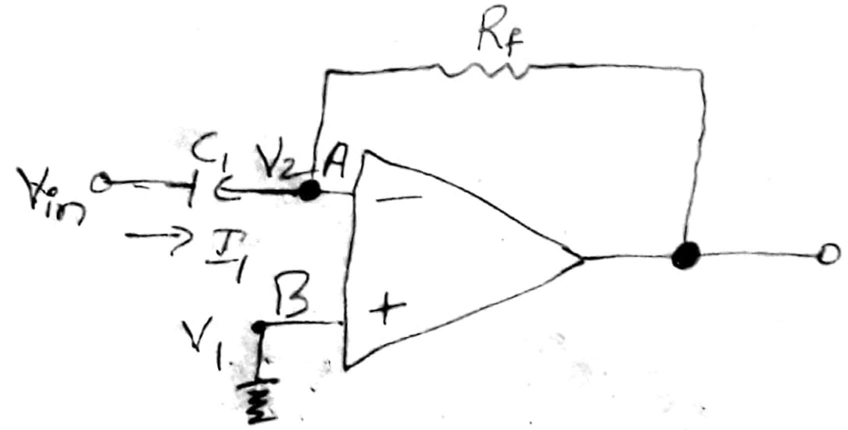
Ideal op-amp Differentiator:-



Due to vertual ground concept $V_1 = V_2 = 0$

Inside the op-amp input is zero

From the input side Differentiator

$$I_1 = C_1 \frac{d}{dt}(V_{in} - V_2) \quad (\because V_2 = 0)$$

$$= C_1 \left[\frac{dV_{in}}{dt}\right] \quad \text{--- (1)}$$

From the output side Differentiator

$$I_1 = \frac{V_2 - V_{out}}{R_f} \quad (\because V_2 = 0)$$

$$I_1 = \frac{-V_{out}}{R_f}$$

Equate the eq ① & ②

$$C_1 \frac{dV_{in}}{dt} = \frac{-V_{out}}{R_f}$$

$$C_1 R_f \frac{dV_{in}}{dt} = -V_{out}$$

$$\boxed{V_{out} = -C_1 R_f \frac{dV_{in}}{dt}}$$

→ Above equation shows that output of differentiator is $C_1 R_f$ times.

→ Negative sign indicates that there is a phase shift of 180° between input & out-put

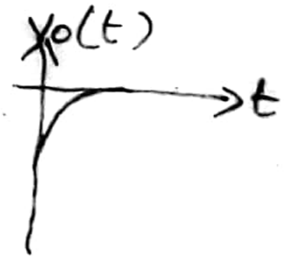
a) Step input signal :-

Let the input wave form is of step type with a magnitude of "A" units is expressed as

$$\boxed{V_{in}(t) = A \text{ for } t \geq 0}$$

Now output of differentiator

$$\boxed{V_o(t) = \frac{-dV_{in}}{dt} = \frac{-d(A)}{dt} = 0}$$



This is because "A" is constant.

Note :- Actually the step input takes limit time to rise from 0 to A volts.

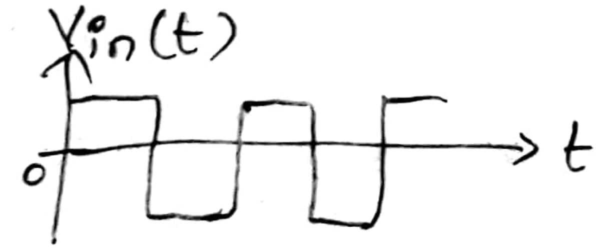
b) Square wave input signal :-

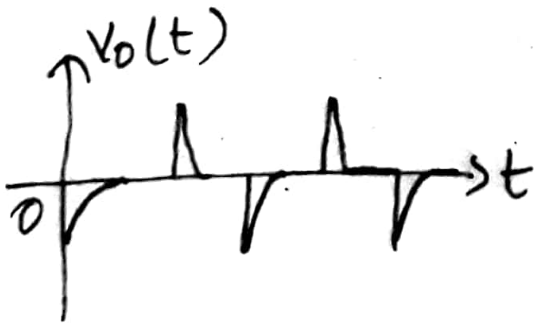
The square wave is made of steps i.e

step of "A" volts from $t=0$ to $t=T/2$ while a step of "$-A$" volts from $t=T/2$ to $t=T$ so

$$\boxed{\begin{aligned} V_{in}(t) &= A \quad ; 0 < t < T/2 \\ &= -A \quad ; T/2 < t < T \end{aligned}}$$

→ so total output in the form of train of impulses or spikes.





<u>Sine wave input</u> :-

Let the input wave form purely sinosoi-dal

$$\boxed{V_{in}(t) = V_m \sin\omega t}$$

V_m = amplitude

T = Time period of wave for "m"

So $V_o(t) = -\dfrac{dV_{in}(t)}{dt}$ for $R_f C_1 = 1$

$$= -\frac{d}{dt}(V_m \sin\omega t)$$

$$V_o(t) = -\frac{d}{dt}(V_m \sin\omega t)$$

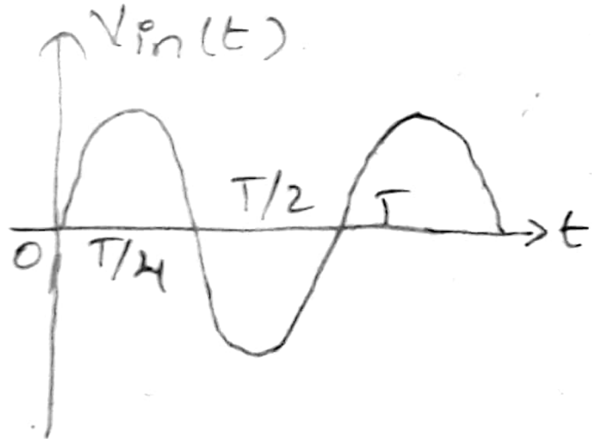
$$V_o(t) = -V_m \cos\omega t \cdot \omega$$

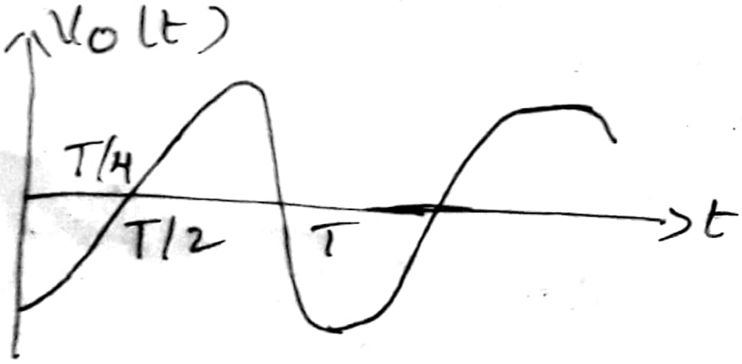
So at $t=0$, $V_o(t) = -V_m \cdot \omega$

$t = \frac{T}{4}$, $V_o(t) = 0$

$t = \frac{T}{2}$, $V_o(t) = +V_m \cdot \omega$

The output of the differentiator is a cosine for a sine wave input





Frequency Response:-

consider the output equation of an ideal differentiator

$$V_o(t) = -R_f C_1 \frac{dV_{in}}{dt}$$

The laplace transform of this equall

$$V_o(S) = -S R_f C_1 V_{in}(S)$$

To get the frequency response, replace "S" by $j\omega$

$$V_o(j\omega) = -j\omega R_f C_1 V_{in}(j\omega)$$

$$\frac{V_o(j\omega)}{V_{in}(j\omega)} = -j\omega R_f C_1$$

Magnitude of gain

$$A = \left| \frac{V_o(j\omega)}{V_{in}(j\omega)} \right| = \left| -j\omega R_f C_1 \right| \quad (\because -j = 1)$$

$$A = \omega R_f C_1$$

$$= 2\pi f R_f C_1$$

at low frequency $(f=0)$ the gain is zero.

and frequency increases gain also increase

$$\boxed{A = \frac{f}{f_a}} \quad \left(\because f_a = \frac{1}{2\pi R_f C_1}\right)$$