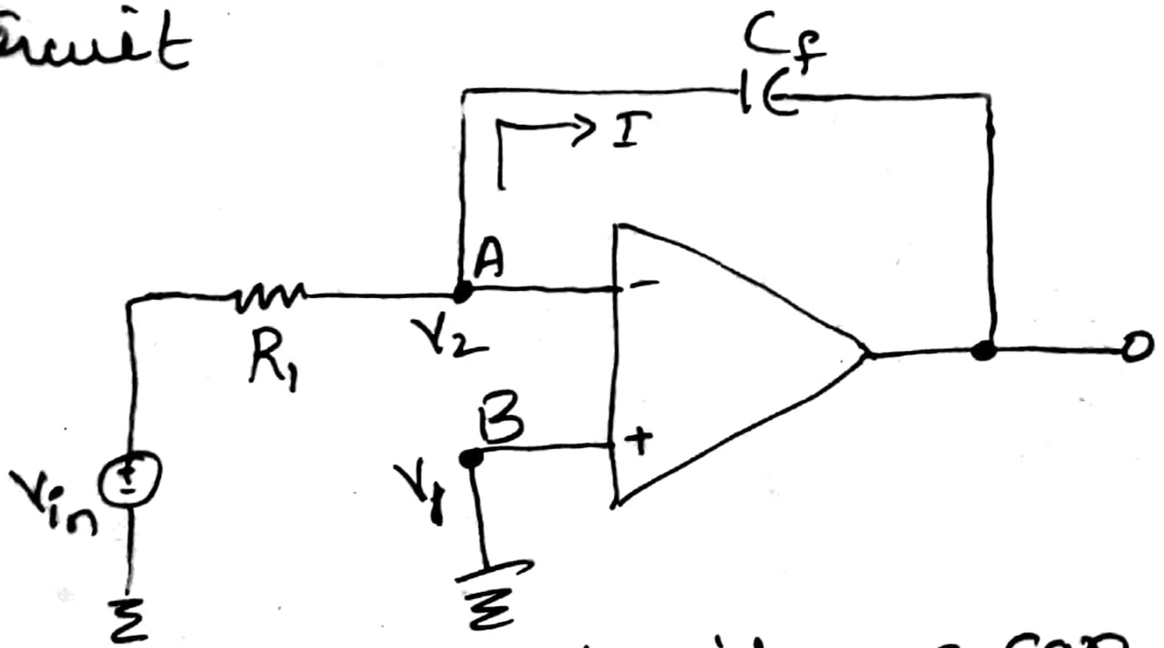
Ideal op-amp Integrator:- ①

→ The integrator circuit can be obtained by exchaning the position of R & C in the basic differentiator circuit



→ From the out put side we can write
→ current through capacitor "C_f"

$$I = C_f \frac{d}{dt}(V_2 - V_{out}) \quad \text{—— ①}$$

~~Applying KCL at node A~~
From the input side we can write

$$I = \frac{V_{in} - V_2}{R_1} \quad \text{—— ②}$$

equating 1 & 2

$$\frac{V_{in} - V_2}{R_1} = C_f \frac{d}{dt}(V_2 - V_{out}) \quad \text{--- (3)}$$

→ As node B is grounded, node A also at grounded potential, from the concept of vertual ground

$(\because V_2 = 0)$

so from eq 3

$$\frac{V_{in}}{R_1} = -C_f \frac{d}{dt}(V_{out})$$

$$-\frac{V_{in}\,dt}{R_1 C_f} = d[V_0]$$

$$dV_0 = -\frac{V_{in}\,dt}{R_1 C_f}$$

(3)

Taking integration on both sides to above equall

$$\int dV_0 = \int_0^t \frac{-1}{R_1 C_f} V_{in}(t) \cdot dt$$

$$V_0 = \frac{-1}{R_1 C_f} \int_0^t V_{in}(t) \cdot dt$$

$$V_0 = \frac{-1}{R_1 C_f} \int_0^t V_{in}(t) dt + V_0(0) \rightarrow (4)$$

where $V_0(0)$ is the constant for integration which indicates initial output voltage.

Input and output waveforms:-

→ let us see the waveforms for various input signals.

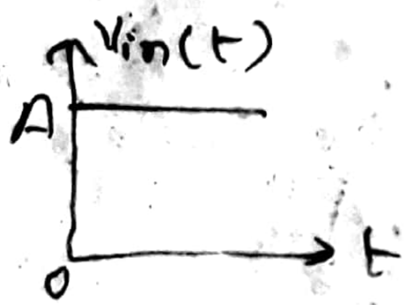
→ assume that the time constant $R_1 C_f > 1$ & initial voltage is $V_0(0) = 0$ Volts.

a) step input signal :-

Let the input waveform is of step type, with magnitude of "A" units

$$V_{in}(t) = A \text{ for } t \geq 0$$

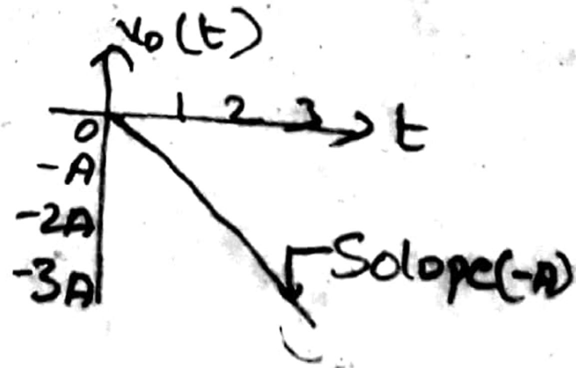
$$V_o(t) = -\int_0^t V_{in}(t)\,dt = -\int_0^t A\,dt$$



$$= A\int_0^t dt$$

$$= -A\left[t\right]_0^t$$

$$\boxed{\therefore V_o(t) = -At}$$



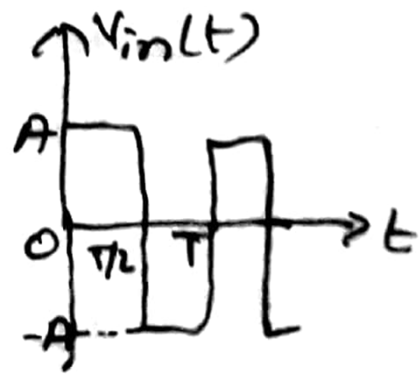
Thus output waveform is a straight line slope of -A where "A" is magnitude of step input

b) Square wave input signal:

→ Let the input wave form is a square wave

→ step or A between time period of 0 to T/2.

→ −A units between time period of T/2 to T

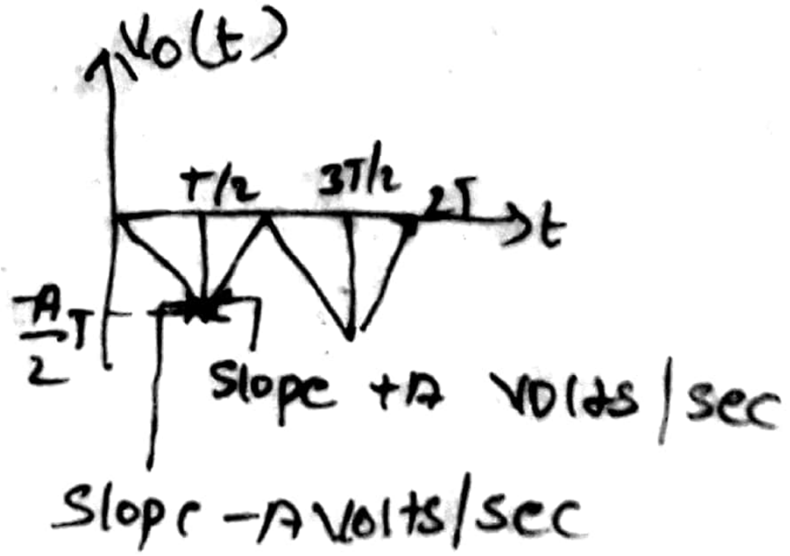


$V_{in}(t) = A \; ; 0 < t < T/2$

$= -A \; ; T/2 < t < T$

$V_o(t) = -At \; ; 0 < t < T/2$

$= +At \; ; T/2 < t < T$



-) Sine wave input signal:-

Let the input wave form is purely sinosoidal.

$$V_{in}(t) = V_m \sin\omega t$$

V_m = amplitude

To find output wave form

$R_1 C_f = 1 \quad V_o(0) = 0$ volts

From the eqn 4 we get

$$V_o(t) = -\frac{1}{R_1 C_f}\int V_{in}\,dt + V_o(0)$$

$$V_o(t) = -\int V_{in}\,dt \quad \text{— (5)}$$

Substitute $V_{in}(t)$ value in eqn 5

$$V_o(t) = -\int V_m \sin\omega t\,dt$$

$$= -V_m \left[\frac{1}{\omega} (-\cos\omega t) \right]$$

$$\boxed{V_0(t) = \frac{-V_m}{\omega} (-\cos\omega t)}$$

Frequency Response of ideal integrator:-

consider the output equation of an ideal integrator

$$V_0(t) = \frac{-1}{R_1 C_f} \int_0^t V_{in} dt$$

$(\because V_0(0) = 0)$

Taking laplace transform to the above eqn

$$V_0(s) = -\frac{1}{sR_1C_f} V_{in}(s)$$

To get the frequency response

replace by $j\omega$

$$V_o(j\omega) = -\frac{1}{j\omega R_1 C_f} V_{in}(j\omega)$$

Hence gain of the integrator

$$\frac{V_o(j\omega)}{V_{in}(j\omega)} = -\frac{1}{j\omega R_1 C_f}$$

To get the frequency obtain the magnitude of the gain

$$A = \left| \frac{V_o(j\omega)}{V_{in}(j\omega)} \right| = \left| -\frac{1}{j\omega R_1 C_f} \right| \quad (\because -\frac{1}{j} = 1$$

$$A = \frac{1}{\omega R_1 C_f} \quad (\because \omega = 2\pi f)$$

$$\boxed{A = \frac{1}{2\pi f R_1 C_f}}$$