motor cycles, rickshaws etc.

Types of springs:-

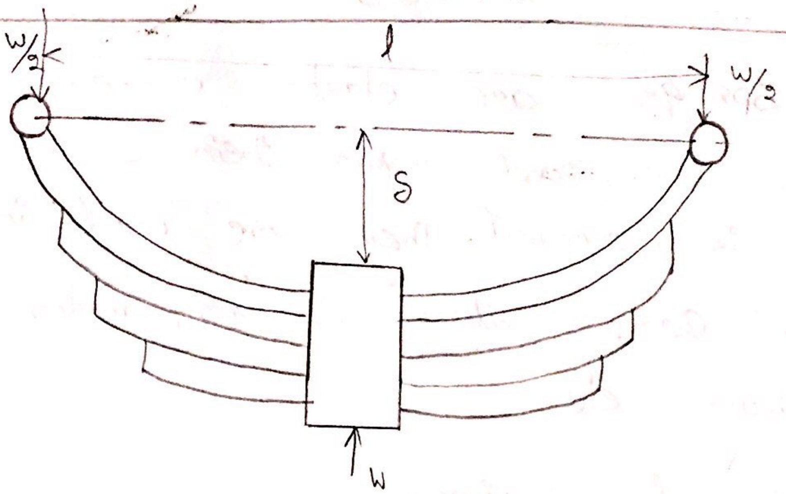
1) Helical Springs
2) Laminated or Leaf spring

Laminated (or) Leaf spring:-

The laminated springs are used to absorb shocks in railway wagons, Cars, Lorries etc. It Consists of a number of parallel strips of spring steel metal having different lengths and same width, placed one over the other & held together at the centre with clamps. Initially all the plates are bent to the same radius and are free to slide one over the other.

The below figure shows the initial position of the spring, which is having some deflection 'S'. The top plate of the spring is ~~pinned~~ pinned jointed to the Chassic of the vehicle.

L = πd^3



Expression for maximum bending stress developed in the plate.

The load W acting at the centre of the lowermost plate, will be shared equally on the two ends of the top plate as shown in above fig.

we know

$$\frac{M}{I} = \frac{\sigma}{y} \quad \text{—} \quad (1)$$

(M) max. bending moment at centre = Load × $\frac{l}{2}$

$$= \frac{W}{2} \times \frac{l}{2} \Rightarrow \frac{Wl}{4}$$

moment of inertia $(I) = \frac{bd^3}{12} = \frac{bt^3}{12}$

$$y = \frac{t}{2}$$

substitute above values in eqn (1)

$\frac{M}{I} = \frac{\sigma}{y}$ $\qquad$ $M = \frac{I}{y} \times \sigma$

$$\frac{\overset{3}{\cancel{12}}\,wl}{\underset{2}{\cancel{4}}\,b\,t^{\cancel{3}}} = \frac{\cancel{2}\sigma}{\cancel{t}}$$

$$\frac{3wl}{\cancel{2}\,b\,t^2} = \sigma$$

$$M = \frac{\sigma \cdot \frac{bt^3}{12}}{t/2} = \frac{\cancel{2}\,\sigma\, b\, t^{\cancel{3}}}{\underset{6}{\cancel{12}}\,\cancel{t}}$$

$$\frac{wl}{4} = \frac{\sigma b t^2}{6}$$

Total resisting moment by n plates

$$\frac{wl}{4} = n \times \frac{\sigma b t^2}{6}$$

~~Bending stress for n plates (...)~~

$$\sigma = \cancel{\frac{n\,3wl}{2bt^2}} = \frac{\overset{3}{\cancel{6}}\,wl}{\underset{2}{\cancel{4}} \times n \cdot b t^2}$$

$$\boxed{\text{max. stress } (\sigma) = \frac{3wl}{2n \cdot bt^2}}$$

w = point load acting at the Centre

l = length of the spring.

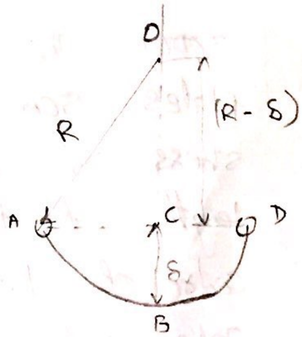
n = Number of plates.

b = width of each plate.

t = Thickness of plate.

σ = max. bending stress developed in the plates

Expression for central deflection of the Leaf spring :-



From Δ^{le} ACO

$$AO^2 = AC^2 + OC^2$$

$$= \left(\frac{l}{2}\right)^2 + (R-\delta)^2$$

$$R^2 = \frac{l^2}{4} + R^2 + \delta^2 - 2R \cdot \delta$$

$$L = \frac{\quad}{\pi d^3}$$

Neglecting δ^2 which is small quantity

$$\cancel{R^2} = \frac{l^2}{4} + \cancel{R^2} - 2R \cdot \delta$$

$$2R \cdot \delta = \frac{l^2}{4}$$

$$\delta = \frac{l^2}{8R}$$

But we know the relation

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$R = \frac{E \times y}{\sigma} = \frac{E \times t}{2\sigma}$$

$$\delta = \frac{l^2}{8^4 \times \frac{E \times t}{\cancel{2}\sigma}} = \frac{\sigma l^2}{4Et}$$

$$\boxed{\text{Central deflection } (\delta) = \frac{\sigma l^2}{4Et}}$$

(9) A leaf spring carries a central load of 3000 N. The leaf spring is to be made of 10 steel plates 5cm wide and 6mm thick. If the bending stress is limited to 150 N/mm² Determine length of the spring and Deflection at the centre of the spring.

Take $E = 2 \times 10^5$ N/mm².

<u>Sol</u> Given data :-

Central length of load (or) point load (W) = 3000 N

No. of steel plates (n) = 10.

width of each plate (b) = 5 cm = 5×10 = 50 mm

Thickness of each plate (t) = 6 mm

Bending stress (σ) = 150 N/mm².

Young's modulus (E) = 2×10^5 N/mm².

we know

$$\sigma = \frac{3Wl}{2nbt^2} = \frac{3\times3000\times l}{2\times10\times50\times(6)^2}$$

$$l = \frac{150\times2\times10\times50\times36}{9000} = \frac{54\times10^5}{9000}$$

$$\boxed{l = 600\,mm}$$

For calculating deflection of the spring we have

$$\delta = \frac{\sigma l^2}{4Et} = \frac{150\times(600)^2}{4\times2\times10^5\times6}$$

$$= \frac{54\times10^6}{48\times10^5}$$

$$\boxed{\delta = 11.25\,mm}$$

Q) A Laminated spring 1m Long is made up of plates each 5cm wide and 1cm thick. If the bending stress in the plate is limited to 100 N/mm^2, how many plates would be required to enable the spring to carry a central point load of 2KN? If $E = 2.1 \times 10^5$ N/mm^2, what is the deflection under the load.

Sol Given data:

Length of the spring(l) = 1m = 1000 mm.

width (b) = 5cm = $5 \times 10 = 50$ mm

Thickness (t) = 1cm = $1 \times 10 = 10$ mm

Bending stress (σ) = 100 N/mm^2.

Central Load(W) = 2KN $\Rightarrow 2 \times 10^3$ N

young's modulus (E) = 2.1×10^5 N/mm^2.

we know

$$\sigma = \frac{3Wl}{2nbt^2} \Rightarrow n = \frac{3Wl}{2\sigma bt^2}$$

$$n = \frac{3 \times 2000 \times 1000}{2 \times 100 \times 50 \times 10^2} = 6$$

$\boxed{\text{No. of plates } (n) = 6}$

Deflection (δ)

$$= \frac{\sigma l^2}{4Et} = \frac{100 \times (1000)^2}{4 \times 2.1 \times 10^5 \times 10}$$

$$= \frac{1 \times 10^8}{84 \times 10^5}$$

$$\boxed{\delta = 11.9 \text{ mm}}$$

Helical Springs:-

There are two types of helical springs.

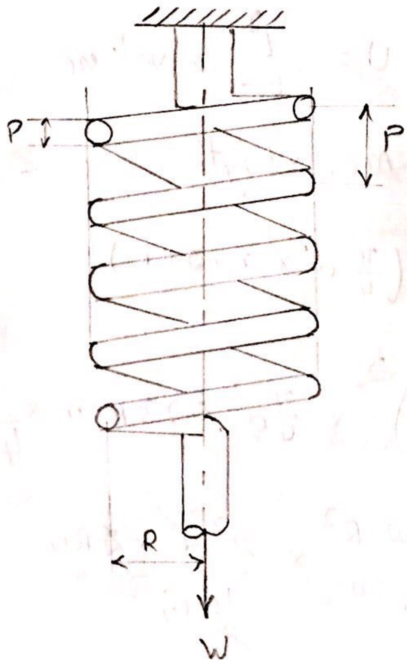
1) closed - Coiled helical springs.

2) open - Coiled helical springs.

Closed - coiled helical springs:-

close coiled helical spring are the springs in which helix angle is very small and the pitch between two adjacent turns is small. A close Coiled helical spring Carries an axial load as shown in below fig. As the helix angle in Case of close-Coiled helical spring are small, hence the bending effect on the spring is ignored and we assume that the Coils of a close-Coiled helical springs are to stand purely torsional stresses.

Expro



$L = \frac{}{\pi d^3}$

Expression for maximum shear stress induced in wire:-

we know

Twisting moment $(T) = F \times R$.

Twisting moment on the wire $(T) = W \times R$. — ①

But twisting moment also given by $T = \frac{\pi d^3}{16} \times \tau$. — ②

Equate eqn ① & ②

$$W \times R = \frac{\pi d^3}{16} \times \tau$$

$$\boxed{\tau = \frac{16 W R}{\pi d^3}} \rightarrow$$ max. shear stress induced in the wire.

Expression for deflection of spring:-

we know

strain energy stored in a solid circular shaft is given by

$$U = \frac{\tau^2}{4G} \times \text{volume}.$$

$\therefore$ Area $= \frac{\pi}{4} d^2$

length of one coil $= 2\pi R$.

Total length of wire $=$ length of one coil $\times$ No. of coils $= n \cdot 2\pi R$

$$U = \frac{\tau^2}{4G} \times \text{Area} \times \text{length}$$

$$= \frac{\tau^2}{4G} \times \left(\frac{\pi}{4} d^2 \times n \times 2\pi R\right)$$

$$= \left(\frac{16 WR}{\pi d^3}\right)^2 \times \frac{\pi}{4} d^2 \times 2\pi R \times n \times \frac{1}{4G}$$

$$U = \frac{\cancel{256}^{\,32}\, W^2 R^2}{\cancel{\pi^2}\, d^{\cancel{6}\,4}} \times \frac{\cancel{\pi^2}\, \cancel{d^2} \times \cancel{2}^{\,1} R n}{\cancel{16}\,\cancel{8}\,1\; G} = \frac{32 W^2 R^3 \times n}{G d^4}.$$

Work done on the spring = Average load × Deflection

$$= \frac{1}{2} W \times \delta$$

Equating work done on the spring to the energy stored

$$U = W$$

$$\frac{32 W^2 R^3 n}{G d^4} = \frac{1}{2} W \times \delta$$

$$\delta = \frac{64 W^{\cancel{2}} R^3 n}{G d^4 \times \cancel{W}}$$

$$\boxed{\delta = \frac{64 R^3 W \times n}{G d^4}}$$

Stiffness of the spring:-

$$\text{Stiffness } (S) = \frac{W}{\delta} = \frac{\cancel{W}}{\frac{64 R^3 \cancel{W} \times n}{G d^4}} = \frac{G d^4}{64 R^3 n}$$

9) A closely coiled helical spring is to carry a load of 500N. Its mean coil diameter is to be 10 times that of the wire diameter. Calculate these diameters. If the maximum shear stress in the material of the spring is to be 80 N/mm^2. and the stiffness is 20N per mm deflection. Take $G = 8.4 \times 10^4$ N/mm^2. Find the number of coils in the closely coiled helical spring.

sol Given data :-

Load $(W) = 500N$

mean Coil diameter $(D_m) = 10d$

Shear stress $(\tau) = 80 N/mm^2$.

stiffness $(S) = 20 N/mm$

modulus of Rigidity $(G) = 8.4 \times 10^4 N/mm^2$

we know

$$\text{Shear stress } (\tau) = \frac{16WR}{\pi d^3}$$

$$80 = \frac{16 \times 500 \times \frac{10d}{2}}{\pi d^3}$$

$$80 = \frac{80000 \cancel{d}}{2\pi d^{\cancel{3}2}}$$

$$d^2 = \frac{80000}{80 \times 2 \times \pi} = 159.15$$

$$d = \sqrt{159.15}$$

$$\boxed{d = 12.61 \text{ mm}}$$

$$D_m = 10d$$
$$= 10 \times 12.61$$

$$\boxed{\text{mean diameter}(D_m) = 126.1 \text{ mm}}$$

we also know

$$\delta = \frac{64WR^3 n}{Gd^4}$$

Knol

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value

N

Q)

From the above eqn we don't know the "δ" value but we know

$$\text{stiffness } (s) = \frac{W}{\delta}$$

$$20 = \frac{500}{\delta}$$

$$\delta = \frac{\cancel{500}^{25}}{\cancel{20}}$$

$$\boxed{\delta = 25 \text{ mm}}$$

Now No. of Coils $(n) = \dfrac{\delta \times G d^4}{64 R^3 W}$ $\qquad \boxed{\therefore \delta = \dfrac{64 R^3 W \times n}{G d^4}}$

$$= \frac{25 \times 8.4 \times 10^{4} \times (12.61)^4}{64 \times (\frac{126.1}{2})^3 \times 500}$$

$$= \frac{\cancel{5.3098 \times 10^{10}}}{\cancel{6.4164 \times 10^{10}}} = \frac{5.3098 \times 10^{10}}{64 \times 500 \times (63.05)^3}$$

$$= \frac{5.3098 \times 10^{10}}{8020570324} = 6.62 \simeq 7$$

$$\boxed{\text{No. of Coils } (n) = 7}$$

Q) A closely Coiled helical spring of round steel wire 10mm in diameter having 10 Complete turns with a mean diameter of 12cm is subjected to an axial load of 200N. Determine (i) Deflection of the spring (ii) maximum shear stress in the wire. iii) stiffness of the spring. Take $G = 8 \times 10^4 \text{ N/mm}^2$.

Sol Given data :-

wire diameter $(d) = 10mm$

No. of turns $(n) = 10$

mean diameter $(D_m) = 12 cm = 120 mm$

Axial Load $(W) = 200N$

modulus of Rigidity $(G) = 8 \times 10^4 \ N/mm^2$.

we know

$$\text{Deflection } (\delta) = \frac{64 W R^3 n}{G d^4}$$

$$= \frac{64 \times 200 \times (60)^3 \times 10}{8 \times 10^4 \times (10)^4} = \frac{2.7648 \times 10^{10}}{8 \times 10^8}$$

$$\boxed{\delta = 34.56 mm}$$

$$\text{maximum shear stress } (\tau) = \frac{16 WR}{\pi d^3} = \frac{16 \times 200 \times 60}{\pi (10)^3}$$

$$= \frac{192000}{3141.59}$$

$$\boxed{\tau = 61.11 \ N/mm^2}$$

$$\text{Stiffness of the spring } (s) = \frac{W}{\delta}$$

$$= \frac{200}{34.56}$$

$$\boxed{S = 5.78 \ N/mm}$$

Q) The stiffness of a close-coiled helical spring is 1.5 N/mm ~~of~~ compression under a maximum load of 60 N. The maximum shearing stress produced in the wire of the spring is 125 N/mm². The solid length of the spring is given as 5 cm. Find.

i) Diameter of wire

ii) mean diameter of the coil.

iii) No. of coils required

Take $G = 4.5 \times 10^4$ N/mm².

Sol Given data :-

stiffness $(S) = 1.5$ N/mm

maximum Load $(W) = 60$ N

max. shear stress $(\tau) = 125$ N/mm².

Solid length $(l) = 5$ cm $= 50$ mm

$d = ?$; $D_m = ?$; $n = ?$

we know

$$\text{stiffness } (S) = \frac{G d^4}{64 R^3 . n}$$

$$\frac{1.5 \times 64 \times R^3 \times n}{4.5 \times 10^4} = d^4$$

$$d^4 = 2.133 \times 10^{-3} \times R^3 \times n \quad \text{——} \quad (1)$$

we also know

$$\tau = \frac{16 W R}{\pi d^3}$$

$$R = \frac{\pi d^3 \times \tau}{16 \times W} = \frac{\pi \times 125 \times d^3}{16 \times 60} = \frac{392.69\, d^3}{960}$$

$$\boxed{R = 0.4090\, d^3} \quad (2)$$

substitute R value in eqn (1)

$$d^4 = 2.133 \times 10^{-3} \times R^3 \times n$$

$$d^4 = 2.133 \times 10^{-3} \times (0.4090 d^3)^3 \times \left(\frac{50}{d}\right)$$

Solid length $= n \times d$

$50 = n \times d$

$n = \frac{50}{d}$

$$d^4 = 2.133 \times 10^{-3} \times 0.06841 \times d^{\cancel{8}\,9} \times \frac{50}{\cancel{d}}$$

$$d^4 = 7.2967 \times 10^{-3}\, d^8$$

$$\frac{1}{7.2967 \times 10^{-3}} = \frac{d^{\cancel{8}\,4}}{\cancel{d^4}}$$

$$d^4 = 137.0469$$

$$d = \sqrt[4]{137.0469}$$

$$\boxed{d = 3.4215\, mm}$$

mean Diameter of the Coil $(D_m) = 2R$

$$= 2 \times 0.4090\, d^3$$

$$= 2 \times 0.4090 \times (3.42)^3$$

$$\boxed{\text{mean Diameter } (D_m) = 32.76\, mm}$$

No. of Coils required $(n) = \frac{50}{d}$

$$= \frac{50}{3.4215}$$

$$= 14.61 \simeq 15$$

$$\boxed{n = 15 \text{ coils}}$$