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UNIT – III

* ATTENUATORS & EQUILIZERS *

Attenuator:- It is a N/W which reduces the i/p signal strength by a given amount.

Attenuators are basically classified into 2 types:-

1) Fixed Attenuators → (PAD's)
2) Variable Attenuators

→ In attenuators we only require attenuation i.e 'no' phase shift is reqd.

∴ Attenuators are designed by purely resistive elements.

→ W.K.T propagation const $P = \alpha + j\beta$

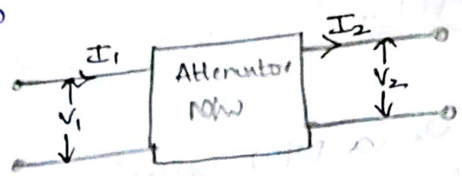
As we know attenuators doesn't consist any reactive elements (L & C) & hence

$\boxed{\beta = 0} \Rightarrow \boxed{\therefore P = \alpha}$

→ Attenuation is measured either in 'Neper' or decibles (dB)

$$\boxed{1 \text{ Neper} = 8.686 \text{ dB}}$$

* Consider an attenuator N/w shown below



i/p power $P_1 = V_1 I_1$

o/p power $P_2 = V_2 I_2$

Attenuation in decibles is

$$\boxed{D = 10 \log_{10}\left(\frac{P_1}{P_2}\right)} \longrightarrow (1)$$

let a parameter

$$N = \sqrt{\frac{P_1}{P_2}} \Rightarrow N^2 = \frac{P_1}{P_2}$$

$$D = 10 \log_{10} N^2$$

$$\boxed{\begin{gathered} D = 20 \log_{10} N \\ \text{(or)} \\ N = 10^{D/20} \end{gathered}} \longrightarrow (2)$$

→ Attenuators can be designed either in Symm. (or) Asymm. form.

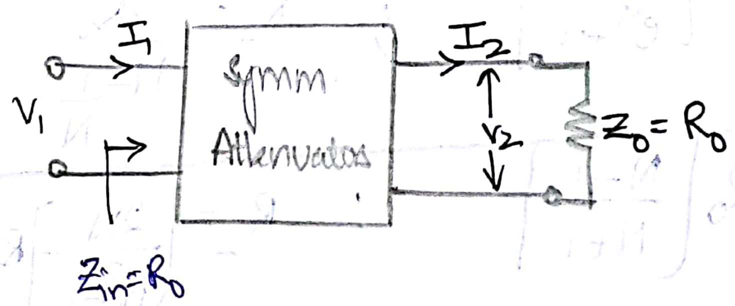
Sym...

* → Symm. T-pad **
* → Symm. π-pad **
* → Symm. Bridged T-pad
* → Symm. Lattice pad.

Asymmetrical Attenuators:-

* → Asymm. T-pad **
* → Asymm. π-pad
* → Asymm L-pad **

Symmetrical Attenuators:-



$P = \alpha$ (only Attenuation const)

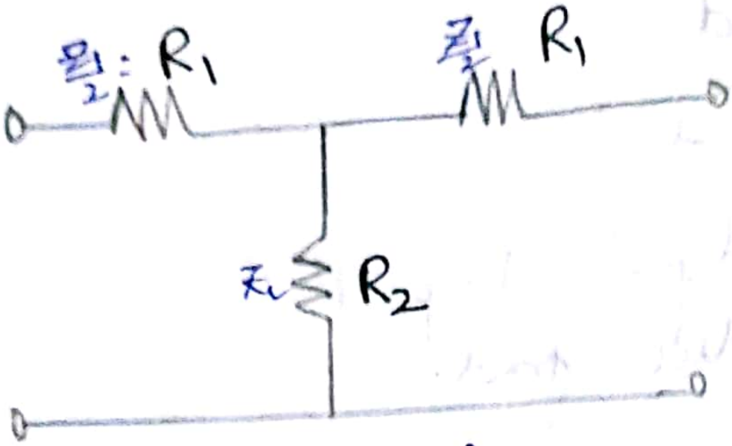
$Z = R_0$ (only pure resistance)

$$\frac{P_1}{P_2} = \frac{V_1 I_1}{V_2 I_2} = \frac{I_1^2 \cancel{R_0}}{I_2^2 \cancel{R_0}}$$

$$\frac{P_1}{P_2} = \frac{I_1^2}{I_2^2} \quad \text{but} \left(N = \sqrt{\frac{P_1}{P_2}}\right)$$

$$\boxed{N = \sqrt{\frac{P_1}{P_2}} = \frac{I_1}{I_2} = e^{\alpha} = 10^{D/20}}$$

Symmetrical T-pad:



Symm. T-pad.

$\boxed{Z_{OT} = R_0}$

$P = \alpha, \quad ; \quad Z_0 = R_0$

$R_1 = R_0 \tanh\left(\frac{\alpha}{2}\right) \quad ; \quad R_2 = \frac{R_0}{\sinh\alpha}$

$= R_0 \cdot \frac{e^{\frac{\alpha}{2}} - e^{-\frac{\alpha}{2}}}{e^{\frac{\alpha}{2}} + e^{-\frac{\alpha}{2}}} \quad ; \quad R_2 = \frac{2R_0}{e^{\alpha} - e^{-\alpha}}$

$\left(\frac{e^{-\alpha}}{2}\right)$ common

$= R_0 \left[\frac{e^{\alpha} - 1}{e^{\alpha} + 1}\right] \quad ; \quad R_2 = \frac{2R_0}{N - \frac{1}{N}}$

$$\boxed{R_1 = R_0 \left[\frac{N-1}{N+1}\right] \quad ; \quad R_2 = \left[\frac{2N}{N^2 - 1}\right] R_0}$$

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$R_1 = R_0 \sinh\alpha$; $R_2 = \frac{R_0}{\tanh(\frac{\alpha}{2})}$

$= R_0\left[\frac{e^{\alpha}-e^{-\alpha}}{2}\right]$; $= R_0\left[\frac{e^{\alpha/2}+e^{-\alpha/2}}{e^{\alpha/2}-e^{-\alpha/2}}\right]$

$= \frac{R_0}{2}\left[N-\frac{1}{N}\right]$; $= R_0\left[\frac{e^{\alpha}+1}{e^{\alpha}-1}\right]$

$$\boxed{R_1 = R_0\left[\frac{N^2-1}{2N}\right] \quad ; \quad R_2 = R_0\left[\frac{N+1}{N-1}\right]}$$

$\boxed{Z_{OT} = R_0}$

problems:-

Q1) Design a T-pad Attenuator to give a attenuation of 20dB. ~~calculate~~ characteristisc impedence is 600Ω

Sol) Given $R_0 = 600\,\Omega$

$D = 20\text{dB}$

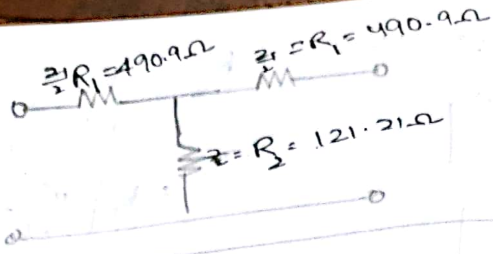
For symm T-pad

$N = 10^{D/20} = 10^{20/20} = 10$ $\boxed{\therefore N = 10}$

$R_1 = R_0\left[\frac{N-1}{N+1}\right]$; $R_2 = \left[\frac{2N}{N^2-1}\right]$

$= 600\left[\frac{10-1}{10+1}\right]$; $R_2 = \left[\frac{2\times 10}{10^2-1}\right]$

$\boxed{R_1 = 490.90\,\Omega}$; $\boxed{R_2 = 121.21\,\Omega}$



Q2> Design a π-type attenuator with 15dB attenuation to match impedance of $75\angle 0^\circ\,\Omega$.

Sol:- Given $D = 15\,dB$

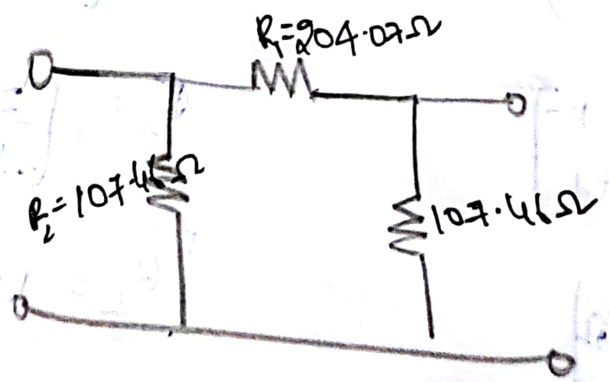
& characteristic imp. $R_0 = 75\angle 0^\circ$
$= 75\,\Omega$.

$$N = 10^{D/20} \Rightarrow 10^{15/20} \Rightarrow \boxed{N = 5.62}$$

$$R_1 = R_0\left[\frac{N^2-1}{2N}\right] \quad ; \quad R_2 = R_0\left[\frac{N+1}{N-1}\right]$$

$$= 75\left[\frac{(5.62)^2-1}{2\times 5.62}\right] \quad ; \quad = 75\left[\frac{5.62+1}{5.62-1}\right]$$

$$\boxed{R_1 = 204.07\,\Omega} \quad ; \quad \boxed{R_2 = 107.46\,\Omega}$$



Q3> Design symm... attenuation ... is 500Ω.

Sol: T-pad:-

Given $\alpha = 2.19...$
& $R_0 = 50...$

$N = e^{\alpha} = e...$

$R_1 = R_0\left[\frac{N-1}{N+1}\right]$

$= 500\left[\frac{9.01-...}{9.01...}\right]$

$\boxed{R_1 = 400\,\Omega}$

π-pad:-

$R_1 = R_0\left[\frac{N^2-1}{2N}\right]$

$= 500\left[\frac{9.0...}{2...}\right]$

$R_1 = 22...$

Q) Design symm T & π-pads with attenuation 2.199 Np & design impedance is 500Ω.

Sol: T-pad:-

Given $\alpha = 2.199\,N_p$

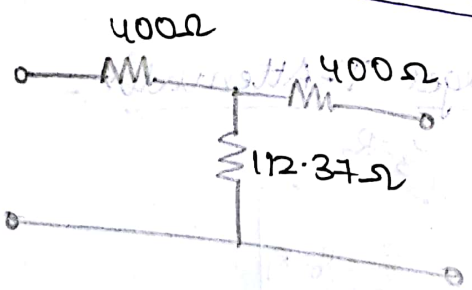
& $R_0 = 500\,\Omega$

$$N = e^{\alpha} = e^{2.199} \Rightarrow \boxed{N = 9.01}$$

$$R_1 = R_0\left[\frac{N-1}{N+1}\right] \quad ; \quad R_2 = \left[\frac{2N}{N^2-1}\right]R_0$$

$$= 500\left[\frac{9.01-1}{9.01+1}\right] \quad ; \quad = \left[\frac{2\times 9.01}{(9.01)^2-1}\right]\times 500$$

$$\boxed{R_1 = 400\,\Omega} \qquad \boxed{R_2 = 112.37\,\Omega}$$



π-pad:-

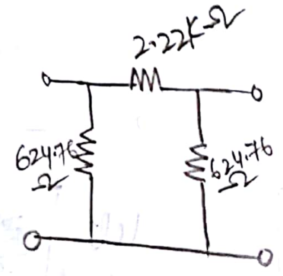
$$R_1 = R_0\left[\frac{N^2-1}{2N}\right] \quad ; \quad R_2 = R_0\left[\frac{N+1}{N-1}\right]$$

$$= 500\left[\frac{9.01^2-1}{2\times 9.01}\right] \quad ; \quad = 500\left[\frac{9.01+1}{9.01-1}\right]$$

$R_1 = 2226.018$

$$\boxed{R_2 = 624.76\,\Omega}$$

$$\boxed{R_1 = 2.22\,k\Omega}$$



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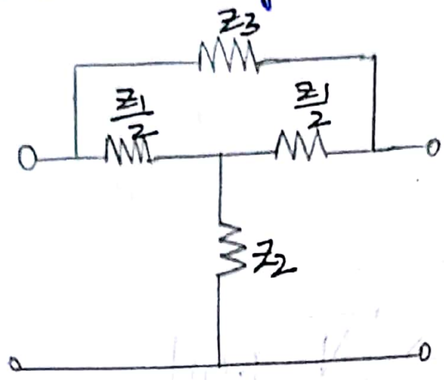
Symm. Bridged T- Attenuator:-

W.K.T Bridged T- N/W

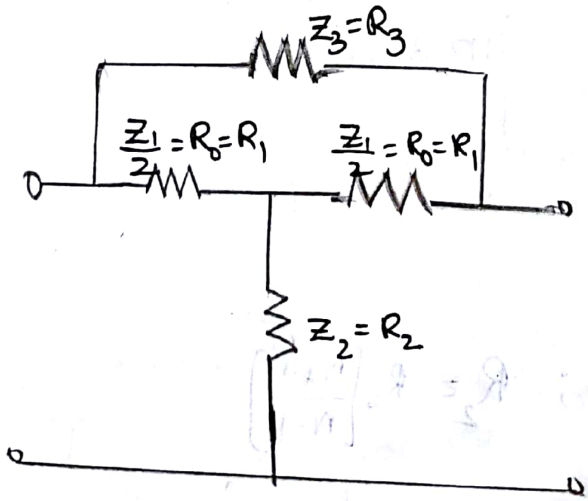


By considering $\boxed{\frac{Z_1}{2} = Z_0}$

then we get $Z_0 = \sqrt{Z_2 Z_3}$

$e^P = 1 + \frac{Z_3}{Z_0}$ (or) $e^P = 1 + \frac{Z_0}{Z_2}$

Standard Bridged T-Attenuator:-



where
$Z_0 = R_0$
&
$P = \alpha$

let $\boxed{\frac{Z_1}{2} = R_0 = R_1}$

let $e^P = 1 + \frac{Z_3}{Z_0}$

$e^{P} = 1 + \frac{R_3}{R_0}$

w.k.T for symm. N/w $e^{\alpha} = N = 10^{D/20} = \frac{I_S}{I_R}$

$\therefore N - 1 = \frac{R_3}{R_0}$

$$\boxed{R_3 = R_0(N-1)}$$

Also consider; $e^{P} = 1 + \frac{Z_0}{Z_2}$

$$e^{\alpha} = 1 + \frac{R_0}{R_2}$$

$$\frac{R_0}{R_2} = e^{\alpha} - 1$$

$$\frac{R_0}{R_2} = N - 1$$

$$\boxed{R_2 = \frac{R_0}{N-1}}$$

$\therefore R_1 = R_0$

$R_2 = \frac{R_0}{(N-1)}$

$R_3 = R_0(N-1)$

$$\boxed{N = e^{\alpha} = 10^{D/20} = \frac{I_S}{I_R}}$$

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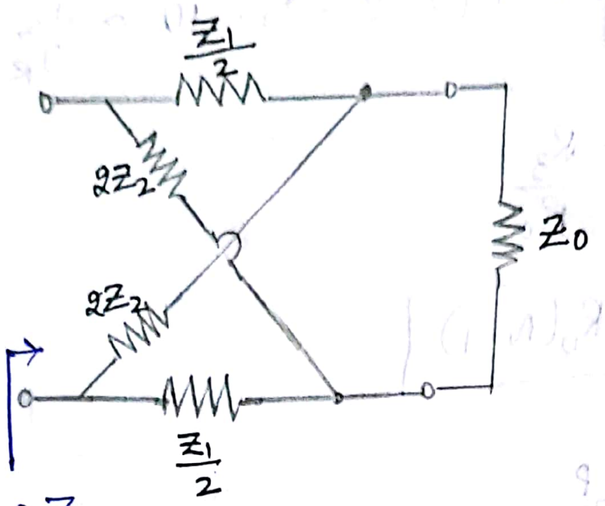
Symm. Lattice Network:-



$Z_{in} = Z_0$

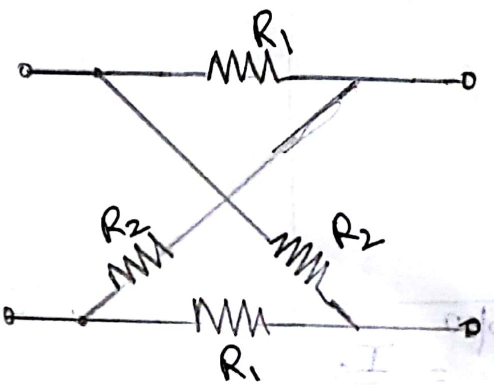
If P & Z_0 are known

$$\frac{Z_1}{2} = Z_0 \tanh\left(\frac{P}{2}\right)$$

$$2Z_2 = \frac{Z_0}{\tanh\left(\frac{P}{2}\right)}$$

Lattice Attenuator:-

let $\left.\begin{aligned} \frac{Z_1}{2} &= R_1 \\ 2Z_2 &= R_2 \end{aligned}\right\} \rightarrow$ In lattice N/W



$Z_0 = R_0$ & $P = \alpha$

W.K.T for lattice N/W

$\frac{Z_1}{2} = Z_0 \tanh\left(\frac{P}{2}\right)$ & $2Z_2 = \frac{Z_0}{\tanh\left(\frac{P}{2}\right)}$

$R_1 = R_0 \tanh\left(\frac{\alpha}{2}\right)$ & $R_2 = \frac{R_0}{\tanh\left(\frac{\alpha}{2}\right)}$

$R_1 = R_0\left[\frac{e^{\alpha/2} - e^{-\alpha/2}}{e^{\alpha/2} + e^{-\alpha/2}}\right]$ & $R_2 = R_0\left[\frac{e^{\alpha/2} + e^{-\alpha/2}}{e^{\alpha/2} - e^{-\alpha/2}}\right]$

$= R_0 \frac{\cancel{e^{-\alpha/2}}}{\cancel{e^{-\alpha/2}}}\left[\frac{e^{\alpha}-1}{e^{\alpha}+1}\right]$ $\qquad = R_0 \frac{\cancel{e^{-\alpha/2}}}{\cancel{e^{-\alpha/2}}}\left[\frac{e^{\alpha}+1}{e^{\alpha}-1}\right]$

$\boxed{R_1 = R_0\left[\frac{N-1}{N+1}\right]}$ $\qquad \boxed{R_2 = R_0\left[\frac{N+1}{N-1}\right]}$

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problem:-

Q1) Design a following symm. Bridged T - attenuator with 40dB attenuation & design impedence 600Ω:

i) Bridged-T

ii) lattice Attenuator.

Sol: $R_0 = 600\Omega$

$D = 40dB$

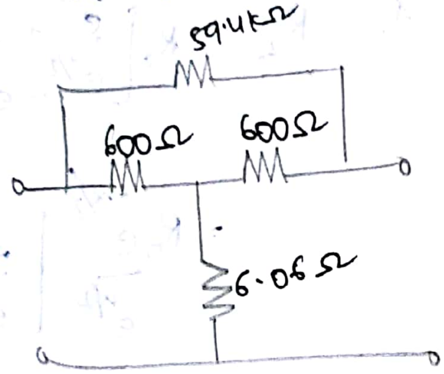
$N = 10^{D/20} \Rightarrow 10^{40/20} = 100$ $\qquad \boxed{\therefore N = 100}$

i, Bridged T- Attenuator :-

$R_1 = R_0 = 600\Omega$

$R_2 = \frac{R_0}{N-1} = \frac{600}{100-1} = 6.06\Omega$

$R_3 = R_0(N-1) = 59.4K\Omega$

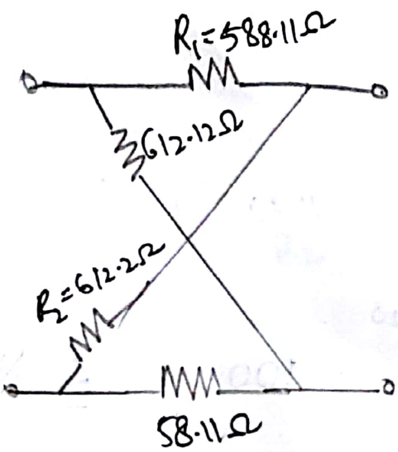


ii, Lattice Attenuator :-

$R_1 = R_0\left[\frac{N-1}{N+1}\right]$; $R_2 = R_0\left[\frac{N+1}{N-1}\right]$

$= 600\left[\frac{100-1}{10+1}\right]$; $= 600\left[\frac{100+1}{100-1}\right]$

$\boxed{R_1 = 588.11\Omega}$; $\boxed{R_2 = 612.12\Omega}$



Design 'balanced' symm. T- Attenuator
Design imp: 500Ω & 20dB attenuation

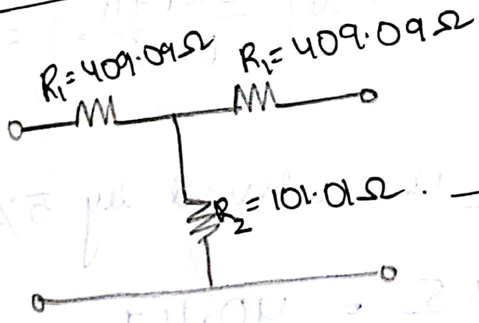
$R_0 = 500\Omega$
$D = 20dB$
$N = 10^{D/20} = 10^{20/20} = 10$

For symm. T- Attenuator :-

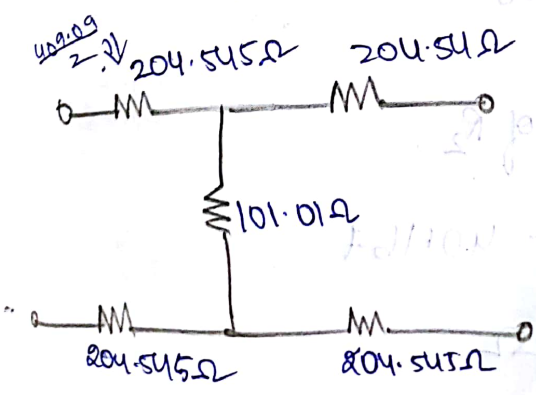
$R_1 = R_0\left[\frac{N-1}{N+1}\right]$; $R_2 = \left[\frac{2N}{N^2-1}\right] R_0$

$= 500\left[\frac{10-1}{10+1}\right]$; $= \left[\frac{2 \cdot 10}{10^2-1}\right] \times 500$

$\boxed{R_1 = 409.09\Omega}$; $\boxed{R_2 = 101.01\Omega}$



→ Unbalanced symm. T-pad



Balanced symm. T-pad.

Imp.

Q3.) Design a Symm. T-pad to operate b/w 600 Ω resistances & to provide an attenuation of 6dB. What would be the error in the attenuation when shunt arm resistor is reduced by 5%.

sol: $R_0 = 600\Omega$

let $D_1 = 6\,dB$

$$N = 10^{D/20} \Rightarrow 10^{6/20} = 1.995$$

$$R_1 = R_0\left(\frac{N-1}{N+1}\right) = 600\left(\frac{1.995-1}{1.995+1}\right) = 199.33\Omega$$

$$R_2 = R_0\left(\frac{2N}{N^2-1}\right) = 600\left(\frac{2\times1.995}{(1.995)^2-1}\right) = 803.34\Omega$$

Given that R_2 is reduced by 5%.

$$803.34 \times \frac{5}{100} = 40.167$$

$$R_2' \Rightarrow R_2 - 5\%\text{ of } R_2.$$

$$= 803.34 - 40.167$$

$$R_2' = 763.17\,\Omega$$

$$R_2' = R_0\left[\frac{2N'}{N'^2-1}\right] \Rightarrow 600\left[\frac{2N'}{N'^2-1}\right]$$

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ymm. T-pad to
0 Ω resistances &
ttenuation of 6dB.
e error in the
shunt arm resistor
5%.

95

$\left(\frac{\ 1}{+1}\right) = 199.33\Omega$

$\left(\frac{995}{-1}\right) = 803.34\Omega$

by 5%.

$\left.\ \right]$

$$163.17 = 600\left[\frac{2N'}{N'^2-1}\right]$$

$$\frac{163.17}{600}\cdot(N'^2-1) = 2N'$$

$$\Rightarrow 1.27N'^2 - 2N' - 1.27 = 0$$

$$N' = 2.06 \text{ or } -0.48$$

$$\boxed{N' = 2.06}$$

now $N' = 10^{D_2/20}$

$$2.06 = 10^{D_2/20}$$

$$D_2 = 20\log N'$$

$$D_2 = 20\log(2.06)$$

$$\boxed{D_2 = 6.27\,dB}$$

now; $Error = D_2 - D_1$

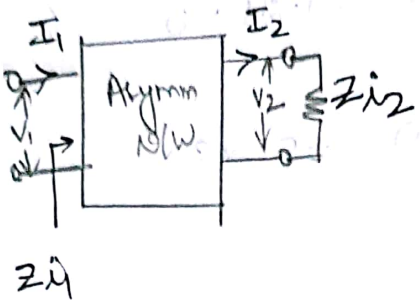
$$= 6.27 - 6$$

$$\boxed{Error = 0.27\,dB}$$

∴ Error detected 27%.

Asymmetrical Attenuators:-

For unequal Impedences:-



W.K.T

$$N = \sqrt{\frac{P_1}{P_2}} = \sqrt{\frac{V_1 I_1}{V_2 I_2}} = \sqrt{\frac{I_1^2 \cdot Z_{i_1}}{I_2^2 Z_{i_2}}} = \frac{I_1}{I_2}\sqrt{\frac{Z_{i_1}}{Z_{i_2}}}$$

$$= e^{\theta i}$$

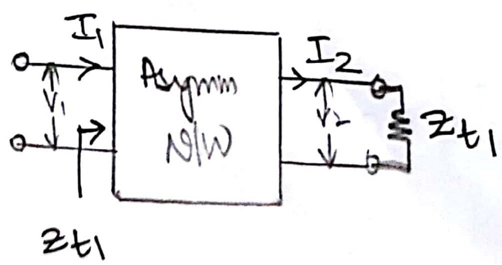
Since Attenuators are pure, resistive N/w's

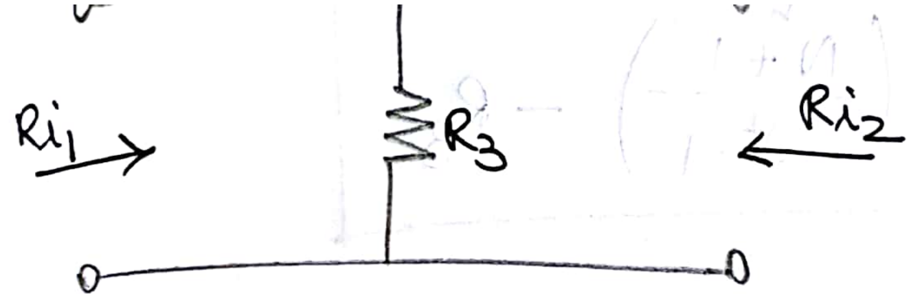
$\theta_i = A_i$ & $\theta_t = A_t$

$Z_{i_1} = R_{i_1}$ & $Z_{i_2} = R_{i_2}$

$$\boxed{N = \sqrt{\frac{P_1}{P_2}} = \frac{I_1}{I_2}\sqrt{\frac{R_{i_1}}{R_{i_2}}} = e^{A_i} = 10^{D/20}}$$

For Equal Impedences:-





let $Z_A = R_1$

$Z_B = R_2$

$Z_C = R_3$

since attenuation is a resistive N/w

$\theta_i = A_i$, $Zi_1 = Ri_1$ & $Zi_2 = Ri_2$

$$R_3 = \frac{\sqrt{Ri_1 Ri_2}}{\sinh A_i} = \frac{\sqrt{Ri_1 Ri_2}}{\frac{e^{A_i} - e^{-A_i}}{2}}$$

$$R_3 = \frac{2\sqrt{R_{i_1} R_{i_2}}}{e^{-Ai}\left[(e^{Ai})^2 - 1\right]} \rightarrow \boxed{R_3 = \frac{2N}{N^2-1}\sqrt{R_{i_1} R_{i_2}}}$$

Now;

$$R_1 = \frac{R_{i_1}}{\tanh(Ai)} - R_3$$

$$= \frac{R_{i_1}\left(e^{Ai} + e^{-Ai}\right)}{e^{Ai} - e^{-Ai}} - R_3$$

$$\boxed{R_1 = R_{i_1}\left(\frac{N^2+1}{N^2-1}\right) - R_3}$$

$$R_2 = \frac{R_{i_2}}{\tanh Ai} - R_3$$

$$\boxed{R_2 = R_{i_2}\left(\frac{N^2+1}{N^2-1}\right) - R_3}$$

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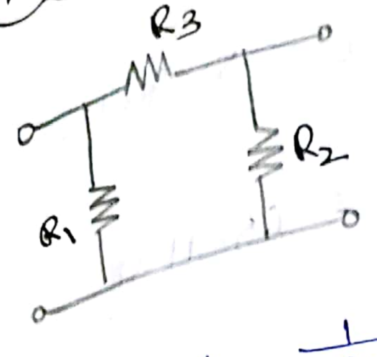
Asymmetrical π - Attenuator :-



$\frac{1}{R_{i_1}} = G_{i_1}$; $\frac{1}{R_{i_2}} = G_{i_2}$ from formula paper

$\theta_i = A_i$

$$G_3 = \frac{\sqrt{G_{i_1} G_{i_2}}}{\sinh A_i} = \frac{2N}{N^2-1}\sqrt{G_{i_1} G_{i_2}}$$

$$G_1 = \frac{G_{i_1}}{\tanh A_i} - G_3 \Rightarrow G_{i_1}\left(\frac{N^2+1}{N^2-1}\right) - G_3$$

$$G_2 = \frac{G_{i_2}}{\tanh A_i} - G_3 \Rightarrow G_{i_2}\left(\frac{N^2+1}{N^2-1}\right) - G_3$$

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problems:-

Q1) Design Asymmetrical T - Attenuators of T-type & π-type to provide an attenuation of 40dB & to work b/w a source of 70Ω & a load of 600Ω.

Sol: Given:- $R_{i_1} = 70\Omega$, $R_{i_2} = 600\Omega$

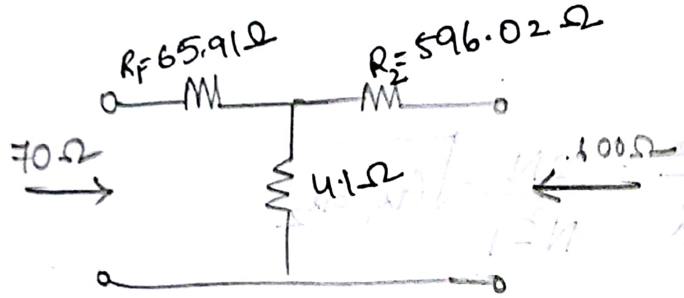
$D = 40\,dB \Rightarrow N = 10^{D/20}$ $\boxed{N = 100}$

(i) Asymmetrical T-pad:-

$$R_3 = \left(\frac{2N}{N^2-1}\right)\sqrt{R_{i_1} R_{i_2}} \Rightarrow 4.1\,\Omega$$

$$R_2 = R_{i_1}\left(\frac{N^2+1}{N^2-1}\right) - R_3 \Rightarrow 65.91\,\Omega$$

$$R_1 = R_{i_2}\left(\frac{N^2+1}{N^2-1}\right) - R_3 \Rightarrow 596.02\,\Omega$$



(ii) Asymmetrical π-pad:-

$$G_{i_1} = \frac{1}{R_{i_1}} = 0.014\,\mho$$

$$G_{i_2} = \frac{1}{R_{i_2}} = 1.6 \times 10^{-3}\,\mho$$

$$G_3 = \frac{2 \times 100}{100^2 - 1} \cdot \sqrt{0.014 \times 1.6 \times 10^{-3}}$$

$$G_3 = 94.6 \times 10^{-6} \Rightarrow \boxed{R_3 = 10.57k\,\Omega}$$

$$G_1 = 0.014\left(\frac{100^2+1}{100^2-1}\right) - 94.6 \times 10^{-6}$$

$$G_1 = 13.908 \times 10^{-3}\,\mho \Rightarrow \boxed{R_1 = 71.9\,\Omega}$$

$$G_2 = 1.505 \times 10^{-3}\,\mho \Rightarrow \boxed{R_2 = 664.45\,\Omega}$$

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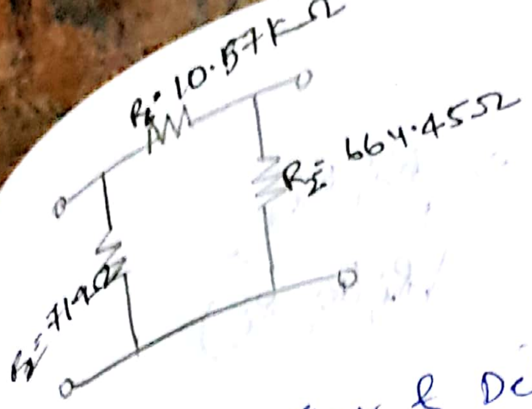
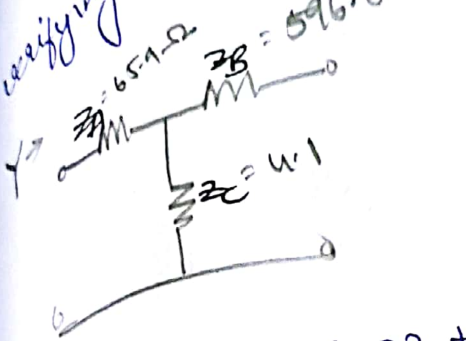


verifying using star & Delta connections.

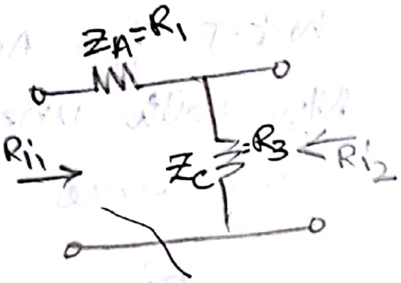


$$Z_3 = \frac{65.9 \times 596.02 + 596.02 \times 4.1 + 4.1 \times 65.9}{4.1}$$

$$Z_3 = 10.24 \times 10^3$$ approx near value.

Asymmeterical L- Attenuator :-

I] Minimum Attenuation L- Attenvator for "Unequal Impedences" :-



$$R_{i_1} = \sqrt{R_1(R_1 + R_3)}$$

$$R_{i_1}^2 = R_1^2 + R_1 R_3.$$

$(\because R_1 R_3 = R_{i_1} R_{i_2})$

$$R_{i_1}^2 = R_1^2 + R_{i_1} R_{i_2}$$

$$\boxed{R_1 = \sqrt{R_{i_1}(R_{i_1} - R_{i_2})}}$$

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W.K.T

$$R_3 = \frac{R_{i_1} R_{i_2}}{R_1} \Rightarrow \frac{R_{i_1} R_{i_2}}{\sqrt{R_{i_1}(R_{i_1} - R_{i_2})}}$$

$$R_3 = \sqrt{\frac{\cancel{R_{i_1}^2} R_{i_2}^2}{\cancel{R_{i_1}}(R_{i_1} - R_{i_2})}}$$

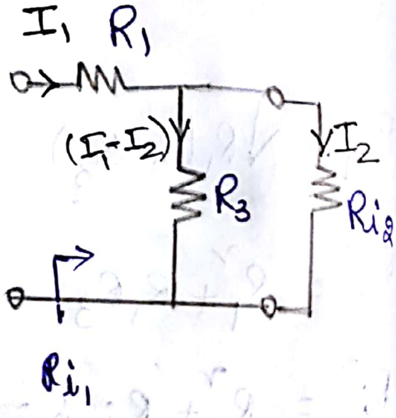
$$\boxed{R_3 = \sqrt{\frac{R_{i_1} R_{i_2}^2}{R_{i_1} - R_{i_2}}}}$$

$$\boxed{\begin{aligned} R_1 &= \sqrt{R_{i_1}(R_{i_1} - R_{i_2})} \\ R_3 &= \sqrt{\frac{R_{i_1} R_{i_2}^2}{(R_{i_1} - R_{i_2})}} \end{aligned}}$$

→ minimum Attenuation L-attenuator

II. L-Attenuator with required attenuation for unequal Impedences :-

W.K.T for Asymmetrical N/w with unequal impedence



$$N = e^{\theta i} = \frac{I_1}{I_2}\sqrt{\frac{Z_{i_1}}{Z_{i_2}}}$$

)

enuation

or

attenuation

$$\boxed{N = e^{\alpha} = \frac{I_1}{I_2}\sqrt{\frac{R_{i_1}}{R_{i_2}}} = 10^{D/20}}$$

from fig:

$$(I_1 - I_2) R_3 = R_{i_2} I_2$$

$$I_1 = I_2 \frac{(R_{i_2}+R_3)}{R_3}$$

$$\frac{I_1}{I_2} = \frac{R_{i_2}}{R_3} + 1$$

wrong

$$\cancel{\frac{I_1}{I_2}\sqrt{\frac{R_{i_1}}{R_{i_2}}} = \sqrt{\frac{R_{i_1}}{R_{i_2}}}\left[R_{i_2}+R_3\right]}$$

$$\cancel{\boxed{\text{let } \sqrt{\frac{R_{i_1}}{R_{i_2}}} = K}}$$

$$\cancel{N = K(R_{i_2}+R_3)}$$

$$\cancel{\boxed{\frac{N}{K} - R_{i_2} = R_3}}$$

$$\frac{I_1}{I_2}\sqrt{\frac{R_{i_1}}{R_{i_2}}} = \left[1+\frac{R_{i_2}}{R_3}\right]\sqrt{\frac{R_{i_1}}{R_{i_2}}}$$

let $K = \sqrt{\frac{R_{i_1}}{R_{i_2}}}$

$$N = \left(1+\frac{R_{i_2}}{R_3}\right)K$$

$$\frac{N}{K} - 1 = \frac{R_{i_2}}{R_3}$$

$$R_{i_2} = \frac{(N-K)R_3}{K}$$

$$\boxed{R_3 = \frac{K R_{i_2}}{N-K}}$$

Also w.k.t: $Z_A Z_C = Z_{i_1} Z_{i_2}$

$$R_1 R_3 = R_{i_1} R_{i_2}$$

$$R_1 = \frac{R_{i_1} R_{i_2}}{R_3}$$

$$\boxed{R_1 = \frac{(N-K) R_{i_1} \cancel{R_{i_2}}}{\cancel{(N-)}\cancel{R_{i_2}} K}}$$

$$\boxed{R_1 = \frac{R_{i_1}(N-K)}{K}}$$

For an Attenuation

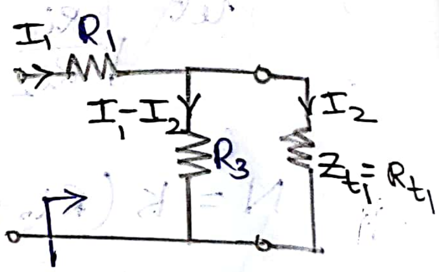
$$N = 10^{D/20} = e^{Ai}$$

$$\boxed{\begin{array}{l} K = \sqrt{\dfrac{R_{i_1}}{R_{i_2}}} \\ R_1 = \left[\cancel{\dfrac{K R_{i_1} R_{i_2}}{N - R_{i_2} K}}\right] \\ \text{wrong} \\ \cancel{R_3 = \dfrac{N}{K} - R_{i_2}} \end{array}}$$

$$\boxed{\begin{array}{l} R_1 = \dfrac{R_{i_1}(N-K)}{K} \\ R_3 = \dfrac{K R_{i_2}}{(N-K)} \end{array}}$$

III. L-Attenuator for equal Impedence:-

for Asymm. N/w with equal impedence



$Z_{t_1} = R_{t_1}$

$$N = e^{\theta t} = \frac{I_1}{I_2} = 10^{D/20}$$

for attenuator $\theta_t = A_t$, $Z_{t_1} = R_{t_1}$

$$(I_1 - I_2) R_3 = R_{t_1} I_2$$

$$R_3 I_1 = I_2 (R_{t_1} + R_3)$$

$$\frac{I_1}{I_2} = N = \frac{R_{t_1} + R_3}{R_3}$$

$$N = 1 + \frac{R_{t_1}}{R_3} \Rightarrow \frac{R_{t_1}}{R_3} = N-1$$

$$\boxed{R_3 = \frac{R_{t_1}}{N-1}}$$

W.K.T

$$Z_{t_1} = \frac{Z_A + \sqrt{Z_A^2 + 4Z_AZ_C}}{2}$$

$$R_{t_1} = \frac{R_1 + \sqrt{R_1^2 + 4R_1R_3}}{2}$$

$$R_{t_1} - \frac{R_1}{2} = \frac{\sqrt{R_1^2 + 4R_1R_3}}{2}$$

$$\left(R_{t_1} - \frac{R_1}{2}\right)^2 = \frac{1}{4}\left(R_1^2 + 4R_1R_3\right)$$

$$R_{t_1}^2 + \cancel{\frac{R_1^2}{4}} - \cancel{2}R_{t_1}\left(\frac{R_1}{\cancel{2}}\right) = \cancel{\frac{R_1^2}{4}} + R_1R_3$$

$$R_{t_1}^2 - R_1R_{t_1} - R_1R_3 = 0$$

$$R_{t_1}^2 = R_1\left(R_{t_1} + R_3\right)$$

$$R_1 = \frac{R_{t_1}^2}{R_{t_1} + R_3}$$

$$R_1 = \frac{R_{t_1}^2}{R_{t_1} + \frac{R_{t_1}}{(N-1)}}$$

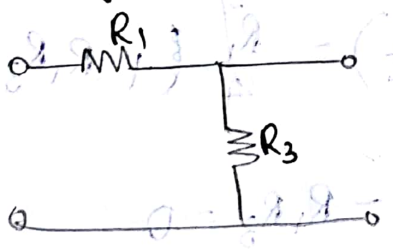
$$R_1 = \frac{R_{t_1}^{\cancel{2}}(N-1)}{\cancel{R_{t_1}}(N-\cancel{1}+\cancel{1})}$$

$$\boxed{R_1 = \frac{R_{t_1}(N-1)}{N}}$$

In general, for equal impedence $R_{t_1} = R_0$

$$\boxed{R_1 = \frac{R_0(N-1)}{N} \qquad R_3 = \frac{R_0}{(N-1)}}$$

Summary of L-attenuator :-



I.] For unequal impedences "R_{i_1} & R_{i_2}" :-

(A) Minimum loss L-attenuator :-

$$R_1 = \sqrt{R_{i_1}(R_{i_1} - R_{i_2})} \qquad R_3 = \sqrt{\frac{R_{i_1} R_{i_2}^2}{R_{i_1} - R_{i_2}}}$$

where $k = \sqrt{\frac{R_{i_1}}{R_{i_2}}}$

ii) with some Attenuation $N = 10^{\alpha/20}$:

$$R_1 = \frac{R_{i_1}}{K}(N - K) \qquad R_3 = \frac{R_{i_2} K}{(N - K)}$$

II] For Equal Impedence:-

$$R_1 = \frac{R_0(N-1)}{N} \quad , \quad R_3 = \frac{R_0}{N-1}$$

problems:-

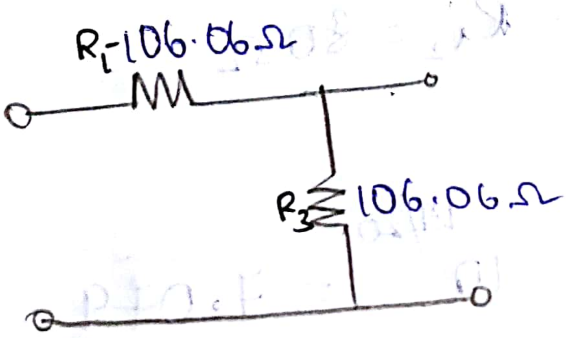
1) Design a minimum loss L-Attenuator to match a source of 150Ω & a load of 75Ω.

Sol: $R_{i_1} = 150\Omega$, $R_{i_2} = 75\Omega$

$$R_1 = \sqrt{R_{i_1}(R_{i_1} - R_{i_2})} \quad ; \quad R_3 = \sqrt{\frac{R_{i_1} R_{i_2}^2}{(R_{i_1} - R_{i_2})}}$$

$$= \sqrt{150(150-75)} \qquad = \sqrt{\frac{150 \cdot 75^2}{(150-75)}}$$

$$R_1 = 106.06\Omega \qquad R_3 = 106.06\Omega$$



$t_1 = R_0$

R_{i_2} 4:-

$$\boxed{K = \sqrt{\frac{R_{i_1}}{R_{i_2}}}}$$

where

Q2.) Design a minimum loss L- Attenuator to match impedences 100Ω of source resistance 75Ω of load resistance with 10dB.

Sol:- $R_{i_1} = 100\Omega$, $R_{i_2} = 75\Omega$

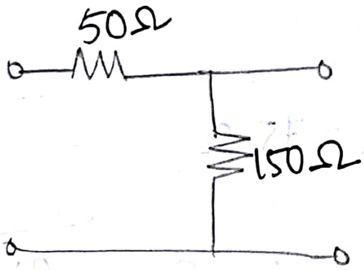
$R_1 = \sqrt{R_{i_1}(R_{i_1} - R_{i_2})}$ $\cancel{N = 10^{10/20}} \Rightarrow$

$= \sqrt{100(100-75)}$

$R_2 = \sqrt{\frac{R_{i_1} R_{i_2}^2}{(R_{i_1} - R_{i_2})}}$

$R_1 = 50\Omega$

$= \sqrt{\frac{100 \times 75^2}{25}} = 150\Omega$



Q3.) Design an L-attenuator to operate b/w two impedences 120Ω & 80Ω & to provide 17 db attenuation.

Sol:- $R_{i_1} = 120\Omega$, $R_{i_2} = 80\Omega$

$D = 17\ dB$

$N = 10^{D/20} \Rightarrow 10^{17/20} = 7.079$

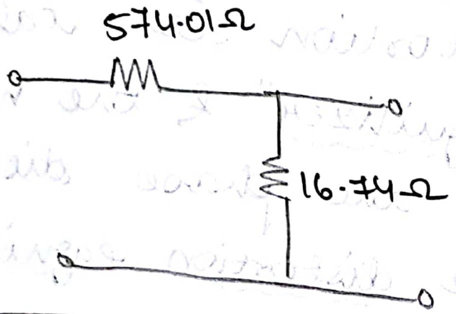
$R_1 = \frac{R_{i_1}(N-K)}{K}$, $R_3 = \frac{R_{i_2} K}{(N-K)}$

$$K = \sqrt{\frac{R_{i_1}}{R_{i_2}}} = \sqrt{\frac{\cancel{120}^{3}}{\cancel{80}_{2}}} = 1.224$$

$$R_1 = \frac{R_i(N-K)}{K} \quad ; \quad R_3 = \frac{R_{i_2} K}{(N-K)}$$

$$= \frac{120(7.079-1.224)}{1.224} \qquad = \frac{80 \times 1.224}{(7.079-1.224)}$$

$$\boxed{R_1 = 574.01\Omega} \qquad \boxed{R_3 = 16.74\Omega}$$



Q4) Design L-attenuator to provide 40dB attenuation & to operate b/w a design impedence of 500Ω.

Sol: $R_{i_1} = R_0 = 500\Omega$

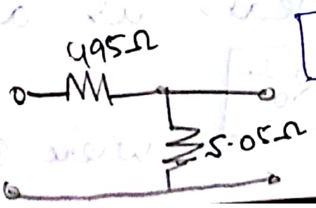
$D = 40dB$

$N = 10^{40/20} = 100$

W.K.T (equal impedence)

$$R_1 = \frac{R_0(N-1)}{N} \Rightarrow \frac{500(100-1)}{100} \quad ; \quad R_3 = \frac{R_0}{N-1} \Rightarrow \frac{500}{99}$$

$$\boxed{R_1 = 495\Omega} \qquad \boxed{R_3 = 5.05\Omega}$$



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EQUILIZERS:-

An equilizer is a network, which is designed to counteract (regain) the attenuation & phase distortions occurred when a signal is passed through a network.

The network that counteract the attenuation distortion is called an "Attenuation Equilizer". & the network that counteract the phase distortion is called "phase distortion equilizer".

→ Based on "Counteraction":

1) Attenuation Equilizer
2) phase Equilizer

→ Based on "Terminals":

1) Two terminal equilizer
2) Four terminal equilizer / constant resistance equilizer

INVERSE IMPEDENCE:-

Two impedences Z_1 & Z_2 are said to be inverse impedences "if their geometric mean is a constant".

$$\boxed{Z_1 Z_2 = R_0^2}$$

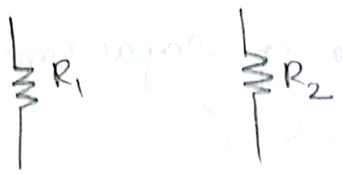
where R_0 - design impedence

Case - (i):-

If Z_1 is a resistance i.e $Z_1 = R_1$ &

$Z_2 = \frac{R_0^2}{Z_1}$ i.e $Z_2 = R_2 = \frac{R_0^2}{R_1}$

$\therefore$ Inverse impedence of a resistor is another resistor

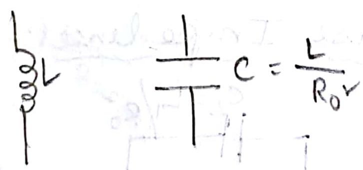


Case - (ii):-

If Z_1 is a inductance. i.e $Z_1 = j\omega L$

then $Z_2 = \frac{R_0^2}{j\omega L}$ & $Z_2 = \frac{1}{j\omega\left(\frac{L}{R_0^2}\right)}$

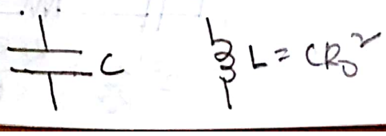
$\therefore$ Inverse impedence of inductor is a capacitor whose capacitance value is $C = \frac{L}{R_0^2}$



Case - (iii):- If Z_1 is a capacitance i.e $Z_1 = \frac{1}{j\omega C}$

then $Z_2 = \frac{R_0^2}{1/j\omega C} = j\omega(CR_0^2)$

$\therefore$ Inverse impedence of capacitor is an inductor whose value $L = CR_0^2$



Summary of Inverse Impedence:-

→ Inverse impedence of Resistor is another resistor whose value is $Z_2 = R_2 = \frac{R_0^2}{R_1}$

→ Inverse impedence of inductance is Capacitor whose value. $C = \frac{L}{R_0^2}$

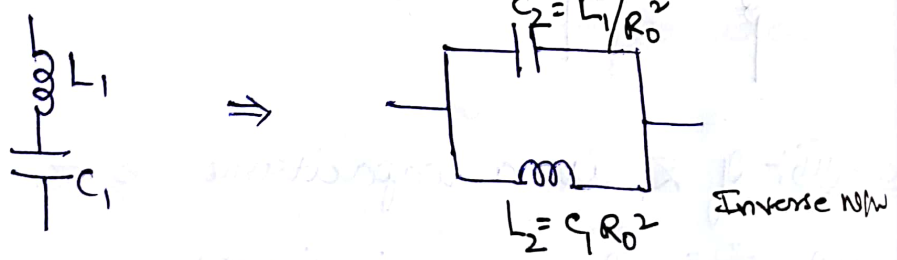
→ Inverse impedence of capacitance is inductor whose value $L = CR_0^2$

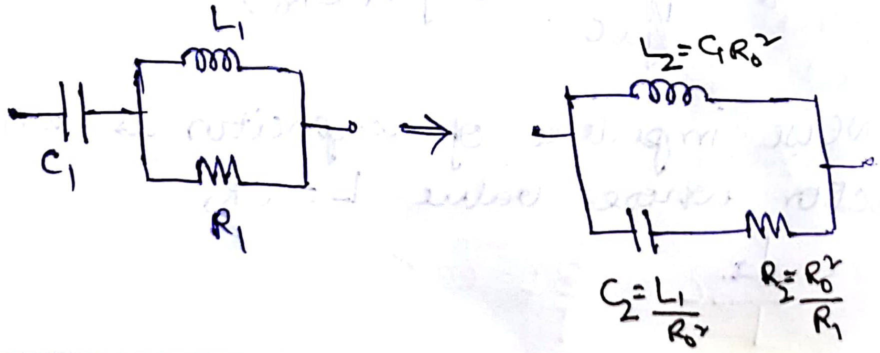
→ Inverse of a series connection is a parallel connection & vice-versa.

NOTE:

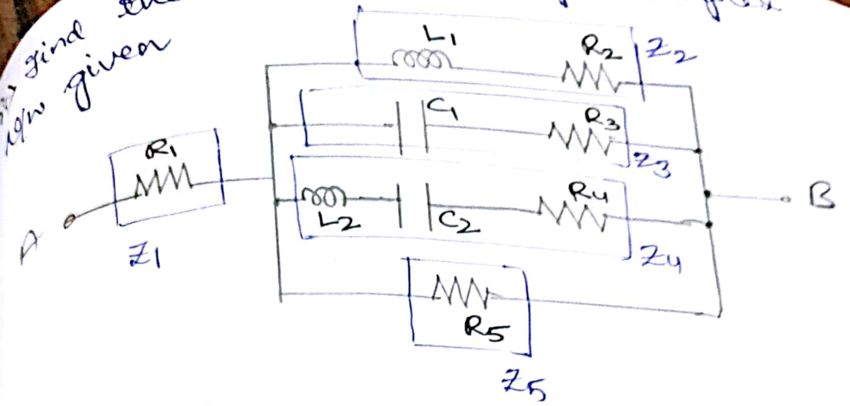
The inverse N/w of a complex (Giant) N/w is obtained by splitting the complex N/w into small units, then find the inverse of each small N/w & we build up ~~total~~ network.

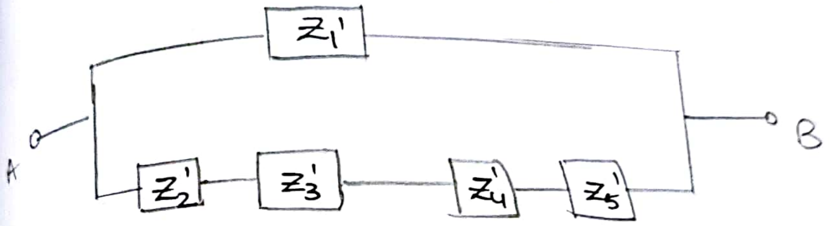
Examples of Inverse Impedences:-

1.) 

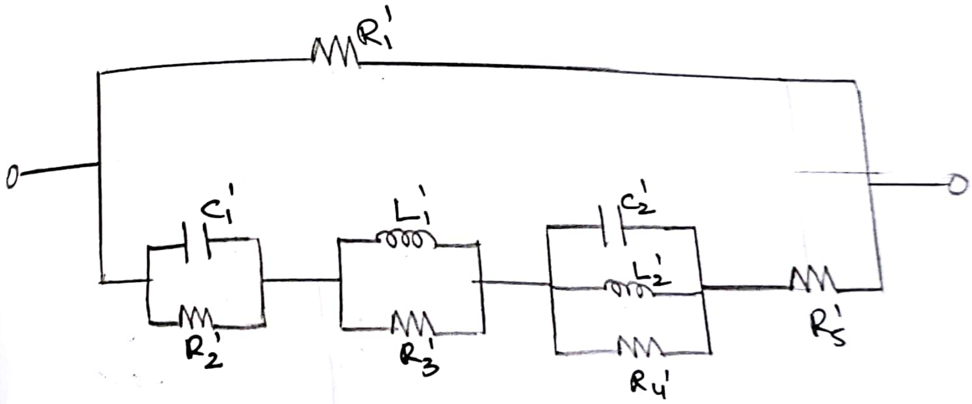
2.) 

Q. Find the inverse N/w of complex N/w given





$$R_1' = Z_1' = \frac{R_0^2}{R_1}$$



$$R_1' = \frac{R_0^2}{R_1}$$

$$C_1' = \frac{L_1}{R_0^2} \quad ; \quad L_1' = C_1 R_0^2$$

$$C_2' = \frac{L_2}{R_0^2} \quad ; \quad L_2' = C_2 R_0^2$$

$$R_2' = \frac{R_0^2}{R_2}$$

$$R_3' = \frac{R_0^2}{R_3} \quad , \quad R_4' = \frac{R_0^2}{R_4} \quad , \quad R_5' = \frac{R_0^2}{R_5}$$

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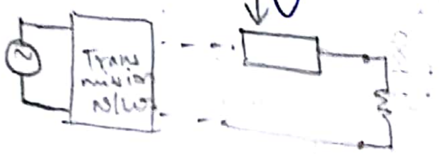
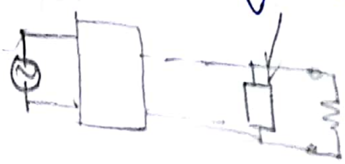
As name suggests two terminal equalizers consists only two terminals & it is connected either in series or in shunt to the transmission network.

There are two types of 2-terminal equalizers

1.> Series equalizer 2.> Shunt Equalizer

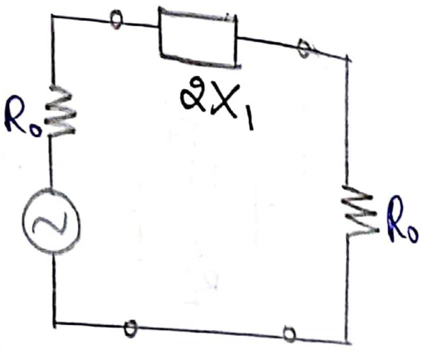




Series equalizer:-

Standard two terminal series equalizer is shown in fig. where X_1 is a reactance.

$$N = \sqrt{\frac{P_1}{P_2}}$$

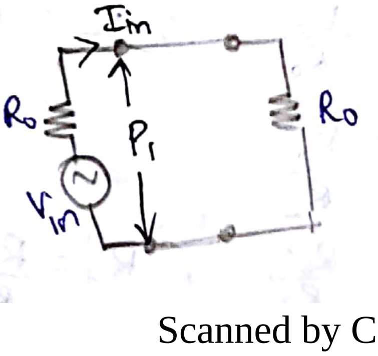


P_1 - power i/p without equalizer

P_2 - power o/p with equalizer.

To calculate input power "without" equalizer: consider fig. shown

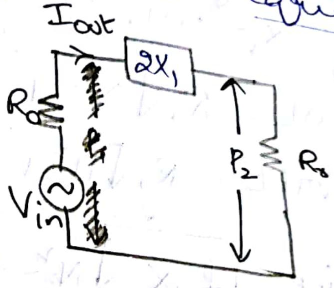
$$|I_{in}| = \frac{V_{in}}{2R_o}$$



$$P_1 = |V_{in}|\,|I_{in}| = |I_{in}|^2 R_0$$

$$P_1 = \left(\frac{V_{in}^2}{4R_0^2}\right) R_0 \longrightarrow \text{i/p power without equilizer}$$

To calculate output power consider the fig. shown 'with' equalizer



$$I_{out} = \frac{V_{in}}{2R_0 + j2X_1}$$

$$|I_{out}| = \left|\frac{V_{in}}{2R_0 + j2X_1}\right|$$

$$|I_{out}| = \frac{V_{in}}{\sqrt{4R_0^2 + 4X_1^2}}$$

$$P_2 = |I_{out}|^2 \cdot R_0$$

$$P_2 = \frac{V_{in}^2 \cdot R_0}{4(R_0^2 + X_1^2)}$$

Now; $$N^2 = \frac{P_1}{P_2}$$

$$= \left(\frac{\cancel{V_{in}^2}}{\cancel{4}R_0^2}\right)\cancel{R_0} \cdot \frac{\cancel{4}(R_0^2 + X_1^2)}{\cancel{V_{in}^2 R_0}}$$

$$N^2 = 1 + \frac{X_1^2}{R_0^2}$$

$$X_1^2 = R_0^2 (N^2-1)$$

$$\boxed{X_1 = R_0 \sqrt{N^2-1}}$$

→ 'X_1' may be an inductor or a capacitor

→ If $X_1 = \omega L$ (inductor), then:

$$\omega L = R_0 \sqrt{N^2-1}$$

$$2\pi f L = R_0 \sqrt{N^2-1}$$

i.e. $\boxed{f \propto \sqrt{N^2-1}}$

∴ If increased attenuation is req'd with increase of frequency then X_1 must be an inductor whose value is $\boxed{L = \dfrac{R_0\sqrt{N^2-1}}{2\pi f}}$

→ If $X_1 = \dfrac{1}{\omega C}$ (capacitor), then:

$$\omega C = \frac{1}{R_0\sqrt{N^2-1}}$$

$$2\pi f C = \frac{1}{R_0\sqrt{N^2-1}}$$

i.e $\boxed{f \propto \dfrac{1}{\sqrt{N^2-1}}}$

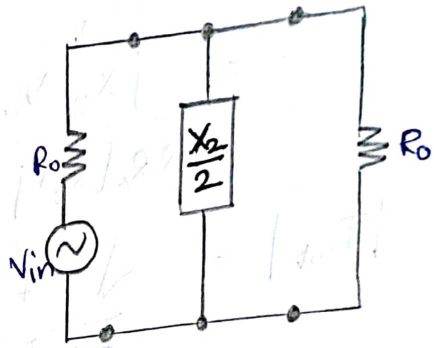
∴ If decreased attenuation is required

... increase of frequency then X_1 ... to be a capacitor whose value is

$$\boxed{C = \frac{1}{R_0 2\pi f_\infty \sqrt{N^2-1}}}$$

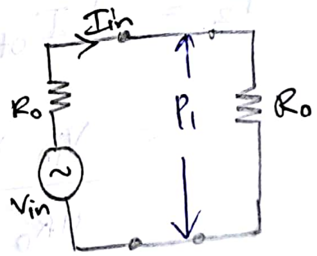
Shunt Equalizer :-

Consider stand. shunt equalizer as shown in fig. where X_2 is reactance.



Input power without Equalizer (P_1) :-

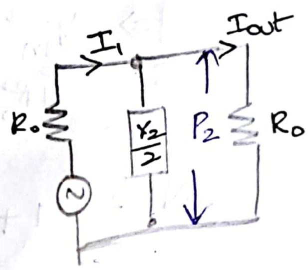
$$I_{in} = \frac{V_{in}}{2R_0}$$



$$P_1 = |I_{in}|^2 R_0$$

$$P_1 = \left|\frac{V_{in}^2}{4R_0^2}\right| R_0$$

O/p power with equalizer (P_2) :-



$$I_1 = \frac{V_{in}}{R_0 + \dfrac{jR_0\frac{X_2}{2}}{R_0 + j\frac{X_2}{2}}}$$

$$I_1 = \frac{V_{in}(2R_0 + jX_2)}{[R_0(2R_0+jX_2)+jX_2R_0]}$$

Now,

current division rule

$$I_{out} = \frac{\left(j\frac{X_2}{2}\right)}{R_0 + j\frac{X_2}{2}} \cdot I_1$$

$$= \frac{jX_2}{\cancel{(2R_0+jX_2)}} \times \frac{V_{in}\,\cancel{(2R_0+jX_2)}}{\left[2R_0^2 + j\,2X_2R_0\right]}$$

$$= \frac{jX_2V_{in}}{2R_0\left[R_0+jX_2\right]}$$

$$|I_{out}| = \frac{\sqrt{0^2+(X_2V_{in})^2}}{2R_0\sqrt{R_0^2+X_2^2}}$$

$$P_2 = |I_{out}|^2 \cdot R_0$$

$$= \frac{V_{in}^2\,X_2^2 \cdot R_0}{4R_0^2\left(R_0^2+X_2^2\right)}$$

Now $N^2 = \dfrac{P_1}{P_2}$

$$\Rightarrow \frac{\cancel{V_{in}^2}}{\cancel{4R_0^2}} \cdot R_0 \times \frac{\cancel{4R_0^2}\left(R_0^2+X_2^2\right)}{\cancel{V_{in}^2}\,X_2^2\,\cancel{R_0}}$$

$$N^2 \Rightarrow 1 + \frac{R_0^2}{X_2^2}$$

$$\frac{R_0^2}{X_2^2} = N^2 - 1$$

$$\boxed{\therefore X_2 = \frac{R_0}{\sqrt{N^2-1}}}$$

X_2 may be a inductor or capacitor

→ If $X_2 = \omega L$ (inductor), then :

$$\omega L = \frac{R_o}{\sqrt{N^2-1}}$$

$$\boxed{f \propto \frac{R_o}{\sqrt{N^2-1}}}$$

∴ Decreased attenuation is obtained with increase of frequency then,

X_2 must be an inductor whose value

ie $$\boxed{L = \frac{R_o}{2\pi f\sqrt{N^2-1}}}$$

→ If $X_2 = \frac{1}{\omega C}$ (capacitor), then ;

$$\frac{1}{\omega C} = \frac{R_o}{\sqrt{N^2-1}}$$

$$\boxed{f \propto \sqrt{N^2-1}}$$

∴ If increased attenuation is req'd with increase of frequency then

X_2 must be a capacitor. whose value

$$\boxed{C = \frac{\sqrt{N^2-1}}{2\pi f R_o}}$$

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Problems:-

Q1) Design a series equaliser with increased attenuation with increase of frequency & to provide 10 dB attenuation at 100 KHz. & $R_0 = 600\Omega$

Sol:- $X_1 = R_0\sqrt{N^2-1}$

$N = 10^{D/20} \Rightarrow 10^{10/20} \Rightarrow \sqrt{10}$

$X_1 = 600\sqrt{(\sqrt{10})^2 - 1}$

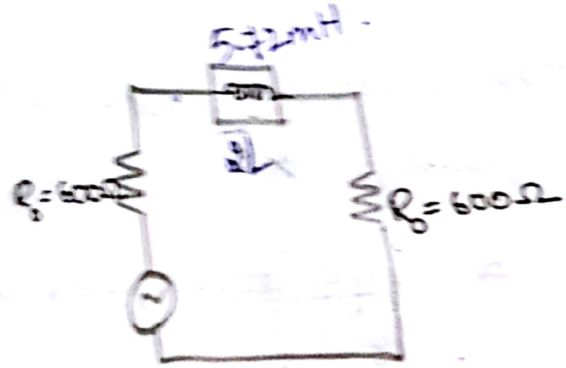
$\boxed{X_1 = 1800}$

∴ X_1 is an inductor $X_1 = \omega L$

$L = \frac{X_1}{\omega}$

$= \frac{1800}{2\pi \times 100KHz}$

$\boxed{L = 2.86mH}$



equalizer with
with increase
provide 10 dB
y. & $R_o = 600\Omega$

Q. Design a shunt equalizer to provide 10 dB attenuation at 100KHz frequency & $R_o = 600\Omega$.

$N = 10^{\alpha/20} \Rightarrow 10^{10/20} \Rightarrow \sqrt{10}$

$R_o = 600\Omega$

i) Increased attenuation with increase of frequency ∴ $X_2 \rightarrow$ capacitor

$$C = \frac{\sqrt{N^2-1}}{2\pi f \cdot R_o} \Rightarrow \frac{\sqrt{(\sqrt{10})^2-1}}{2\times\pi\times100\times10^3\times600}$$

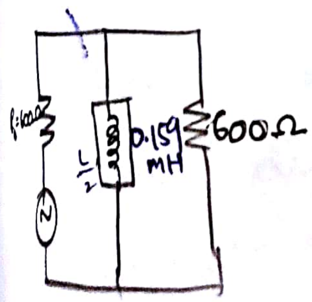
$\boxed{C = 7.95 nF}$ $\Rightarrow 2C = 15.97 nF$

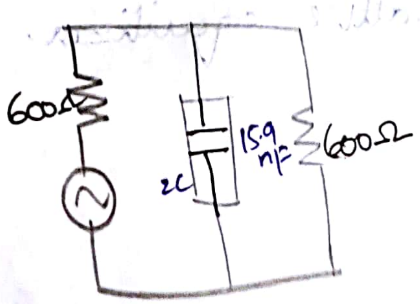
ii) Decreased attenuation with increase of frequency ∴ $X_2 \rightarrow$ Inductor

$$L = \frac{R_o}{2\pi f\sqrt{N^2-1}}$$

$$L = \frac{600}{2\pi\times100\times10^3\times\sqrt{(\sqrt{10})^2-1}} \Rightarrow 0.318 mH$$

$\frac{L}{2} = 0.159 mH$





Drawbacks of Two-terminal equalizer:-

→ The impedence of two-terminal equalizers changes ~~to~~ w.r.t frequency i.e not maintaining a constant impedence in the circuit. Due to this impedence mismatch occurs so that maximum is not transferred from source to load.

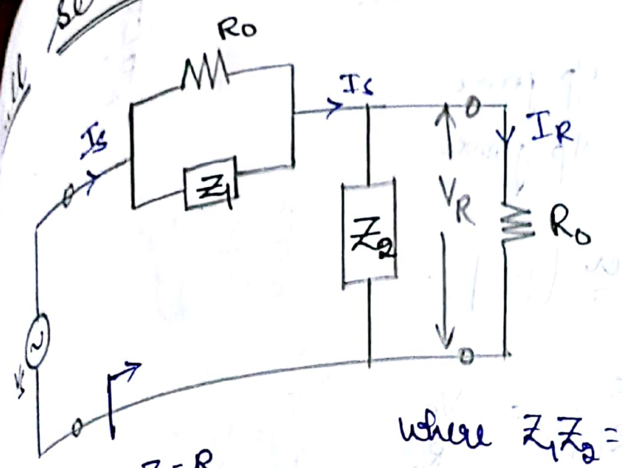
FOUR-terminal Equilizers:-

Four terminal equilizers provides constant impedence irrespective of frequency. Hence, they are also called "constant-Resistance Equilizers".

Types of 4-terminal equilizers;

1) Full-series equilizer
2) Full-shunt equilizer
3) Bridged-T equilizer
4) Lattice equilizer.

Full series Equilizer :-



$Z_{in} = Z_0 = R_0$

where $Z_1 Z_2 = R_0^2$ (Inverse Impedences)

$$Z_{in} = \frac{R_0 Z_2}{R_0 + Z_2} + \frac{R_0 Z_1}{R_0 + Z_1}$$

$$= \frac{R_0 Z_2 (R_0 + Z_1) + R_0 Z_1 (R_0 + Z_2)}{(R_0 + Z_2)(R_0 + Z_1)}$$

$$= \frac{R_0 [R_0 Z_2 + Z_1 Z_2 + Z_1 R_0 + Z_1 Z_2]}{R_0^2 + R_0 Z_2 + R_0 Z_1 + Z_1 Z_2}$$

$$= \frac{R_0 [R_0 (Z_1 + Z_2) + 2 Z_1 Z_2]}{[R_0 (Z_1 + Z_2) + 2 Z_1 Z_2]}$$

$Z_{in} = R_0$.

i.e Insertion of full series equalizer does not disturb the impedence of circuit

Now, w.k.T $N = \sqrt{\dfrac{P_1}{P_2}} \Rightarrow N^2 = \dfrac{P_1}{P_2}$

where $P_1 \rightarrow$ i/p power

$P_2 \rightarrow$ o/p power

<u>Input power $[P_1]$</u>:-

$$I_S = \frac{V_S}{Z_{in}} = \frac{V_S}{R_0}$$

$$|I_S| = \frac{V_S}{R_0}$$

$$P_1 = |I_S|^2 \cdot R_0 \Rightarrow \left(\frac{V_S^2}{R_0^2}\right) \cdot R_0$$

<u>Output power $[P_2]$</u>:-

$I_2 = \left(\dfrac{R_1}{R_1+R_2}\right) I$

$$I_R = \left(\frac{Z_2}{R_0 + Z_2}\right) I_S$$

$$|I_R| = \left|\frac{Z_2}{(R_0 + Z_2)} \cdot I_S\right|$$

$$P_2 = |I_R|^2 \cdot R_0$$

$$P_2 = \left|\frac{Z_2 I_S}{R_0 + Z_2}\right|^2 \cdot R_0$$

Now w.k.T $N^2 = \dfrac{P_1}{P_2}$

$$N^2 = \frac{|I_s|^2 R_0}{\left|\frac{Z_2 I_s}{Z_2+R_0}\right|^2 \cdot R_0}$$

$$N^2 = \left\{\left|\frac{I_s(Z_2+R_0)}{Z_2 I_s}\right|\right\}^2$$

$$\boxed{N^2 = \left\{\left|\frac{Z_2+R_0}{Z_2}\right|\right\}^2 \quad (\text{or}) \quad N^2 = \left\{\left|\frac{R_0+Z_1}{R_0}\right|\right\}^2} \longrightarrow (1)$$

where Z_1 & Z_2 are considered as Reactances

case-(i): If $Z_1 = j\omega L_1$ (inductor)

w.k.T $Z_1 Z_2 = R_0^2$

$$Z_2 = \frac{R_0^2}{j\omega L_1} \Rightarrow \frac{1}{j\omega C_2}$$

$$\boxed{C_2 = \frac{L_1}{R_0^2}}$$

from eqn (1)

$$N^2 = \left\{\left|\frac{R_0 + j\omega L_1}{R_0 + 0}\right|\right\}^2$$

$$N^2 = \left\{\frac{\sqrt{R_0^2+\omega^2 L_1^2}}{\sqrt{R_0^2+0^2}}\right\}$$

$N^2 = \frac{P_1}{P_2}$

R_0

CDR

$I_2 = \left(\frac{R_1}{R_1+R_2}\right) I$

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$$N^2 = \frac{R_0^2 + \omega^2 L_1^2}{R_0^2}$$

$$N^2 = 1 + \frac{\omega^2 L_1^2}{R_0^2}$$

$$N^2 - 1 = \frac{\omega^2 L_1^2}{R_0^2} \Rightarrow X_1$$

$$X_1^2 = R_0^2 (N^2 - 1)$$

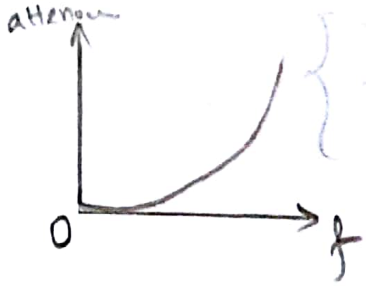
$$X_1 = R_0\sqrt{N^2-1} \longrightarrow X_1 \propto \sqrt{N^2-1}$$

$$\boxed{L_1 = \frac{R_0\sqrt{N^2-1}}{\omega}}$$

$$\Rightarrow C_2 = \frac{L_1}{R_0^2} = \frac{\cancel{R_0}\sqrt{N^2-1}}{R_0^{\cancel{2}}\,\omega}$$

$$\boxed{C_2 = \frac{\sqrt{N^2-1}}{R_0\omega}}$$

→ If Increased attenuation is req'd with increase of frequency then;
$Z_1 \rightarrow$ inductor & $Z_2 \rightarrow$ capacitor.



$$\boxed{\begin{aligned} L_1 &= \frac{R_0\sqrt{N^2-1}}{\omega} \\ C_2 &= \frac{\sqrt{N^2-1}}{R_0\omega} \end{aligned}}$$

$\rightarrow x_1 \alpha \sqrt{N^2-1}$

req'd with

n;

pacitor.

$$\boxed{L_1 = \frac{R_0\sqrt{N^2-1}}{\omega} \qquad C_2 = \frac{\sqrt{N^2-1}}{R_0\omega}}$$

case-iii: If $Z_1 = \frac{1}{j\omega C}$ (capacitor)

w.r.t $Z_1 Z_2 = R_0^2 \Rightarrow Z_2 = j\omega(C_1 R_0^2)$

$$\boxed{\therefore L_2 = C_1 R_0^2}$$

from eqn ①

$$N^2 = \left\{\left|\frac{R_0 + \frac{1}{j\omega C_1}}{R_0}\right|\right\}^2$$

$$N^2 = \left\{\left|\frac{1 + j\omega R_0 C_1}{0 + j\omega R_0 C_1}\right|\right\}^2$$

$$N^2 = \left\{\frac{\sqrt{1+\omega^2 R_0^2 C_1^2}}{\sqrt{0^2+\omega^2 R_0^2 C_1^2}}\right\}^2$$

$$N^2 = \frac{1+\omega^2 R_0^2 C_1^2}{\omega^2 R_0^2 C_1^2}$$

$$N^2 = 1 + \frac{1}{\omega^2 R_0^2 C_1^2}$$

$$C_1^2 = \frac{1}{\omega^2 R_0^2 (N^2-1)} \rightarrow x_1 \alpha$$

$$\boxed{C_1 = \frac{1}{R_0\omega\sqrt{N^2-1}}}$$

$$L_2 = C_1 R_0^2 \Rightarrow \boxed{L_2 = \frac{R_0}{\omega\sqrt{N^2-1}}}$$

→ Decreased attenuation is req'd with increase of frequency then

$Z_1 \rightarrow$ capacitor & $Z_2 \rightarrow$ inductor.

$$\therefore \boxed{\begin{aligned} C_1 &= \frac{1}{\omega R_0\sqrt{N^2-1}} \\ L_2 &= \frac{R_0}{\omega\sqrt{N^2-1}} \end{aligned}}$$

problems :-

Q1) Design a Full series equilizer to provide 15dB attenuation at 20kHz & with a design impedence 600Ω. Assume Increased attenuation with increase of frequency.

Sol :- $N = 10^{D/20} \Rightarrow 10^{15/20} \Rightarrow 5.623$

$f = 20kHz$

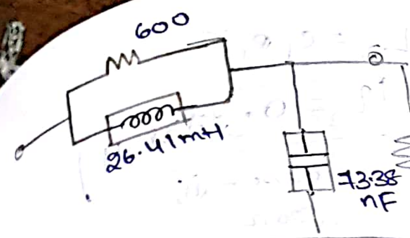
$R_0 = 600\Omega$

$\therefore Z_1 \rightarrow$ inductor & $Z_2 =$ capacitor.

$$\therefore L_1 = \frac{R_0\sqrt{N^2-1}}{2\pi f} \quad \& \quad C_2 = \frac{\sqrt{N^2-1}}{R_0 \cdot 2\pi f}$$

$$= \frac{600\sqrt{(5.623)^2-1}}{2\pi \times 20kHz} \qquad = \frac{\sqrt{(5.623)^2-1}}{2\pi \times 20kHz \times 600}$$

$$\boxed{L = 26.41mH} \qquad \boxed{C_2 = 73.38nF}$$



Q2) Design Full ser[ies] ... impedence 500Ω ... attenuation of 2[0] ...

Sol) $N = 10^{D/20} \Rightarrow 10^{20/\ldots}$

$f = 12kHz$

$R_0 = 500\Omega$

→ Increased attenu[ation] ...

$\therefore Z_1 \rightarrow$ inductor & ...

$$L_1 = \frac{R_0\sqrt{N^2-1}}{2\pi f}$$

$$L_1 = \frac{500\sqrt{(17.78)^2-1}}{2\times\pi\times 12k\ldots}$$

$$\boxed{L_1 = 117.72m\ldots}$$

→ Decreased atten[uation] ...

$Z_1 \rightarrow$ capacit[or] ...

$$C_1 = \frac{1}{2\pi f R_0\sqrt{\ldots}}$$

is req'd with
then

inductor.

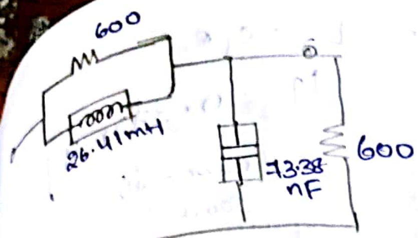
equilizer to
at 20kHz
ce ; 600Ω.
ation with

Z_2 = capacitor.

$\frac{N^2-1}{R_0 \cdot 2\pi f}$

$\frac{(5.623)^2-1}{2\pi\times 20kHz\times 600}$

$\boxed{38\,nF}$



Q. Design Full series equilizer with design impedance 500Ω & to provide a attenuation of 25dB at 12 KHz.

Sol. $N = 10^{\alpha/20} \Rightarrow 10^{25/20} \Rightarrow 17.78$

$f = 12KHz$

$R_0 = 500\Omega$

→ Increased attenuation with increase of frequency

∴ Z_1 → inductor & Z_2 → capacitor

$L_1 = \frac{R_0\sqrt{N^2-1}}{2\pi f}$; $C_2 = \frac{\sqrt{N^2-1}}{2\pi f \cdot R_0}$

$L_1 = \frac{500\sqrt{(17.78)^2-1}}{2\times\pi\times 12KHz}$ $= \frac{\sqrt{(17.78)^2-1}}{2\pi\times 12KHz\times 500}$

$\boxed{L_1 = 117.72\,mH}$ $\boxed{C_2 = 470.88\,nF}$

→ Decreased attenuation with increase of frequency

Z_1 → capacitor & Z_2 → inductor

$C_1 = \frac{1}{2\pi f R_0\sqrt{N^2-1}}$, $L_2 = \frac{R_0}{2\pi f\sqrt{N^2-1}}$

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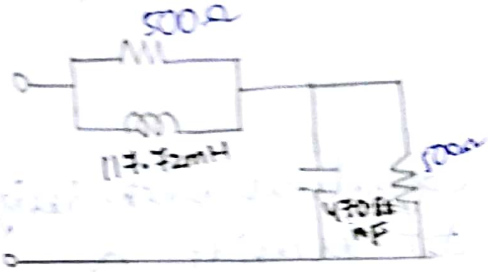
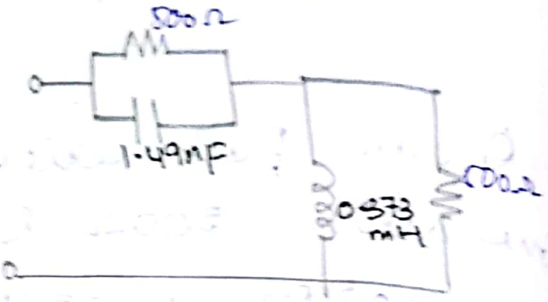
$\boxed{C_1 = 1.49\,nF}$; $L_2 = C_1 R_0^2$

$\boxed{L_2 = 0.373\,mH}$

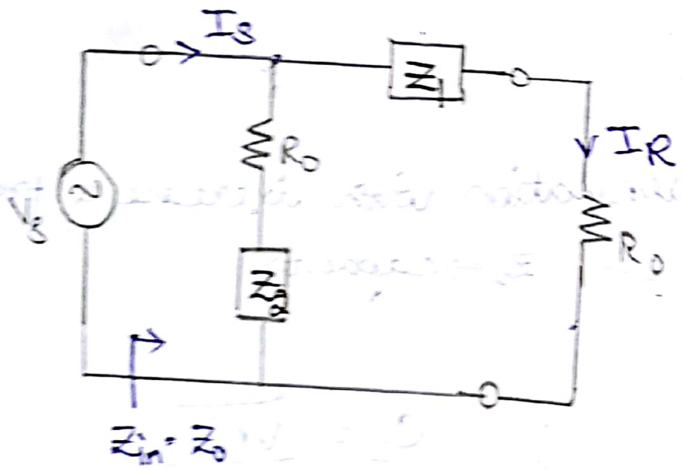
Case - i,



Case - ii,



Full - Shunt Equalizer :-



$$Z_{in} = (Z_1 + R_0) \,||\, (Z_2 + R_0)$$

$$= \frac{(Z_1 + R_0)(Z_2 + R_0)}{Z_1 + Z_2 + 2R_0}$$

$$\Rightarrow \frac{Z_1 Z_2 + Z_1 R_0 + Z_2 R_0 + R_0^2}{Z_1 + Z_2 + 2R_0}$$

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$$\frac{R_0^2 + R_0(Z_1+Z_2) + R_0^2}{2R_0 + Z_1 + Z_2}$$

$$= \frac{2R_0^2 + R_0(Z_1+Z_2)}{2R_0 + Z_1 + Z_2}$$

$$= \frac{R_0(\cancel{2R_0+Z_1+Z_2})}{\cancel{2R_0+Z_1+Z_2}}$$

$$\boxed{Z_{in} = R_0}$$

w.k.T $\boxed{N^2 = \frac{P_1}{P_2}}$

Input power $[P_1]$:-

$$I_S = \frac{V_S}{Z_{in}} = \frac{V_S}{R_0}$$

$$|I_S| = \frac{V_S}{R_0}$$

$$P_1 = |I_S|^2 \cdot R_0$$

Output power $[P_2]$:-

$$I_R = \left(\frac{R_0+Z_2}{(R_0+Z_2)+Z_1+R_0}\right) \cdot I_S$$

$$|I_R| = \left|\frac{R_0+Z_2}{(R_0+Z_2)+(Z_1+R_0)} \cdot I_S\right|$$

$$|I_R| = \left| \frac{R_0 + Z_2}{2R_0 + Z_1 + Z_2} \cdot I_S \right|$$

$$P_2 = |I_R|^2 \cdot R_0$$

$$P_2 = \left| \frac{R_0 + Z_2}{2R_0 + Z_1 + Z_2} \cdot I_S \right|^2 \cdot R_0$$

Now ; $N^2 = \frac{P_1}{P_2}$

$$N^2 = \frac{|I_S|^{\cancel{2}} \cancel{R_0}}{\left| \frac{(R_0 + Z_2)\cancel{I_S}}{2R_0 + Z_1 + Z_2} \right|^2 \cdot \cancel{R_0}}$$

$$N^2 = \left[\left| \frac{2R_0 + Z_1 + Z_2}{(R_0 + Z_2)} \right| \right]^2$$

& $Z_1 Z_2 = R_0^2 \Leftrightarrow Z_2 = \frac{R_0^2}{Z_1}$

$$N^2 = \left\{ \left| \frac{2R_0 + Z_1 + \frac{R_0^2}{Z_1}}{R_0 + \frac{R_0^2}{Z_1}} \right| \right\}^2$$

$$N^2 = \left\{ \left| \frac{2R_0 Z_1 + Z_1^2 + R_0^2}{R_0 Z_1 + R_0^2} \right| \right\}^2$$

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(iii): If $Z_1 = j\omega L_1 \Rightarrow Z_2 = \dfrac{R_0^2}{Z_1} = \dfrac{R_0^2}{j\omega L_1}$

$$= \frac{1}{j\omega\left(\frac{L_1}{R_0^2}\right)}$$

$$\simeq \frac{1}{j\omega(C_2)}$$

where $\boxed{C_2 = \dfrac{L_1}{R_0^2}}$

$$N^2 = \left\{\left|\frac{2R_0(j\omega L_1) + R_0^2 + (j\omega L_1)^2}{R_0(j\omega L_1) + R_0^2}\right|\right\}^2$$

$$N^2 = \left\{\left|\frac{(R_0^2-\omega^2L_1^2) + j(2\omega L_1 R_0)}{R_0^2 + j\omega R_0 L_1}\right|\right\}^2$$

$$= \frac{(R_0^2-\omega^2L_1^2)^2 + (2\omega L_1 R_0)^2}{R_0^4 + (R_0\omega L_1)^2}$$

$$= \frac{(R_0^2)^2 + (\omega^2L_1^2)^2 - 2R_0^2\omega^2L_1^2 + 4R_0^2\omega^2L_1^2}{R_0^4 + R_0^2\omega^2L_1^2}$$

$$= \frac{(R_0^2)^2 + (\omega^2L_1^2)^2 + 2R_0^2\omega^2L_1^2}{R_0^4 + R_0^2\omega^2L_1^2}$$

$$= \frac{\left[(R_0^2)^2 + R_0^2\omega^2L_1^2\right] + \omega^2L_1^2\left[R_0^2+\omega^2L_1^2\right]}{(R_0^2)^2 + R_0^2\omega^2L_1^2}$$

$$= \frac{(R_0^2)^2 + \cancel{R_0^2\omega^2L_1^2}}{(R_0^2)^2 + \cancel{R_0^2\omega^2L_1^2}} + \frac{\omega^2L_1^2\left[\cancel{R_0^2+\omega^2L_1^2}\right]}{R_0^2\left(\cancel{R_0^2+\omega^2L_1^2}\right)}$$

$$N^2 = 1 + \frac{\omega^2 L_1^2}{R_0^2}$$

$$(N^2-1)R_0^2 = \omega^2 L_1^2 \longrightarrow \omega = \frac{R_0\sqrt{N^2-1}}{L}$$

$$f \propto \sqrt{N^2-1}$$

$$L_1 = \frac{R_0\sqrt{N^2-1}}{\omega}$$

Now $C_2 = \frac{L_1}{R_0^2} = \frac{R_0\sqrt{N^2-1}}{R_0^2\omega} = \frac{\sqrt{N^2-1}}{R_0\omega}$

$$\therefore \boxed{L_1 = \frac{R_0\sqrt{N^2-1}}{\omega} \; ; \; C_2 = \frac{\sqrt{N^2-1}}{R_0\omega}}$$

From Above eqn's it is evident that $f \propto \sqrt{N^2-1}$ i.e increased attenuation is required with increase of frequency, then $\therefore$ 'Z_1' must be an Inductor.

Case-(ii):- If $Z_1 = \frac{1}{j\omega C_1}$ $\Rightarrow Z_2 = \frac{R_0^2}{Z_1} = \frac{R_0^2}{\frac{1}{j\omega C_1}}$

$$Z_2 = j\omega(C_1R_0^2) \approx j\omega L_2$$

$$\therefore \boxed{L_2 = C_1R_0^2}$$

Now $N^2 = \frac{2R_0\left(\frac{1}{j\omega C_1}\right) + R_0^2 + \left(\frac{1}{j\omega C_1}\right)^2}{\frac{R_0}{j\omega C_1} + R_0^2}$

$$N^2 = \left\{\left|\frac{\left[(j\omega C_1)2R_0 + R_0^2(j\omega C_1)^2 + 1\right]/\cancel{(j\omega C_1)^2}}{(R_0 + R_0^2 j\omega C_1)/\cancel{j\omega C_1}}\right|\right\}^2$$

$\omega = \frac{R_0\sqrt{N^2-1}}{4}$

$f \propto \sqrt{N^2-1}$

$\frac{\sqrt{N^2-1}}{R_0\omega}$

$\boxed{\frac{\sqrt{N^2-1}}{\dots\omega}}$

...t that $f \propto \sqrt{N^2-1}$
... required with
'Z_1' must be

$\frac{R_0^2}{Z_1} = \frac{R_0^2}{\frac{1}{j\omega C_1}}$

$j\omega(C_1R_0^2) \approx j\omega L_2$

$N^2 = \left\{\left|\frac{2R_0(j\omega C_1)+1-\omega^2C_1^2R_0^2}{R_0 j\omega C_1 - R_0^2\omega^2C_1^2}\right|\right\}^2$

$$N^2 = \frac{(1-\omega^2C_1^2R_0^2)^2+(2R_0\omega C_1)^2}{(R_0\omega C_1)^2+(R_0^2\omega^2C_1^2)^2}$$

$$N^2 = \frac{1+(\omega^2C_1^2R_0^2)^2-2\omega^2C_1^2R_0^2+4R_0^2\omega^2C_1^2}{(R_0\omega C_1)^2+(R_0^2\omega^2C_1^2)^2}$$

$$N^2 = \frac{1+(\omega^2C_1^2R_0^2)^2+2R_0^2\omega^2C_1^2}{R_0^2\omega^2C_1^2+(R_0^2\omega^2C_1^2)^2}$$

$$= \frac{1+R_0^2\omega^2C_1^2+R_0^2\omega^2C_1^2+(\omega^2C_1^2R_0^2)^2}{R_0^2\omega^2C_1^2+(R_0^2\omega^2C_1^2)^2}$$

$$= \frac{1+\cancel{R_0^2\omega^2C_1^2}}{R_0^2\omega^2C_1^2\left[\cancel{1+R_0^2\omega^2C_1^2}\right]} + \frac{\cancel{R_0^2\omega^2C_1^2+(\omega^2C_1^2R_0^2)^2}}{\cancel{R_0^2\omega^2C_1^2+(R_0^2\omega^2C_1^2)^2}}$$

$$N^2 = \frac{1}{R_0^2\omega^2C_1^2}+1$$

$$N^2-1 = \frac{1}{R_0^2\omega^2C_1^2}$$

$$R_0^2\omega^2C_1^2 = \frac{1}{N^2-1}$$

$\therefore f \propto \frac{1}{\sqrt{N^2-1}}$

$$\therefore \boxed{C_1 = \frac{1}{R_0 \omega \sqrt{N^2-1}} \qquad L_2 = C_1 R_0^2 = \frac{R_0}{\omega\sqrt{N^2-1}}}$$

from the above eq's it is evident that

$$f \propto \frac{1}{\sqrt{N^2-1}}$$

$\therefore$ If decreased attenuation is required with increase of frequency then 'Z_1' must be a "capacitor"

<u>Alternative Method</u>:-

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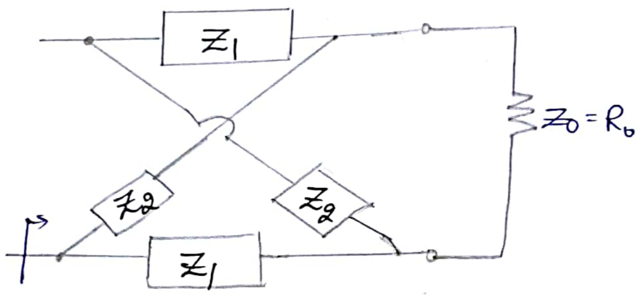
Lattrice Attenuator Egquilizer:-



$Z_{in} = Z_0 = R_0$

W.K.T $Z_1 Z_2 = R_0^2$

(inverce impedence)

& $\boxed{Z_1 = R_1 + jX_1}$

Design Egquations:-

case-i, :- If $X_1 = \omega L_1$

$$N^2 = \frac{A^2 + f^2}{B^2 + f^2}$$

where $A = \frac{R_0 + R_1}{2\pi L_1}$ & $B = \frac{R_0 - R_1}{2\pi L_1}$

case-ii, :- If $X_1 = \frac{-1}{\omega C_1}$

$$N^2 = \frac{P^2 f^2 + 1}{Q^2 f^2 + 1}$$

where $P = (R_0 + R$

$Q = (R_0 -$

Note:- If attenuation known then calculated. can calcula

Design a latt 17dB attenuatio & $R_0 = 600\Omega$.

Sol:- Given:

17db a

3db a

W.K.T [case-i];

$N^2 = A^2$ / B^2

where $A = \frac{R_0 + R_1}{2\pi L}$

→ At $f = 50$

so $N =$

where

$P = (R_0 + R_1) 2\pi C_1$

$Q = (R_0 - R_1) 2\pi C_1$

If attenuation at two frequencies are known then A, B & P, Q values are calculated. From these constants we can calculate Z_1 & Z_2.

Design a lattice attenuation equalizer with 17 dB attenuation at 50Hz & 3dB at 2KHz

$R_0 = 600\Omega$.

Sol: Given:

17db at 50 Hz

3db at 2 KHz

w.k.T [case-i];

$N^2 = \frac{A^2 + f^2}{B^2 + f^2}$; $Z_1 = R_1 + jX_1$

where $A = \frac{R_0 + R_1}{2\pi L_1}$, $B = \frac{R_0 - R_1}{2\pi L_1}$

→ At $f = 50Hz$; $D = 17db$

so $N = 10^{D/20} \Rightarrow 10^{17/20} = 7.079$

→ At $f = 2KHz$, $D = 3db$

$$N = 10^{D/20} \Rightarrow 10^{3/20} = 1.4125$$

Now subⁿ in $N^2 = \frac{A^2 + f^2}{B^2 + f^2}$

$$(7.079)^2 = \frac{A^2 + 50^2}{B^2 + 50^2}$$

$$(50.11)(B^2 + 50^2) = A^2 + 50^2$$

$$50.11B^2 + 125275 = A^2 + 50^2$$

$$A^2 - 50.11B^2 = 122775 \longrightarrow (1)$$

→ At $f = 2KHz$, $D = 3db$

$$N^2 = \frac{A^2 + f^2}{B^2 + f^2}$$

$$(1.4125)^2 = \frac{A^2 + (2\times10^3)^2}{B^2 + (2\times10^3)^2}$$

$$(1.993)(B^2 + (2\times10^3)^2) = A^2 + (2\times10^3)^2$$

$$B^2(1.993) + 3986000 = A^2 + (2\times10^3)^2$$

$$A^2 - B^2(1.993) = 3972000 \longrightarrow (2)$$

on solving (1) & (2)

$A^2 = 4131434.408$; $B^2 = 79997.194$

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$\boxed{A = 2032.59}$; $\boxed{B = 282.84}$

WKT $A = \frac{R_0 + R_1}{2\pi L_1}$ & $B = \frac{R_0 - R_1}{2\pi L_1}$

$2032.59 = \frac{R_0 + R_1}{2\pi L_1}$; $282.84 = \frac{R_0 - R_1}{2\pi L_1}$

$R_0 + R_1 = (2032.59) 2\pi L_1 \longrightarrow (3)$; $R_0 - R_1 = (282.84) 2\pi L_1 \longrightarrow (4)$

Solving (3) & (4)

$$R_0 + \cancel{R_1} = (2032.59)\, 2\pi L_1$$
$$R_0 - \cancel{R_1} = (282.84)\, 2\pi L_1$$
$$\cancel{2}R_0 = \cancel{2}\pi L_1 [2032.59 + 282.84]$$

$$\frac{600}{\pi(2032.59 + 282.84)} = L_1$$

$$\boxed{L_1 = 82.48\,mH}$$

$$C_2 = \frac{L_1}{R_0^2} = \frac{82.48mH}{(600)^2} \Rightarrow \boxed{C_2 = 229.12nF}$$

Inorder to get 'R_1'?

sub^n L_1 in eqn (3) i.e $R_1 = (2032.59)\, 2\pi L_1 - R_0$

$$\boxed{R_1 = 453.36\,\Omega}$$

W.K.T $R_1 R_2 = R_0^2$.

$$R_2 = \frac{R_0^2}{R_1}$$

$$R_2 = \frac{600^2}{453.36}$$

$$\boxed{R_2 = 794.07\,\Omega}$$

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