UNIT – I

5M × 3 = 15M
2M × 2 = 4M
19M

Sequences and Series

Necessary condition for convergence :- (2M)

If a series $\sum u_n$ is convergent then

$$\lim_{n\to\infty} u_n = 0$$

$\lim_{n\to\infty} u_n = 0$, then $\sum u_n$ is convergent

$\lim_{n\to\infty} u_n \neq 0$, then $\sum u_n$ is divergent

① Test for convergence of the series $\sum \frac{4n^3+2}{7n^3+2n}$.

Sol:- Given $\sum \frac{4n^3+2}{7n^3+2n}$

Let $u_n = \frac{4n^3+2}{7n^3+2n}$

$\frac{\infty}{\infty}$ not defined

$$\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{4n^3+2}{7n^3+2n}$$

$$= \lim_{n\to\infty} \frac{\cancel{n^3}\left(4+\frac{2}{n^3}\right)}{\cancel{n^3}\left(7+\frac{2}{n^2}\right)}$$

$\frac{1}{\infty} = 0$

$$= \frac{4+\frac{2}{\infty}}{7+\frac{2}{\infty}}$$

$$= \frac{4+0}{7+0}$$

$\therefore \lim_{n\to\infty} u_n = \frac{4}{7} \neq 0$

$\therefore$ $\sum u_n$ is divergent.

$\therefore$ Given series $\sum \frac{4n^3+2}{7n^3+2n}$ is divergent.

P - Series Test :

(5m)

The series $\sum \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots$

if $p > 1$ then $\sum u_n$ is convergent, if $p \leq 1$ then $\sum u_n$ is divergent.

Comparison Test :

If $\sum u_n$ and $\sum v_n$ are positive term series, if $\lim_{n\to\infty} \frac{u_n}{v_n} = \text{finite} = k$ then both $\sum u_n$ and $\sum v_n$ have same property.

1 Pb) Test for Convergence of $\sum \frac{2n^3+5}{4n^5+1}$.

Sol:- Given $\sum \frac{2n^3+5}{4n^5+1}$.

Let $u_n = \frac{2n^3+5}{4n^5+1}$

$v \frac{n^3}{n^5}$

$v_n = \frac{n^3}{n^5} = \frac{1}{n^2} \quad \left[\sum \frac{1}{n^p} \right]$

By series series Test, $p = 2 > 1$, $\sum v_n$ is convergent

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{\frac{2n^3+5}{4n^5+1}}{\frac{1}{n^2}}$$

$$= \lim_{n\to\infty} \frac{2n^3+5}{4n^5+1} \times \frac{n^2}{1}$$

$$= \lim_{n\to\infty} \frac{\cancel{n^3}\left(2+\frac{5}{n^3}\right)}{\cancel{n^5}\left(4+\frac{1}{n^5}\right)} \frac{\cancel{n^2}}{1}$$

$$= \frac{2+0}{4+0}$$

$$= \frac{2}{4} = \frac{1}{2} = \text{finite}$$

$\therefore$ By using Comparison Test, both $\sum u_n$ and $\sum v_n$ have same property,

Here $\sum v_n$ is convergence.

$\therefore$ $\sum u_n$ is also convergence

$\therefore$ Given series $\sum \frac{2n^3+5}{4n^5+1}$ is convergent.

2(b)) find whether the series $\sum \frac{1}{n\sqrt{n^2-1}}$ is Convergent or divergent.

Sol:- Given $\sum \frac{1}{n\sqrt{n^2-1}}$

Let $u_n = \frac{1}{n\sqrt{n^2-1}}$

$v_n = \frac{1}{n^2}$ $\left(\sum \frac{1}{n^p}\right)$

Here $p = 2 > 1$,

$\therefore$ By using P-series Test, $\sum v_n$ is convergent.

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{\frac{1}{n\sqrt{n^2-1}}}{\frac{1}{n^2}}$$

$$= \lim_{n\to\infty} \frac{1}{\cancel{n}\sqrt{n^2-1}} \cdot \frac{n^{\cancel{2}}}{1}$$

$$= \lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}\sqrt{1-\frac{1}{n^2}}}$$

$$= \frac{1}{\sqrt{1-0}}$$

$\therefore \lim_{n\to\infty} \frac{u_n}{v_n} = 1 = \text{finite}$

D'Alembert's ratio Test :- If $\sum u_n$ is a series of positive terms and $\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = l$, then

(i) If $l > 1$, then $\sum u_n$ is convergent.

(ii) If $l < 1$, then $\sum u_n$ is divergent.

(iii) If $l = 1$, then test fails.

Note :-

(i) If $\sum u_n$ involves x^n terms

$$u_n = \frac{x^{2n}}{(n+1)\sqrt{n}}$$

(ii) If $\sum u_n$ involves factorial terms

$$u_n = 1 + \frac{2^2}{2!} + \frac{3^2}{3!} + \frac{4^2}{4!} + \cdots + \frac{n^2}{n!} + \cdots$$

(iii) If u_n involves power terms (or) same base terms.

$$u_n = \frac{3n-1}{2^n}$$

$2^2 = 2.2$

$2^4 = 2.2.2.2$

(iv) If u_n involves infinite numbers.

$$u_n = \frac{1.2.3.4 \ldots n}{3.5.7 \ldots (3n+1)}$$

① Test for convergence of the Series $\sum \frac{n^3+a}{2^n+a}$.

Sol:- Given $\sum \frac{n^3+a}{2^n+a}$.

Here $u_n = \frac{n^3+a}{2^n+a}$

$$u_{n+1} = \frac{(n+1)^3+a}{2^{n+1}+a}$$

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$$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = \lim_{n\to\infty} \frac{\frac{n^3+a}{2^n+a}}{\frac{(n+1)^3+a}{2^{n+1}+a}}$$

$$= \lim_{n\to\infty} \frac{n^3+a}{2^n+a} \cdot \frac{2^{n+1}+a}{(n+1)^3+a}$$

$$= \lim_{n\to\infty} \frac{n^3\left(1+\frac{a}{n^3}\right)}{2^n\left(1+\frac{a}{2^n}\right)} \cdot \frac{2^n \cdot 2+a}{n^3\left(1+\frac{1}{n}\right)^3+a}$$

$$= \lim_{n\to\infty} \frac{\cancel{n^3}\left(1+\frac{a}{n^3}\right)}{\cancel{2^n}\left(1+\frac{a}{2^n}\right)} \cdot \frac{\cancel{2^n}\left(2+\frac{a}{2^n}\right)}{\cancel{n^3}\left[\left(1+\frac{1}{n}\right)^3+\frac{a}{n^3}\right]}$$

$$= \frac{(1+0)\left(2+\frac{a}{\infty}\right)}{\left(1+\frac{a}{\infty}\right)(1+0+0)} = \frac{1\cdot 2}{1\cdot 1}$$

$$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = 2 > 1,$$

$\therefore$ By using D'Alemberts ratio test,

Σu_n is convergent.

$\therefore$ Given ~~ser~~ is $\Sigma \frac{n^3+a}{2^n+a}$ is convergent.

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Geometric Series Test:

The Geometric Series $1 + r + r^2 + r^3 + \dots + r^n$

if $r \geq 1$ Then given series is diverges

if $r < 1$, Then given series is converges.

Here r is common ratio.

(Pb) Test for convergence of $\sum \frac{1}{2^n + 3^n}$

Sol:- Given $\sum \frac{1}{2^n + 3^n}$

Here $u_n = \frac{1}{2^n + 3^n} = \frac{1}{3^n\left(1 + \frac{2^n}{3^n}\right)}$

Here $v_n = \frac{1}{3^n}$ $\quad 1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^n}$, $\frac{1}{3}$

Here $r = \frac{1}{3} < 1$

$\therefore$ By using Geometric series series

$\sum v_n$ is convergent.

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{1}{2^n + 3^n} \times \frac{3^n}{1}$$

$$= \lim_{n\to\infty} \frac{1}{\cancel{3^n}\left(1 + \frac{2^n}{3^n}\right)} \times \frac{\cancel{3^n}}{1}$$

$$= \lim_{n\to\infty} \frac{1}{1 + \left(\frac{2}{3}\right)^n}$$

$$= \frac{1}{1+0} = 1 = \text{finite.}$$

$\left(\frac{2}{3}\right)^n = (0.66)^n$

$\lim_{n\to\infty} \left(\frac{2}{3}\right)^n = \lim_{n\to\infty} (0.66)^n =$

$\therefore$ By using comparision test $\sum u_n$ & $\sum v_n$ have same property.

Here $\sum v_n$ is convergent

$\therefore$ $\sum u_n$ is also convergent

$\therefore$ The given series $\sum \frac{1}{2^n + 3^n}$ is convergent

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If $\sum u_n$

Cauchy's n^{th} root test :-

and (a) of positive terms. $\lim_{n\to\infty} (u_n)^{1/n} = l$, and

(i) If $l<1$, then $\sum u_n$ is convergent

(ii) If $l>1$, then $\sum u_n$ is divergent.

Problems

1. Test for convergence of the series $\sum \left(\frac{n}{n+1}\right)^{n^2}$ (or) $\sum \left(1+\frac{1}{n}\right)$

Sol:- Given $\sum \left(\frac{n}{n+1}\right)^{n^2}$

$\boxed{\frac{n+1}{n} = (}$

let $u_n = \left(\frac{n}{n+1}\right)^{n^2}$

$\boxed{(x^2)^4 = x^{2\cdot 4} = x^8}$

$$(u_n)^{1/n} = \left[\left(\frac{n}{n+1}\right)^{n^2}\right]^{\frac{1}{n}} = \left(\frac{n}{n+1}\right)^{n^2\cdot\frac{1}{n}}$$

$$= \left(\frac{n}{n+1}\right)^{n} = \left(\left(\frac{n+1}{n}\right)^{-1}\right)^{n}$$

$$= \left(\frac{n+1}{n}\right)^{-n} \qquad \left[\left(1+\frac{1}{n}\right)^{n(-1)}\right] = \left[\left(1+\frac{1}{n}\right)^{n}\right]^{-1}$$

$$= \left(1+\frac{1}{n}\right)^{-n}$$

$$(u_n)^{1/n} = \left[\left(1+\frac{1}{n}\right)^{n}\right]^{-1}$$

$\boxed{\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{n} = e}$

$$\lim_{n\to\infty} (u_n)^{1/n} = \lim_{n\to\infty} \left[\left(1+\frac{1}{n}\right)^{n}\right]^{-1}$$

$$= e^{-1}$$

$e = 2.71 > 1$

$$\lim_{n\to\infty} (u_n)^{\frac{1}{n}} = \frac{1}{e} < 1.$$

$\therefore$ By using Cauchy's n^{th} root test, $\sum u_n$ is convergent

$\therefore$ Given series $\sum \left(\frac{n}{n+1}\right)^{n^2}$ is Convergent.

Raabe's Test :-

If $\sum u_n$ is a series of positive terms $\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}} - 1\right) = l$,

(i) if $l > 1$, then $\sum u_n$ is convergent.

(ii) If $l < 1$, then $\sum u_n$ is divergent.

(iii) if $l = 1$, then test fails.

Pb Discuss the convergence of $\sum \frac{4\cdot 7\cdot 10 \cdots (3n+1)}{1\cdot 2\cdot 3 \cdots n} x^n$

sol:- Given $\sum \frac{4\cdot 7\cdot 10\cdot 13 \cdots (3n+1)}{1\cdot 2\cdot 3 \cdots n} \cdot x^n$

Here $u_n = \frac{4\cdot 7\cdot 10\cdot 13 \cdots (3n+1)}{1\cdot 2\cdot 3\cdot 4 \cdots n} \cdot x^n$

$$u_{n+1} = \frac{4\cdot 7\cdot 10\cdot 13 \cdots (3n+1)\cdot(3n+4)}{1\cdot 2\cdot 3\cdot 4 \cdots n\cdot(n+1)} \cdot x^{n+1}$$

$$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = \lim_{n\to\infty} \cancel{\frac{4\cdot 7\cdot 10 \cdots (3n+1)}{1\cdot 2\cdot 3 \cdots n}} \cdot x^n \times \frac{\cancel{1\cdot 2\cdot 3 \cdots} n(n+1)}{\cancel{4\cdot 7\cdot 10 \cdots (3n+1)}(3n+4)} \frac{1}{x^{n+1}}$$

$$= \lim_{n\to\infty} \frac{\cancel{x^n}\cdot(n+1)}{\cancel{x^n}\cdot x\cdot(3n+4)} \longrightarrow 1$$

$$= \lim_{n\to\infty} \frac{1}{x}\cdot \frac{\cancel{n}\left(1+\frac{1}{n}\right)}{\cancel{n}\left(3+\frac{4}{n}\right)}$$

$$= \frac{1}{x}\cdot \frac{(1+0)}{(3+0)}$$

$$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = \frac{1}{3x}$$

$\therefore$ By using D'Alembert's ratio test, if $\frac{1}{3x} > 1$, then $\sum u_n$ is convergent, if $\frac{1}{3x} < 1$, then $\sum u_n$ is divergent

if $\frac{1}{3x} = 1$, then Test fails.

put $\frac{1}{3x} = 1$ in

put $x = \frac{1}{3}$ in u_n (1)

Here $\frac{u_n}{u_{n+1}} = \frac{1}{x} \cdot \frac{(n+1)}{(3n+4)}$

$$= \frac{1}{\frac{1}{3}} \cdot \frac{n+1}{3n+4}$$

$$= \frac{3(n+1)}{3n+4}$$

$$= \frac{\cancel{3n}\left(1+\frac{1}{n}\right)}{\cancel{3n}\left(1+\frac{4}{3n}\right)}$$

$$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = \lim_{n\to\infty} \frac{\left(1+\frac{1}{n}\right)}{\left(1+\frac{4}{3n}\right)}$$

$$= \frac{1+0}{1+0}$$

$$\therefore \lim_{n\to\infty} \frac{u_n}{u_{n+1}} = 1$$

$\therefore$ D' Alemberts ratio test fails.

using Raabes Test.

$$\frac{u_n}{u_{n+1}} = \frac{3(n+1)}{3n+4}$$

$$\frac{u_n}{u_{n+1}} - 1 = \frac{3n+3}{3n+4} - 1$$

$$= \frac{\cancel{3n}+3-\cancel{3n}-4}{3n+4}$$

$$\frac{u_n}{u_{n+1}} - 1 = \frac{-1}{3n+4}$$

$$n\left(\frac{U_n}{U_{n+1}}-1\right)=\frac{-n}{3n+4}$$

$$\lim_{n\to\infty} n\left(\frac{U_n}{U_{n+1}}-1\right)=\lim_{n\to\infty}\left[\frac{-n}{n\left(3+\frac{4}{n}\right)}\right]$$

$$=\lim_{n\to\infty}\left(\frac{-1}{3+\frac{4}{n}}\right)$$

$$=\frac{-1}{3+0}$$

$$=-\frac{1}{3}<1$$

$\therefore$ By using Raabe's test $\sum u_n$ is divergent.

$\therefore$ Given series $\sum \frac{4\cdot 7\cdot 10\cdots(3n+1)}{1\cdot 2\cdot 3\cdots n}x^n$ is divergent.

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Leibnitz Test :-

The Alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}u_n$ = $u_1 - u_2 + u_3 - u_4 + \cdots + (-1)^{n-1}u_n$ is converges if

(i) $u_n \geq u_{n+1}$ (ii) $\lim_{n\to\infty} u_n = 0$

① Examine the convergence of the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$

Sol:- Given $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\cdots+(-1)^{n-1}\frac{1}{n}+\cdots$

Here $u_n = \frac{1}{n}$

Given given series $\sum(-1)^{n-1}.u_n$ is alternating series

$u_n = \frac{1}{n}$, $u_{n+1} = \frac{1}{n+1}$

$u_1 = \frac{1}{1} = 1$

$u_2 = \frac{1}{2} = 0.5$

$u_3 = \frac{1}{3} = 0.33$

$u_4 = \frac{1}{4} = 0.25$

$u_1 > u_2 > u_3 > u_4 > \cdots > u_n > u_{n+1}$

$\lim_{n\to\infty} u_n = \lim_{n\to\infty}\frac{1}{n} = \frac{1}{\infty} = 0$

∴ Two conditions of Leibnitz Test satisfied.

∴ Given alternating series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$ is convergent.

② Test for convergence of $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{2n-1}$.

$1-\frac{2}{3}+\frac{3}{5}-\frac{4}{7}$

Sol:- Given $\sum(-1)^{n-1}\frac{n}{2n-1} = \sum(-1)^{n-1}.u_n$

Given series an a

$u_n = \frac{n}{2n-1}$, $u_{n+1} = \frac{n+1}{2n+1}$

Given series is an alternating series

$u_1 = \frac{1}{1} = 1$

$u_2 = \frac{2}{3} = 0.66$

$u_3 = \frac{3}{5} = 0.6$

$u_4 = \frac{4}{7} = 0.57$

$u_1 > u_2 > u_3 > u_4 \cdots u_n > u_{n+1} \cdots$

$$\therefore \lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{n}{2n-1} = \cancel{\lim_{n\to\infty} \frac{n}{n(2-\frac{1}{n})}}$$

$$\lim_{n\to\infty} u_n = \frac{1}{2} \neq 0$$

$\therefore$ second condition of leibnitz test fails

$\therefore$ Giving Alternating series $\sum (-1)^{n-1} \frac{n}{2n-1}$ is divergence.

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Absolutely convergence :- Σu_n is a series of positive terms. If Σu_n is convergent and $\Sigma |u_n|$ is also convergent, then Σu_n is said to be Absolutely convergent.

Conditionally convergence :- If Σu_n is a series of positive terms. If Σu_n is convergent and $\Sigma |u_n|$ is divergent then Σu_n said to be conditionally convergence.

Pb.) Test whether the series $\Sigma \frac{(-1)^n}{n\sqrt{n}}$ converges absolutely or not.

Sol:- Given $\Sigma \frac{(-1)^n}{n\sqrt{n}} = \Sigma u_n$ is an alternating series.

Here $u_n = \frac{(-1)^n}{n\sqrt{n}} = (-1)^n \frac{1}{n\sqrt{n}}$ is alternating series

Let $v_n = \frac{1}{n\sqrt{n}}$ $\quad v_{n+1} = \frac{1}{(n+1)\sqrt{n+1}}$

$v_1 = \frac{1}{1\sqrt{1}} = 1$

$v_2 = \frac{1}{2\sqrt{2}} = 0.353$

$v_3 = \frac{1}{3\sqrt{3}} = 0.192$

$v_4 = \frac{1}{4\sqrt{4}} = 0.125$

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$\therefore v_1 > v_2 > v_3 > \cdots v_n > v_{n+1}$

$$\lim_{n\to\infty} v_n = \lim_{n\to\infty} \frac{1}{n\sqrt{n}}$$

$$= \frac{1}{\infty} = 0.$$

$\therefore$ Two conditions of Leibnitz Test satisfy

$\therefore$ Given alternating series $\sum \frac{(-1)^n}{n\sqrt{n}}$ is convergent

(i) $\therefore \sum u_n$ is convergent; $u_n = \frac{(-1)^n}{n\sqrt{n}}$

(ii) $|u_n| = \frac{1}{n\sqrt{n}}$

$$\sum |u_n| = \sum \frac{1}{n\sqrt{n}}$$

$$|u_n| = \frac{1}{n\sqrt{n}}$$

$$v_n = \frac{1}{n.\sqrt{n}} = \frac{1}{n^{3/2}} \quad \left[\sum \frac{1}{n^p}\right]$$

Here $p = \frac{3}{2} > 1$,

$\therefore$ By using p-series Test, $\sum v_n$ is convergent.

$$\lim_{n\to\infty} \frac{|u_n|}{v_n} = \lim_{n\to\infty} \frac{\frac{1}{n\sqrt{n}}}{\frac{1}{n\sqrt{n}}} = 1 = \text{finite}$$

$\therefore$ By using Comparison test, $\sum |u_n|$ and $\sum v_n$ have same property,

Here $\sum v_n$ is convergent.

$\therefore \sum |u_n|$ is also convergent

$\therefore \sum u_n$ is convergent and $\sum |u_n|$ is also convergent

$\therefore$ Given series is $\sum \frac{(-1)^n}{n\sqrt{n}}$ is Absolutely Convergence.

Ratio :-

) $\sum \frac{1}{2^n+3^n}$

Sol:- Given $\sum \frac{1}{2^n+3^n}$

Here $U_n = \frac{1}{2^n+3^n} = \frac{1}{3^n\left(1+\frac{2^n}{3^n}\right)}$

Here $V_n = \frac{1}{3^n}$

Here $r = \frac{1}{3} < 1$

$\therefore$ By using geometric series $\sum V_n$ is convergent

$$\lim_{n\to\infty} \frac{U_n}{V_n} = \lim_{n\to\infty} \frac{1}{2^n+3^n} \times \frac{3^n}{1}$$

$$= \lim_{n\to\infty} \frac{1}{\cancel{3^n}\left(1+\frac{2^n}{3^n}\right)} \times \frac{\cancel{3^n}}{1}$$

$$= \frac{1}{1}$$

$$= 1 = \text{finite}$$

$\therefore$ By using Comparision test $\sum U_n$ & $\sum V_n$ have same property.

$\therefore$ $\sum U_n$ is also convergent.

$\therefore$ The given series $\frac{1}{2^n+3^n}$ is convergent

(4) Cauchy's :-

(1) Given : $\sum \left(\frac{n}{n+1}\right)^{n^2}$

let $u_n = \left(\frac{n}{n+1}\right)^{n^2}$

$$(u_n)^{1/n} = \left[\left(\frac{n}{n+1}\right)^{n^2}\right]^{1/n}$$

$$= \left(\frac{n}{n+1}\right)^{n^2 \times \frac{1}{n}}$$

$$= \left(\frac{n}{n+1}\right)^{n}$$

$$= \left[\left(\frac{n+1}{n}\right)^{-1}\right]^{n}$$

$$= \left(\frac{n+1}{n}\right)^{-n}$$

$$= \left(1+\frac{1}{n}\right)^{-n}$$

$$\lim_{n\to\infty} (u_n)^{1/n} = \lim_{n\to\infty} \left[\left(1+\frac{1}{n}\right)^{n}\right]^{-1}$$

$$= e^{-1}$$

$$\lim_{n\to\infty} (u_n)^{1/n} = \cancel{\lim} \frac{1}{e} < 1$$

∴ By using Cauchy's n^{th} root test, $\sum u_n$ is convergent.

∴ Given series $\sum \left(\frac{n}{n+1}\right)^{2}$ is convergent.

Examine the following series for absolutely convergence or conditional convergence of

$\frac{1}{5\sqrt{2}} - \frac{1}{5\sqrt{3}} + \frac{1}{5\sqrt{4}} + \cdots + (-1)^n \frac{1}{5\sqrt{n}} + \cdots$

<u>Sol:-</u> Given $\frac{1}{5\sqrt{2}} - \frac{1}{5\sqrt{3}} + \frac{1}{5\sqrt{4}} - \cdots + (-1)^n \frac{1}{5\sqrt{n}} + \cdots$

Given series an alternating series

Let $u_n = (-1)^n \frac{1}{5\sqrt{n}}$

Let $v_n = \frac{1}{5\sqrt{n}}$, $v_{n+1} = \frac{1}{5\sqrt{n+1}}$

$v_1 = \frac{1}{5} = 0.2$

$v_2 = \frac{1}{5\sqrt{2}} = 0.141$

$v_3 = \frac{1}{5\sqrt{3}} = 0.115$

$v_4 = \frac{1}{5\sqrt{4}} = 0.1$

$v_1 > v_2 > v_3 > \cdots \quad \underline{v_n > v_{n+1}}$

$\lim_{n\to\infty} v_n = \lim_{n\to\infty} \frac{1}{5\sqrt{n}} = \frac{1}{5(\infty)} = \underline{0}$

$\therefore$ Two conditions of Leibnitz are satisfied

$\therefore \Sigma u_n$ is convergent.

$\Sigma |u_n| = \Sigma \frac{1}{5\sqrt{n}}$

$|u_n| = \frac{1}{5\sqrt{n}}$

$v_n = \frac{1}{\sqrt{n}} = \frac{1}{n^{1/2}} =$

$p = \frac{1}{2} < 1$

$\therefore$ By p-test, Σv_n is divergent.

$\lim_{n\to\infty} \frac{|u_n|}{v_n} = \lim_{n\to\infty} \frac{\frac{1}{5\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \frac{1}{5} =$ finite

$\therefore$ By using comparison test, $\Sigma|u_n|$ and Σv_n both have same property.

Here Σv_n is divergent.

$\therefore$ $\Sigma|u_n|$ is divergent.

$\therefore$ Σu_n is convergent and $\Sigma|u_n|$ is divergent.

$\therefore$ Given series is conditionally convergent.

P3) Test whether the series is absolutely convergent or $\Sigma \frac{\cos n\pi}{n^2+1}$.

$\cos n\pi = (-1)^n$

Sol:- Given $\Sigma \frac{\cos n\pi}{n^2+1} = \Sigma \frac{(-1)^n}{n^2+1}$.

Here $u_n = \frac{(-1)^n}{n^2+1} = (-1)^n \frac{1}{n^2+1}$

$\therefore$ Given series is an alternating series.

$v_n = \frac{1}{n^2+1}$, $v_{n+1} = \frac{1}{(n+1)^2+1}$

$v_1 = \frac{1}{1+1} = \frac{1}{2} = 0.5$

$v_2 = \frac{1}{4+1} = \frac{1}{5} = 0.2$

$v_3 = \frac{1}{9+1} = \frac{1}{10} = 0.01$

$v_4 = \frac{1}{16+1} = \frac{1}{17} = 0.05$

$v_1 > v_2 > v_3 > \cdots$ $\quad$ $v_n > v_{n+1}$

$\lim_{n\to\infty} v_n = \lim_{n\to\infty} \frac{1}{n^2+1} = \frac{1}{\infty} = 0$

$\therefore$ Two conditions of Leibnitz test satisfy.

$\therefore$ Σu_n is convergent.

$\Sigma |u_n| = \Sigma \frac{1}{n^2+1}$

$|u_n| = \frac{1}{n^2+1}$

$V_n = \frac{1}{n^2}$

$P = 2 > 1$

$\therefore$ By ~~Ps~~ series test, ΣV_n is convgt.

$\therefore$ By using comparison both $\Sigma |u_n|$ and ΣV_n have same property.

Hen ΣV_n is convergent.

$\therefore$ $\Sigma |u_n|$ is also convergent.

Hen Σu_n is convergent and $\Sigma |u_n|$ is also convergent

$\therefore$ Given Ser is $\Sigma \frac{\cos n\pi}{n^2+1}$ is Absolutely convergent