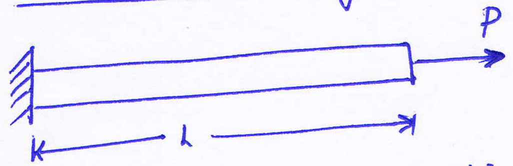
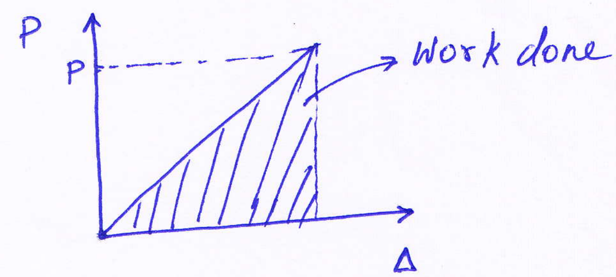
STRESSES DUE TO GRADUAL, SUDDEN AND IMPACT LOADS.

A) Gradual Loading.



Let a bar of cross-sectional area A be subjected to a gradually applied axial load P due to which the bar deforms by Δ.

P-Δ curve



The work done by external load

$$= \frac{1}{2} P\Delta \qquad —①$$

The internal energy stored in the bar = internal work done by the internal resistance

$$= \frac{1}{2} R\Delta = \frac{1}{2} \sigma A \Delta \qquad —②$$

From ① and ② we get,

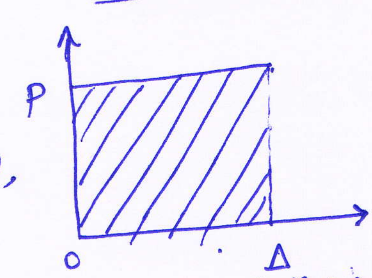
$$\frac{1}{2} P\Delta = \frac{1}{2} \sigma A \Delta$$

$$\Rightarrow \boxed{\sigma = \frac{P}{A}}$$

B) Sudden Loading

If the load P is suddenly applied instead of gradually then the full load P exists through the deformation.

P-Δ curve



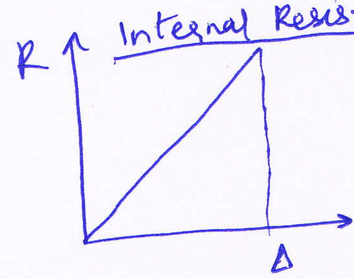
∴ Work done by external load

$$= P\Delta \qquad —①$$

But, internal resistance R develops gradually along with the deformation, from O to the maximum value.

∴ Internal work done by $R = \frac{1}{2} R\Delta$ —②

Internal Resistance



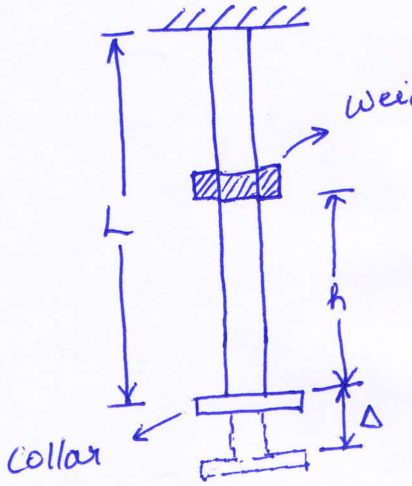
From ① and ② we get

$$P\Delta = \frac{1}{2} R\Delta = \frac{1}{2} \sigma A \Delta$$

$$\Rightarrow \boxed{\sigma = 2P/A}$$

the case of sudden loading is twice the value of the max. stress developed for gradual loading of the same load P.

(c) IMPACT LOADING.



Consider the dropping of a weight P through a height 'h' on the collar fixed at the end of a vertically supported bar as shown in the figure.

After dropping of the weight, let the bar deform by Δ.

Assuming that whole energy of the falling weight is used in stretching the bar by Δ, we have:

Work done by the falling weight, $P = P(h+\Delta)$ —— ①

Work stored in the bar as internal strain energy

$$= \frac{1}{2} R\Delta$$

$$= \frac{1}{2} \sigma A \frac{\sigma L}{E} \qquad \left[\because \sigma = E\frac{\Delta}{L}\right]$$

$$= \frac{\sigma^2}{2E} AL \quad \text{—— ②}$$

From ① and ②, we get

$$P(h+\Delta) = \frac{\sigma^2}{2E} AL \quad \text{—— ③}$$

$$\Rightarrow P\left(h + \frac{\sigma L}{E}\right) = \frac{\sigma^2}{2E} AL$$

$$\Rightarrow \sigma^2\left(\frac{AL}{2E}\right) - \sigma\left(\frac{PL}{E}\right) - Ph = 0.$$

$$\Rightarrow \sigma^2 - \left(\frac{2P}{A}\right)\sigma - \frac{2PEh}{AL} = 0$$

$$\Rightarrow \sigma = \frac{\left(\frac{2P}{A} \pm \sqrt{\frac{4P^2}{A^2} + 4\frac{2PEh}{AL}}\right)}{2}$$

$$\Rightarrow \sigma = \frac{P}{A} \pm \sqrt{\frac{P^2}{A^2} + \frac{P^2}{A^2}\left(\frac{2AEh}{PL}\right)}.$$

omitting the negative root,

$$\Rightarrow \boxed{\sigma = \frac{P}{A}\left(1 + \sqrt{1 + \frac{2AEh}{PL}}\right)}. \quad \text{--- (4)}$$

If however, Δ is considered negligible in comparison to h then the equation (3) becomes.

$$Ph = \frac{\sigma^2}{2E} AL.$$

$$\Rightarrow \boxed{\sigma = \sqrt{\frac{2EPh}{AL}}} \quad \text{--- (5)}.$$

Equations in terms of deformation, Δ.

External Work done, $W = P(h+\Delta)$.

Internal Work done, $U = \frac{1}{2}R\Delta$

where R is the internal resistance, which is zero initially and increases to P at final static equilibrium.

Thus, $U = \frac{1}{2}R\Delta = \frac{1}{2}P\Delta = \frac{1}{2}\frac{\Delta EA}{L}\Delta$ $\left[\because \frac{P}{A} = E\frac{\Delta}{L}\right]$.

Thus we can rewrite equation (3) as.

$$W = U$$

$$\Rightarrow P(h+\Delta) = \frac{1}{2}P\Delta = \frac{1}{2}\frac{EA\Delta^2}{L} \quad \text{--- (6)}.$$

$$\Rightarrow \Delta^2 - \frac{2PL}{EA}\Delta - \frac{2PLh}{EA} = 0 \quad \text{--- (7)}$$

$$\Delta_{st} = \frac{PL}{AE}$$

Thus equation (7) can be rewritten as

$$\Delta^2 - 2\Delta_{st}\Delta - 2\Delta_{st}h = 0$$

$$\Rightarrow \Delta = \frac{2\Delta_{st} \pm \sqrt{4\Delta_{st}^2 + 8\Delta_{st}h}}{2}$$

$$\Rightarrow \Delta = \Delta_{st} \pm \sqrt{\Delta_{st}^2 + 2h\Delta_{st}}$$

Omitting -ve sign.

$$\Rightarrow \Delta = \Delta_{st} + \Delta_{st}\sqrt{1 + \frac{2h}{\Delta_{st}}}$$

$$\Delta = \Delta_{st}\left[1 + \sqrt{1 + \frac{2h}{\Delta_{st}}}\right] \quad \text{--- (8)}$$

If Δ_{st} is ~~compared~~ considered small in comparison to h, we get equation

$$\Delta \simeq \sqrt{2h\Delta_{st}}$$

$$\frac{\Delta}{\Delta_{st}} = \sqrt{\frac{2h}{\Delta_{st}}} \quad \text{--- (9)}$$

$\frac{\Delta}{\Delta_{st}}$ is called as the impact factor.