Helical spring :-

1) Wire diameter [d]: It is the diameter of wire.

2) Coil diameter (D) : It is the mean dia of coil.

$$D = \frac{D_o + D_i}{2}$$

3) Outer diameter (D_o) :

$$D_o = D + d$$

4) Inner diameter (D_i) :

$$D_i = D - d$$

5) Free length of the spring :→ It is the total length of the spring when it is uncompressed.

$L_f = L_s$ + max compression + clearance b/w adjacent coils.

$$= n_t d + \delta_{max} + \delta_{max}^{0.15}$$

6) Solid length of the spring :

It is the length of spring measured along the axis of the spring when it is compressed state. It is denoted by $L_s = n_t d$.

7) No. of coils : It is the total no. of turns, it is denoted by

$$n_t = n_a + n_{in}$$

8) No. of active coils : These coils contribute the spring motion.

It's limiting value $3 \leq n_a \leq 15$

9) No. of Inactive coils :→

It is denoted by p. The coils do not contribute the spring action.

10) Spring Index [C] :

It is the ratio of mean dia. to the wire dia. It's limiting value is $4 \leq C \leq 12$.

Note :→

1. If the $C < 4$ then it is difficult to the manufacture of spring.
2. If C value exceeding 12 than springs are subjected to buckling.

11) Spring rate [stiffness of spring] (K) :→

It is the ratio of force to unit deflection. It's units is N/mm or N/m.

12) Pitch :→

It is the axle distance b/w two successive coils.

It is denoted by p.

$$P = \frac{L_f}{n_t - 1}$$

13) Helix angle (α) :

It is the angle b/w ground & coil of spring.

$$\alpha = \tan^{-1}\left[\frac{P}{\pi D}\right]$$

14) Figure of merit (Relative cost) :→

$$m = \rho V$$

$$V = A l$$

$$= \frac{\pi}{4} d^2 \times \pi D n_t$$

$$m = \frac{\rho \pi^2 d^2 D n_t}{4}$$

15. End of conditions :-

1. Plane end :

These ends are formed by making the spring two half (the no. of turns is same as n_t)

2. Grounded ends :-

These ends are formed by grinding the ends of the spring with this better transformation of load is possible. (no. of active turns = $n_t - 0.5$)

3. Square end :

These ends are formed by making helix angle as a zero at the spring with this better sitting is possible. (no. of active coils = $n_t - 2$)

4. Grounded and Square end :

These are formed by grinding the ends along with making helix angle zero with this better sitting and transformation load is possible (no. of active coils = $n_t - 2$)

Design for stresses in Helical spring :

$$T = \frac{FD}{2} \text{ N-mm}$$

$$\frac{T}{J} = \frac{\tau_2}{r}$$

$$\tau_2 = \frac{\left(\frac{FD}{2}\right)\left(\frac{d}{2}\right)}{\frac{\pi}{32} d^4}$$

$$\tau_2 = \frac{8FD}{\pi d^3}$$

$$\tau_1 = \frac{F}{A} = \frac{4F}{\pi d^2}$$

$$\tau_{max} = \tau_1 + \tau_2$$

$$= \frac{4F}{\pi d^3} + \frac{8FD}{\pi d^3}$$

$$= \frac{8FD}{\pi d^3}\left[1 + \frac{d}{2D}\right]$$

$$= \frac{8FD}{\pi d^3}\left[1 + \frac{1}{2c}\right] \quad \because c = \frac{D}{d}$$

$$\boxed{\tau_{max} = \frac{8FD}{\pi d^3}[k_s]}$$

$$\boxed{\tau_{max} = \frac{8FD}{\pi d^3} KW} \qquad KW = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

Design for deflection in Helical Spring:

$$\frac{T}{J} = \frac{G\theta}{l}$$

$$\theta = \frac{Tl}{JG}$$

$$= \frac{\frac{FD}{2} \times \pi D n_t}{\frac{\pi}{32} d^4 G}$$

$$\theta = \frac{16FD^2 n_t}{Gd^4}$$

$$\delta = \theta \times \perp^{r} \text{ distance}$$

$$= \frac{16FD^2 n_t}{Gd^4} \times \frac{D}{2}$$

$$\boxed{\delta_{max} = \frac{8FD^3 n_t}{Gd^4} \text{ mm}}$$

Strain energy stored in a spring

$$E = \frac{1}{2} OB \times AB$$

$$= \frac{1}{2}\delta F$$

$$= \frac{1}{2}\left[\frac{8FD^3 n_t}{Gd^4}\right][F]$$

$$\boxed{E = \frac{4F^2 D^3 n_t}{Gd^4} \text{ N-mm}}$$

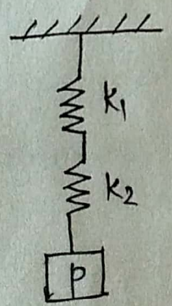
Design for rate of spring :→

$$K = \frac{F}{\delta}$$

$$= \frac{\cancel{F} G d^4}{8 \cancel{F} D^3 n_t}$$

$$\boxed{K = \frac{Gd^4}{8D^3 n_t} \text{ N/mm}}$$

Series in parallel connection :→



$$\frac{F}{K} = \frac{F}{K_1} + \frac{F}{K_2}$$

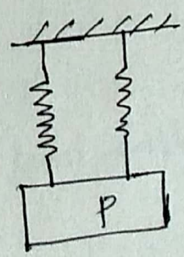
$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$\boxed{K = \frac{K_1 K_2}{K_1 + K_2}}$$

From fig (1)

$$K = \frac{F}{\delta}$$

$$\delta_{12} = \frac{F}{K_{12}}$$



$$F = K_{12}\,\delta$$

$$K\delta = K_1\delta + K_2\delta$$

$$\boxed{K = K_1 + K_2}$$

Design procedure for Helical spring:

1. For the given application estimate the maximum spring force & corresponding required deflection of the spring. In some cases maximum force & stiffness are given.

2. Select suitable material & find out ultimate tensile strength from the table. Calculate the permissible shear stress for the spring wire.

$$\boxed{\tau = 0.3 S_{ut} - 0.5 S_{ut}}$$

3. Assume the suitable value of spring index its limiting value is

$$\boxed{4 \le C \le 12}$$

4. Calculate wahl correction factor. It is denoted by Kw

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

5. Determine the wire dia. by load stress eqn

$$\tau = \left[\frac{8PD}{\pi d^3}\right] K_w$$

$$\tau = \left[\frac{8PC}{\pi d^2}\right] K_w \qquad \left[\because C = \frac{D}{d}\right]$$

6. Determine the coil dia. by the following relations.

$$C = \frac{D}{d}$$

$$\boxed{D = Cd}$$

7. Determine the no. of active coils by load deflection eqn

$$\delta = \frac{8PD^3}{Gd^4} \qquad G = 81370 \text{ N/mm}^2$$

8. Decide the style of end condition for the spring depending upon the configuration of the application determine the no. of inactive coils, total no. of coils.

9. Determine the solid length of the spring by following relations

$$\boxed{L_s = n_t d}$$

Scanned with CamScanner

10. Determine actual deflection of spring by load deflection eqn

$$\delta = \frac{8PD^3 n_a}{Gd^4}$$

11. Assume a gap of 0.52 - 2mm b/w the adjacent coils. When the spring is under action of max load. The total axial length of the spring is calculated by free length of spring.

$$L_f = L_s + \delta_{max} + \text{gap b/w adj. coils}$$

$$= 0.15\ \delta_{max}$$

$$\text{gap b/w coils} = (n_t - 1) \times \text{Assumed gap.}$$

12. Pitch of the spring

$$P = \frac{L_f}{n_t - 1}$$

13. Determine the rate of the spring

$$k = \frac{Gd^4}{8D^3 n_a}$$

14. Prepare any bit specification.

A helical compression i.e., too long compared to the mean dia. of coil then it acts as a flexible column & may buckel at a comparitively low load application the spring should be preferrably designed buckel proof. If it is not buckel proof & a guide (sleeve/Arbor) must be provided.

Ratio of free length to mean dia ≤ 2.6 → guide is not necessary.

If $\frac{L_f}{D} > 2.6$ → guide is necessary

Relation b/w ultimate tensile strength & Exponential:

$$\sigma_{ut} = \frac{A}{d^m}$$

Where A & m are constants.

* It is required to design helical compression spring subject to max. force of 1250 N. The deflection of the spring corresponding to the max. force should be approx 30 mm. The spring index may be taken as 6. The spring is made of cold form steel wire of grade 1. The constants are A & m are 1753 & 0.182 respectively. consider G as 81370 N/mm². The permissible shear stress for string wire should be taken as 50% of uts. Design the spring & calculate i) wire dia. ii) D, iii) n_a, iv) N_t, v) free length of spring, vi) pitch dia, vii) Draw the neat sketech of the spring & show various dimensions of the spring.

Sol. Given data

$F = 1250\,N$

$\delta = 30\,mm$

$C = 6\,mm$

$m = 0.182$

$G = 81370\,N/mm^2$

$\tau = 0.5$

Grade 1 $\sigma_{ut} = 1090\,N/mm^2$

$\tau = 0.5\,\sigma_{ut}$

$= 0.5 \times 1090 = 545$

$\sigma_{ut} = \frac{A}{d^m} = \frac{1753}{d^{0.182}}$

$\tau = 0.5\left(\frac{1753}{d^{0.182}}\right) \rightarrow (1)$

(2) $K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$

$= \frac{4(6)-1}{4(6-1)} + \frac{0.615}{6}$

$= 1.2525$

$\tau = \frac{8PC}{\pi d^2} \times K_w$

$= \frac{8(1250)(6)}{\pi d^2} \times 1.2525 = 0.5\left(\frac{1753}{d^{0.182}}\right)$

$d = 6.16 \simeq 7\,mm$

(3) $\tau = K_w\left(\frac{8PC}{\pi d^2}\right)$

$= 1.2525\left(\frac{8 \times 1250 \times 6}{\pi (7)^2}\right)$

$= 488.583\,N/mm^2$

$\tau_{all} \propto \tau$ permissible

Design is safe.

10. D 4) $C = \frac{D}{d}$

$6 = D \times 7$

11. A $D = 42$ mm

5) $n_a = ?$

$\delta = \frac{8PD^3 n_a}{Gd^4}$

$30 = \frac{8 \times 1250 \times (42)^3 \times n_a}{81370 \times (7)^4}$

$n_a = 7.91 \simeq 8$ mm

12. Pi

6) $n_t = ?$

$= n_a + n_{in}$

$= 8 + 2$

13. De

$= 10$

9) $k = \frac{Gd^4}{8D^3 n_a}$

14. Pre

$k = \frac{81370 \times 7^4}{8 \times 42^3 \times 8}$

me

$k = 41.202$ N/mm

at

des

mc

7) $L_f = L_s + \delta_{max} +$ gap b/w adjacent coils

$(n_t - 1) \times$ Assumed gap

$(10-1) \times 0.5 = 4.5$

$L_f = n_t \times d = 10 \times 7 = 70$ mm

$\delta = 8PD^3 n_a / Gd^4$

$= \frac{8 \times 1250 \times (42)^3 \times 8}{81370 \times (7)^4}$

$= 30.337$

$L_s = 70 + 30.33 + 4.5$

$= 104.83 = 105$ mm

8) $P = \frac{L_f}{n_t - 1} = \frac{105}{10-1}$

$= 11.6$ mm

* A helical compression spring is made of circular wire is subjected to axial force 2000 N. The deflection of the spring should be approx. 5 mm. The spring index can be taken as square and grounded ends. The spring is made of cold form steel wire of grade 1. The constants are 1753 & 0.182 respectively. The permissible shear stress of spring wire should be taken as 30% UTS. Design the spring & calculate required spring rate & actual spring rate.

G1 [$\sigma_{ut} = 1090$ N/mm^2]

Sol: Given data

$P = 2000$ N $C = 5$

$\delta = 5$ mm square & grounded ends [$n_a = 2$]

$\tau = 30\% \, \sigma_{ut} \Rightarrow 0.3 \sigma_{ut}$

$A = 1753$

$m = 0.182$

$\tau = 0.3 f_{ut}$

$f_{ut} = \frac{A}{d^m} = \frac{1753}{d^{0.182}} \Rightarrow \tau = 0.3 \frac{1753}{d^{0.182}}$

1) $\tau_m = 0.3 f_{ut}$

$= 0.3 \times 1090$

$= 327 \ N/mm^2$

2) $k_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$

$= \frac{4\times5-1}{4\times5-4} + \frac{0.615}{5}$

$= 1.3105$

3) $\tau = \left[\frac{8PD}{\pi d^3}\right] k_w$

$0.3 \times \frac{1753}{d^{0.182}} = \left[\frac{8\times2000\times5}{\pi d^2}\right] \times 1.3105$

$d = 9.8 \simeq 10mm$

$\tau_{in} = \frac{8(2000)\times5}{\pi(10)^2} \times 1.3105$

$= 333.7 \ N/mm^2$

4) coil diameter

$C = \frac{D}{d}$

$D = Cd$

$D = 5\times10 = 50mm$

5) $n_a = ?$

$\delta = \frac{8PD^3 n_a}{Gd^4}$

$5 = \frac{8\times2000\times50^3\times n_a}{81370\times10^4}$

$n_a = 2$

6) Total no of active coils

$n_t = n_a + n_{in}$

$= 2+2$

$= 4$

7) $L_f = L_s + \delta_{max}$ + Assumed gap

$L_s = n_t d$

$= 4\times10 = 40$

$\delta_{max} = \frac{8\times2000\times50^3\times2}{81370\times10^4}$

$= 4.9 \simeq 5mm$

gap $= (n_t - 1) \times A.G$

$= (4-1)\times1 = 3$

$L_f \Rightarrow 40+5+3 = 48mm$

8) $P = \frac{L_f}{n_t - 1}$

$= \frac{48}{4-1} = 16$

9) $K_a = \frac{Gd^4}{8D^3 n_a}$

$= \frac{81370\times10^4}{8\times50^3\times2}$

$= 406.85 \ N/mm^2$

10) $K_r = \frac{F}{\delta}$

$= \frac{2000}{5}$

$= 400 \ N/mm$

* It is required to design a helical compression spring subjected to max force of 7.5 N/mm². The mean coil should be 150 mm from the space consideration. The spring rate is 75 N/mm. The spring is made of oil hardend & tempered steel with τ_{ut} of 1250 N/mm². The permissible shear stress for the spring wire is 30% τ_{ut}. Calculate wire dia & no. of active coils.

Sol: Given data

$P = 7.5$ kN

$D = 150$ mm

$K = 75$ N/mm

$\tau_{ut} = 1250$ N/mm²

$\tau = 0.3\,\tau_{ut}$

$d = ?,\ n_a = ?$

G 7 material oil hardend & tempered steel

$$\tau = \left(\frac{8PC}{\pi d^2}\right) K_w$$

$$C = \frac{D}{d} \Rightarrow d = \frac{D}{C}$$

$$\tau = \frac{8PC^3}{\pi (150)^2} K_w \rightarrow (1)$$

$$\tau = 0.3\,\tau_{ut}$$

$$= 0.3 \times 1250 = 375 \text{ N/mm}^2 \rightarrow (2)$$

$$(1) = (2)$$

$$375 = \frac{8PC^3}{\pi (150)^2} K_w$$

$$375 = \frac{8 \times 7.5 \times 10^3 C^3}{\pi (150)^2} K_w$$

$$C^3 K_w = 441.78$$

$$C = \frac{D}{d}$$

$$d = \frac{150}{7.1}$$

$$d = 21.12 \simeq 22 \text{ mm}$$

$$K = \frac{Gd^4}{8D^3 n_a}$$

$$75 = \frac{81370 \times (22)^4}{8(150)^3 \times n_a}$$

$$n_a = 9$$

C	K	KC^3
5	1.311	163.387
6	1.253	270.648
7	1.184	606.208
8	1.213	416.059
7.5	1.197	504.98
7.1	1.210	433.07
7.2	1.200	447.89
7.3	1.203	467.98

Trial and error method of Design of spring

G1 Steel wire

d(mm)	f_{ut} (N/mm^2)
0.3	1720
0.6	1650
1	1570
2	1420
3	1320
4	1250
5	1190
8	1050

From the above table it is observed that f_{ut} of the material is not constant. But it varies with wire diameter i.e., Tesile strength is inversely proportional to diameter.

$$f_{ut} \propto \frac{1}{d}$$

From the eqn of permissible shear strength it can be conclude that

$$\tau = 0.5\, f_{ut}$$

$$\tau \propto \frac{1}{d}$$

Also from the eqn of load stress it can be conclude that

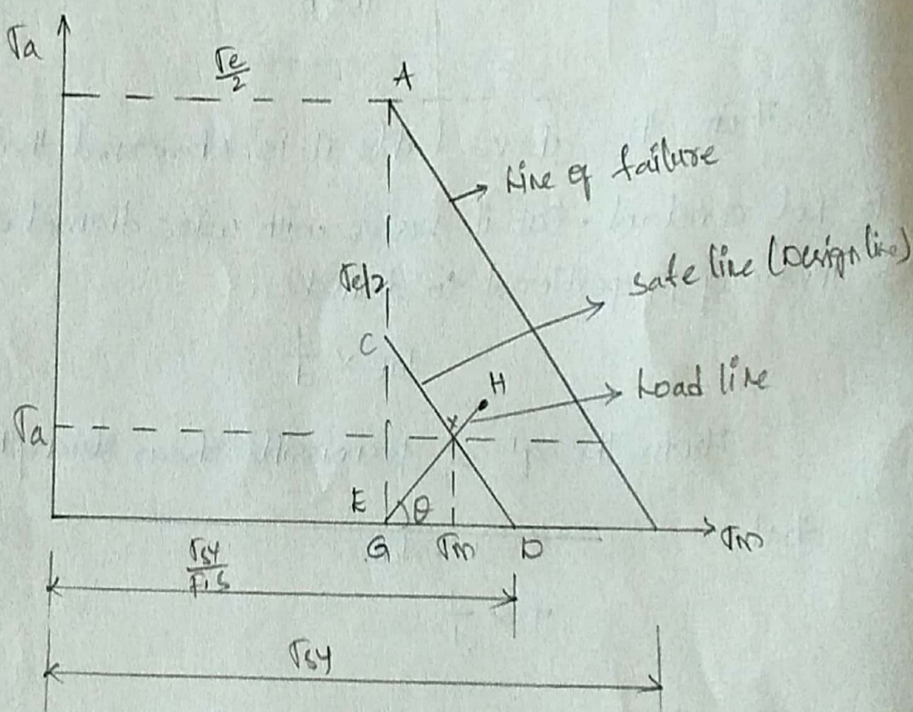
$$\tau = \left(\frac{8PD}{\pi d^3}\right) k_w$$

$$\tau \propto \frac{1}{3}$$

From above relations it is observed that both stresses (Permissible & induced) depends on wire dia. Such problems can be solved by trial & error method it consists of following steps.

1. Assume the suitable wire diameter (d)
2. Find out the corresponding τ_{ut} from the data table & using this value find out permissible shear strength.
3. Find out the induced stress.
4. Check up whether permissible shear stress is more than induced stress. If not increase the wire dia. & repeat above procedure.
5. The above mentioned procedure is repeated till the value of induced stress comes out to be less than permissible stress.

Design for fluctuating loads :→



By similar Δ^{les},

ΔAEB & ΔXFD

$$\frac{AE}{EB} = \frac{XF}{FD}$$

$$\frac{\tau_e/2}{\tau_{sy} - \frac{\tau_e}{2}} = \frac{\tau_a}{\frac{\tau_{sy}}{F.S} - \tau_m}$$

$$\tau_m = k_s\left[\frac{8P_m D}{\pi d^3}\right]$$

$$k_s = \left[1 + \frac{1}{2C}\right]$$

$$\tau_a = k_w\left[\frac{8P_a D}{\pi d^3}\right]$$

$$P_m = \frac{1}{2}\left[P_{max} + P_{min}\right]$$

$$P_a = \frac{1}{2}\left[P_{max} - P_{min}\right]$$

endurance strength for the cold form of steel wire from Grade 1-4

1) $\tau_e = 0.21\,\tau_{ut}$ & $\tau_{sy} = 0.42\,\tau_{ut}$

2) $\tau_e = 0.22\,\tau_{ut}$ & $\tau_{sy} = 0.45\,\tau_{ut}$

oil hardened tempered SW & VW Grade.

* In fatigue testing of spring wire the load changing its magnitude from tension to compression & is passing through zero w.r.t. time. A helical compression spring is subjected purely compressive forces & tensile forces but the spring wire is purely subjected to pulsating shear stress which vary from 0 to τ_e.

Sol: Given data

* A helical tension spring is used to spring balance to measure the weight one end of the spring is attached to rigid support while the other end which is free carries the weights to measured. The max. weight attached to spring balance is 1500 units. The length of the scale should be approx. 100 mm. the spring index can be taken as 6. The spring is made of oil hardened tempered steel wire SW Grade [$G = 81370\ N/mm^2$, $\sigma_{ut} = 1360$, A, m can be taken as 1855 & 0.187] spring wire should be taken as 50% of σ_{ut} design the spring.

Sol: Given data

$W = 1500\ N$

$\delta = 100mm$

$C = 6$

$$\delta = \frac{8PD^3 n_a}{Gd^4}$$

$$\sigma_{ut} = \frac{A}{d^m} = \frac{1855}{d^{0.187}}$$

$$\tau = 0.5\left(\frac{1855}{d^{0.187}}\right) \rightarrow (1)$$

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$= \frac{4(6)-1}{4(6)-1} + \frac{0.615}{6}$$

$$= 1.2525$$

$$\tau = \frac{8PC}{\pi d^2} K_w$$

$$\tau = \frac{8 \times 1500 \times 6}{\pi d^2} \times 1.2525 \rightarrow (2)$$

$$\frac{8 \times 1500 \times 6}{\pi d^2} \times 1.2525 = 0.5\left(\frac{1855}{d^{0.187}}\right)$$

$$d = 6.64 \simeq 7mm$$

$$\tau = K_w\left[\frac{8PC}{\pi d^2}\right]$$

$$= 1.2525\left[\frac{8 \times 1500 \times 6}{\pi (7)^2}\right]$$

$$= 585\ N/mm$$

$$C = \frac{D}{d}$$

$$6 \times 7 = D$$

$$D = 42\ mm$$

* $\sigma_{ut} = \frac{1855}{7^{0.187}} = 1289.17$

$\tau = 0.5 \times 1289.17$

$= 644.58\ N/mm^2$

$n_a = \frac{8Gd^4}{8PD^3}$

$= \frac{100 \times 81370 \times (7)^4}{8 \times 1500 \times (42)^3} = 22$

$k = \frac{Gd^4}{8D^3 n_a} \Rightarrow \frac{81370 \times (7)^4}{8 \times (42)^3 \times 22} = 15\ N/mm$

* It is required to design a helical compression spring subjected to a force of 500 N. The deflection of the spring corresponding to this force is approx. 20 mm. The spring index should be 6. The spring is made of cold drawn steel wire with σ_{ut} of 1000 N/mm^2. The permissible shear stress for the spring wire can be taken as 50% of the σ_{ut} ($G = 81370\ N/mm^2$). Design the spring & calculate.

i) wire dia, ii) mean coil dia, iii) no. of active coils, iv) total no. of coils, v) free length of the spring, & vi) pitch of the coils.

Assume a gap of 1mm b/w adjacent coils under maximum load condition. spring has square & ground ends.

Sol: Given data

$P = 500N$

$\delta = 20mm$

$C = 6$

$\sigma_{ut} = 1000\ N/mm^2$

$\tau = 0.5\ \sigma_{ut}$

$G = 81370\ N/mm^2$

$\delta = \frac{8PD^3 n_a}{Gd^4}$

$\tau = \frac{8PC}{\pi d^2} \times k_w$

$k_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$

$= \frac{4 \times 6-1}{4 \times 6-4} + \frac{0.615}{6} = 1.2525$

$\tau = \frac{8 \times 500 \times 6}{\pi d^2} \times 1.2525$

$0.5 \times 1000 = \frac{8 \times 500 \times 6}{\pi d^2} \times 1.2525$

$d = 4.37 \simeq 5\,mm$

$C = \frac{D}{d}$ $\boxed{d = 5\,mm}$

$D = 6 \times 4.37$

$= 26.22 \simeq 30\,mm$

$$20 = \frac{8 \times 500 (26.22)^3 n_a}{81370 \times (4.37)^4}$$

$n_a = 9.4 \simeq 10$

$\boxed{n_a = 10}$

$n_t = n_a + n_{in}$

$= 10 + 2$

$\boxed{n_t = 12}$

$L_f = L_s + \delta_{max} + gap$

$L_s = n_t d$

$= 12 \times 5 = 60$

$gap = (n_t - 2) \times 1$

$= (12-2) \times 1$

$= 10$

$L_f = 60 + 20 + 10$

$= 90\,mm$

$$\delta_{max} = \frac{8 \times 500 \times (30)^3 \times 10}{81370 \times (5)^4}$$

$= 21.236 \simeq 22\,mm$

$L_f = 60 + 22 + 10 = 92\,mm$

$p = L_f / n_t - 1$

$= 92/12-1$

$\boxed{p = 8.36\,mm}$

* In an automotive plate clutch, six helical compression springs arranged in parallel, provide the axial thrust of 1500N. The springs are compressed by 10mm to provide this thrust force. The springs are identical and the spring index is 6. The springs are made of cold-drawn steel wires with σ_{ut} of 1200 N/mm². The permissible shear stress for the spring wire can be taken as 50% of σ_{ut} ($G = 81370$ N/mm²). Springs have square & ground ends. There should be a gap of 1mm b/w adjacent coils when springs are subjected to the max. force. Design the springs and calculate: i) wire dia, ii) mean coil dia, iii) no. of active coils, iv) total no. of active coils, v) solid length, vi) free length, vii) required spring rate, and viii) actual spring rate.

Sol: Given data

no. of helical springs = 6

end conditions = 12

$P = 1500\,N$

$\delta = 10\,mm$

$C = 6$

$\sigma_{ut} = 1200\,N/mm^2$

$\tau = 0.5\,\sigma_{ut}$

$G = 81370\,N/mm^2$

square & grounded ends

gap of adjacent coils = 1 mm

$$K_w = \frac{4C-1}{4C-4} - \frac{0.615}{C}$$

$$= \frac{4\times6-1}{4\times6-4} - \frac{0.615}{6}$$

$$= 1.2525$$

$$\tau = \frac{8PC}{\pi d^2} K_w$$

$$0.5\times1200 = \frac{8\times1500\times6}{\pi d^2}\times1.2525$$

$$d = 2.8 \simeq 3mm$$

$$\boxed{d = 3mm}$$

$$C = \frac{D}{d}$$

$$D = 6\times3$$

$$\boxed{D = 18mm}$$

$$\delta = \frac{8PD^3 n_a}{Gd^4}$$

$$10 = \frac{8\times250\times(18)^3\times n_a}{81370\times(3)^4}$$

$$\boxed{n_a = 6}$$

$$n_t = n_a + n_{ip}$$

$$= 6+2$$

$$\boxed{n_t = 8}$$

$$L_s = n_t d$$

$$= 8\times3$$

$$\boxed{L_s = 24mm}$$

$$L_f = L_s + \delta_{max} + gap$$

$$gap = (n_t-2)\times1 \qquad \delta_{max} = \frac{8\times250\times18^3\times6}{81370\times(3)^4}$$

$$= (8-2)\times1 \qquad = 10.61\ mm$$

$$= 6$$

$$L_f = 24+11+6$$

$$\boxed{L_f = 41mm}$$

$$K_a = \frac{Gd^4}{8D^3 n_a} = \frac{81370\times(3)^4}{8\times(18)^3\times6}$$

$$\boxed{K_a = 23.54\ N/mm}$$

$$K_r = \frac{F}{\delta} = \frac{250}{10}$$

$$\boxed{K_r = 25\ N/mm}$$

Scanned with CamScanner

$$\tau_m = K_s\left[\frac{8P_m C}{\pi d^2}\right]$$

$$= \left(1+\frac{1}{2\times 6}\right)\left[\frac{8\times 75\times 6}{\pi\times 3^2}\right]$$

$$= 137.93 \simeq 138$$

$$\boxed{\tau_m = 138\ N/mm^2}$$

$$\tau_a = K_w\left[\frac{8P_m C}{\pi d^2}\right]$$

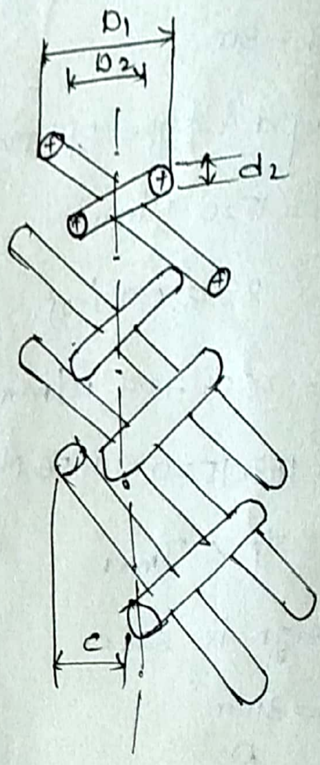
$$= 1.2525\left[\frac{8\times 50\times 60}{\pi(3)^2}\right]$$

$$\boxed{\tau_a = 1063.15\ N/mm^2}$$

$$\frac{334.4/2}{684-\frac{334.4}{2}} = \frac{1063.15}{\frac{684}{F.S}-138}$$

$$\boxed{F.S = 0.1}$$

Design of concentric helical spring:



$$\tau = K\left[\frac{8PD}{\pi d^3}\right]$$

$$\tau_1 = \tau_2$$

$$K_1\left[\frac{8P_1D_1}{\pi d_1^2}\right] = K_2\left[\frac{8P_2D_2}{\pi d_2^3}\right]$$

$$K_1 = K_2$$

$$\frac{P_1D_1}{d_1^3} = \frac{P_2D_2}{d_2^3} \rightarrow (1)$$

$$\delta_1 = \delta_2$$

$$\frac{8P_1D_1^3N_1}{Gd_1^4} = \frac{8P_2D_2^3N_2}{Gd_2^4}$$

$$\frac{P_1D_1^3N_1}{d_1^4} = \frac{P_2D_2^3N_2}{d_2^4} \rightarrow (2)$$

$$N_1d_1 = N_2d_2 \rightarrow (3)$$

$$\frac{P_1D_1^3\cancel{N_1d_1}}{d_1^5} = \frac{P_2D_2^3\cancel{N_2d_2}}{d_2^5} \rightarrow (4)$$

$$\frac{P_1D_1^3}{d_1^5} = \frac{P_2D_2^3}{d_2^5} \rightarrow (5)$$

Divide (4) by (1)

$$\frac{\frac{\cancel{P_1}D_1^3}{d_1^5}}{\frac{\cancel{P_1}D_1}{d_1^3}} = \frac{\frac{\cancel{P_2}D_2^3}{d_2^5}}{\frac{\cancel{P_2}D_2}{d_2^3}} \rightarrow (6)$$

$$\frac{D_1^2}{d_1^2} = \frac{D_2^2}{d_2^2}$$

$$c_1 = c_2 = c$$

From eq. (1)

$$\frac{P_1 d}{d_1^2} = \frac{P_2 d}{d_2^2}$$

$$\frac{P_1}{d_1^2} = \frac{P_2}{d_2^2}$$

$$\frac{P_1}{P_2} = \frac{\frac{\pi}{4} d_1^2}{\frac{\pi}{4} d_2^2} \rightarrow ⑥$$

$$D_1 = \frac{d_1}{2} + C + \frac{d_2}{2} + D_2 + \frac{d_2}{2} + C + \frac{d_1}{2}$$

$$D_1 = 2C + d_1 + d_2 + D_2$$

$$2C = D_1 - D_2 - (d_1 + d_2)$$

$$C = \left(\frac{D_1 - D_2}{2}\right) - \left(\frac{d_1 + d_2}{2}\right) \rightarrow ⑦$$

25

$$C = \frac{d_1 - d_2}{2} \rightarrow ⑧$$

$$⑦ = ⑧$$

$$\frac{d_1 - d_2}{2} = \frac{D_1 - D_2}{2} - \left(\frac{d_1 + d_2}{2}\right)$$

$$D_1 - D_2 = d_1 - \cancel{d_2} + d_1 + \cancel{d_2}$$

$$D_1 - D_2 = 2d_1 \rightarrow ⑨$$

$$Cd_1 - Cd_2 = 2d_1$$

$$d_1(C-2) = Cd_2$$

$$\boxed{\frac{d_1}{d_2} = \frac{C}{C-2}} \rightarrow ⑩$$

* A concentric spring is used as a valve spring in heavy diesel engine. It consist of two helical comp. springs having the same free length & solid length. The composite spring is subjected to max. force of 6,000 N & corresponding deflection is 50 mm max. torsional shear stress induced in spring 800 N/mm² spring index for each spring is 6. Assume same material for the two springs & modulus of rigidity. The diametral clearance b/w the two springs is equal to the diff. b/w their wire dia's. calculate i) axial force transmitted by each spring, ii) d, D of each spring, iii) Na in each spring.

Sol. Given data

2 spring

$P = 6000 N$

$\delta = 50 mm$

$\tau = 800 N/mm^2$

$C = 6$

$2c = d_1 - d_2$

$G = G_1 = G_2$

$$P = P_1 + P_2$$

$$\frac{d_1}{d_2} = \frac{C}{C-2}$$

$$= \frac{6}{6-2} = 1.5$$

$$\frac{P_1}{P_2} = \frac{d_1^2}{d_2^2} \Rightarrow \frac{P_1}{P_2} = (1.5)^2$$

$$P_1 = (1.5)^2 P_2$$

$$(1.5)^2 P_2 + P_2 = 6000$$

$P_2 = 1846.15\ N$

$P_1 = 4153.84\ N$

$d_1 D_1,\ d_2 D_2$

$$\tau = K\left[\frac{8 P_1 C_1}{\pi d_1^2}\right]$$

$d_1 = 9.96\ mm \quad C = \frac{D_1}{d_1}$

$D_1 = 6 \times 9.96$

$D_1 = 59.80\ mm$

$$\tau = K_2\left[\frac{8 P_2 C_2}{\pi d_2^2}\right]$$

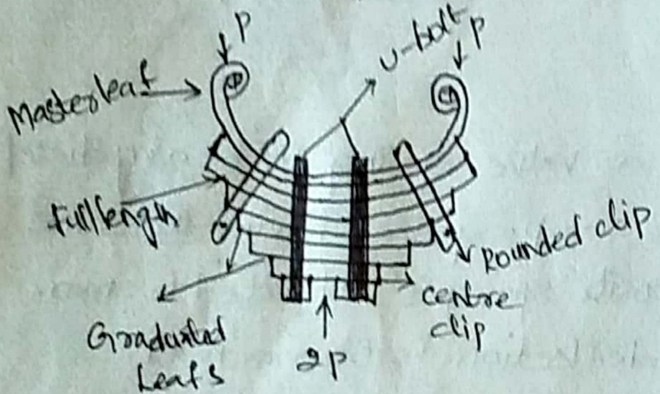
$$800 = 1.2525\left[\frac{8 \times 1846.15 \times 6}{\pi d_2^2}\right]$$

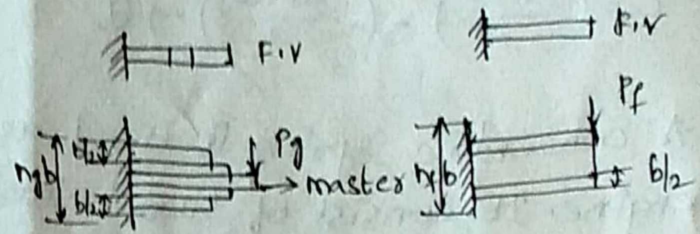
$d_2 = 6.64\ mm \quad C = \frac{D_2}{d_2}$

$D_2 = C d_2$

$D_2 = 39.84\ mm$

Multi Leaf Spring:





$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\sigma_{bg} = \frac{MY}{I}$$

$$= \frac{(P_g L)(t/2)}{\frac{1}{2}[n_g b \times t^3]}$$

$$\boxed{\frac{6 P_g L}{n_g b t^2} = \sigma_{bg}} \rightarrow (1)$$

$$\delta = \frac{P_g L^3}{3EI}$$

$$= \frac{P_g L^3}{3E\left[\frac{1}{12} n_g b t^3\right]}$$

$$\boxed{\delta_g = \frac{6 P_g L^3}{E n_g b t^3}} \rightarrow (2)$$

$$\frac{M}{I} = \frac{\sigma_b}{Y}$$

$$\sigma_{bf} = \frac{MY}{I}$$

$$\sigma_{bf} = \frac{(P_f L)\left(\frac{t}{2}\right)}{\frac{1}{2}[n_f b t^3]}$$

$$\boxed{\sigma_{bf} = \frac{6 P_f L}{n_f b t^2}} \rightarrow (3)$$

$$\delta = P_f L^3 / 3EI$$

$$= \frac{P_f L^3}{3E\left[\frac{1}{12} n_f b t^3\right]}$$

$$\boxed{\delta_f = \frac{4 P_f L^3}{E n_f b t^3}} \rightarrow (4)$$

$$\delta g = \delta f$$

$$\frac{6 P_g L^{\cancel{3}}}{\cancel{E} n_g b t^{\cancel{3}}} = \frac{4 P_f L^{\cancel{3}}}{\cancel{E} n_f b t^{\cancel{3}}}$$

$$\frac{3 P_g}{n_g} = \frac{2 P_f}{n_f}$$

$$\frac{P_g}{P_f} = \frac{2 n_g}{3 n_f}$$

$$\frac{P_g}{P_f} + 1 = \frac{2 n_g}{3 n_f} + 1$$

$$\frac{P_g + P_f}{P} = \frac{3 n_f + 2 n_g}{3 n_f}$$

$$\frac{P}{P_f} = \frac{3 n_f + 2 n_g}{3 n_f}$$

$$\boxed{P_f = \frac{3 n_f P}{3 n + 2 n_g}} \rightarrow 5$$

$$P = P_g + P_f$$

$$P_f = \frac{P - 3 n_f P}{3 n_f + 2 n_g}$$

$$P_f = \frac{P(3 n_f + 2 n_g) - 3 n_f P}{3 n_f + 2 n_g}$$

$$\frac{3 n_f P + 2 n_g P - 3 n_f P}{3 n_f + 2 n_g}$$

$$\boxed{P_f = \frac{2 n_g P}{3 n_f + 2 n_g}} \rightarrow 6$$

subs eq 5 & 6 in eq 1 & 3

$$\sigma_{bg} = \frac{6 P_g L}{n_g b t^2}$$

$$\sigma_{bg} = \frac{6 [2 n_g P] L}{[3 n_f + 2 n_g] n_g b t^2}$$

$$\boxed{\sigma_{bg} = \frac{12 P L}{(3 n_f + 2 n_g) b t^2}} \rightarrow 7$$

$$\sigma_{bf} = \frac{6 P_f L}{n_f b t^2} \qquad \frac{3 n_f P + 3 n_g P - 3 n_f P}{3 n_f + 2 n_g}$$

$$= \frac{18 n_g P_L}{n_g (3 n_f + 2 n_g) b t^2} \qquad \boxed{P_f = \frac{3 n_g P}{3 n_f + 2 n_g}}$$

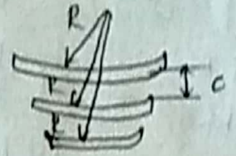
$$\boxed{\sigma_{bf} = \frac{18 P L}{(3 n_f + 2 n_g) b t^2}} \rightarrow 8$$

$$\delta = \delta g = \delta f = \frac{6 P_g L^3}{E n_g b t^3}$$

$$= \frac{6 (2 n_g P) L^3}{(3 n_f + 2 n_g) E n_g b t^3}$$

$$\boxed{\delta = \frac{12 P L^3}{E (3 n_f + 2 n_g) b t^3}} \rightarrow 9$$

Equalised stresses in springs:



$$\sigma_{bg} = \sigma_{bf}$$

$$\frac{6P_gL}{n_g b t^2} = \frac{6P_fL}{n_f b t^2}$$

$$\frac{P_g}{n_g} = \frac{P_f}{n_f}$$

$$\frac{P_g}{P_f} + 1 = \frac{n_g}{n_f} + 1$$

$$\frac{P_g + P_f}{P_f} = \frac{n_g + n_f}{n_f}$$

$$\frac{P}{P_f} = \frac{n}{n_f}$$

$$\boxed{P_f = \frac{Pn_f}{n}} \rightarrow (1)$$

$$P_g = P - P_f$$

$$= P - \frac{Pn_f}{n}$$

$$\boxed{P_g = \frac{Pn - Pn_f}{n}}$$

$$= \frac{P[n - n_f]}{n}$$

$$\boxed{P_g = \frac{Pn_g}{n}} \rightarrow (2)$$

$$C = \delta_g - \delta_f$$

$$= \frac{6P_gL^3}{En_gbt^3} - \frac{4P_fL^3}{En_fbt^3}$$

$$= \frac{2L^3}{Ebt^3}\left[\frac{3P_g}{n_g} - \frac{2P_f}{n_f}\right]$$

$$= \frac{2L^3}{Ebt^3}\left[\frac{3(P\cancel{n_g})}{n\cancel{n_g}} - \frac{2(P\cancel{n_f})}{n\cancel{n_f}}\right]$$

$$C = \frac{2L^3}{Ebt^3}\left[\frac{P}{n}\right]$$

$$\boxed{C = \frac{2PL^3}{Ebt^3n}} \rightarrow (3)$$

$$C = \delta_{gi} + \delta_{fi}$$

$$\delta_{gi} = \frac{6\left(\frac{P_i}{2}\right)L^3}{En_gbt^3}$$

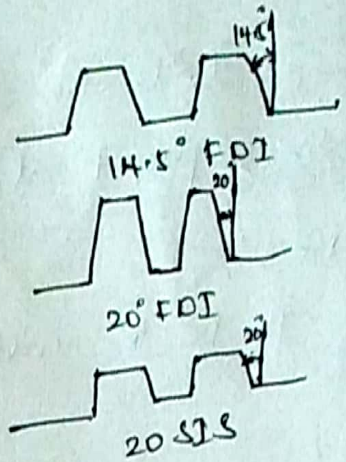
$$\delta_{fi} = \frac{4\left(\frac{P_i}{2}\right)L^3}{En_fbt^3}$$

$$\frac{2PL^3}{Ebt^3n} = \frac{3P_iL^3}{En_gbt^3} + \frac{2P_iL^3}{En_fbt^3}$$

$$\boxed{P_i = \frac{2n_gn_fP}{n(3n_f+2n_g)}} \rightarrow (4)$$

Standard system of Gear tooth:

1) 14.5° full depth Involute system
2) 22° full depth involute system
3) 20° Stub involute system.



Full depth Involute system:

1) 14½° composite system:

The basic rack for the system is composed of straight sides except for the fillet arcs in this system interference occur when the no. of teeth on pinion is less than 32. This system is satisfactory when the no. of teeth on gear is large if the no. of teeth is small & if the gears are made by generating process undercutting is unavoidable.

2) Full depth involute system 22°:

The basic rack for the system consist of straight sides except for the fillet arcs. In the system interference occurs when the no. of teeth on pinion less than 17. The 20° pressure angle system with full depth involute teeth is widely used in practise.

Increasing the pr. angle improves the tooth strength but shortends the duration of contact decreasing the pressure angle requires more no. of teeth on to avoid under cutting.

Note :- 20° pressure angle system has the following advantages over 14.5° pressure angle system.

1. It reduces the risk of under cutting.
2. It reduces the interference due to increased pressure angle tooth becomes slightly wider at the root. This makes the tooth stronger & increasing the load carrying capacity.
3. It has greater length of contact.

The main advantages of 14.5° pressure angle system is it's quiteness of operation.

3. 20° stub involute system:

It is gear system having shorter addendum & shorter deddendum the interfering portion of the teeth i.e., a part of addendum is thus removed. Therefore these teeth have still smaller interference. This also reduces the under cutting. In this system minimum no. of teeth on pinion to avoid interference is 14.

Teeth profile:

1) cycloidal tooth profile
2) Involute tooth profile.

cycloidal tooth profile:

Advantages of Involute teeth profile:

1. For the involute gears as the centre distance for a pair of involute gears can be varid with in the limits without changing the velocity ratio but this is not true for cycloidal gears which requires exact centre distance to maintain constant V.R.

2. In Involute gears the pressure angle from the start of engagement to end of engagement remains constant. It is necessary for smooth running and less wear in gears. But in cycloidal gears the pr. angle is max. at the begining of the engagement reduces to zero at pitch point & start increasing become maximum at the end of engagement. this results less smooth running.

3. The face & flank of involute gear teeth are generated by single curve (involute) where as in cycloidal gears double curve (Hypocycloid, epicycloid) are required for the face & flank.

4. The involute teeths are easily manufactured than cycloidal teeth.

Advantages of cycloidal gears:

1. Since the cycloidal teeth have wider flank therefore cycloidal gears stronger than involute gears for the same pitch point. Due to this reason cycloidal teeths are preferred for cast teeth.

2. In cycloidal gears contact takes place b/w convex flank & concave surface where as in involute gears the convex surfaces are in contact for this condition less wear in cycloidal geare. as compared to involute gears.

3. In cycloidal gears interference may not ocur

Interference in Involute gears

Diagram

If the radius of addendum circle of pinion is increased to O_1 to N. The pt. of contact L will move from L to N. When again further increasing in radius of addendum wheel the pt. of contact L will be inside of the base circle.

Beam strength of tooth gear (Lewis eqn):

$$\sigma_w = \frac{My}{I}$$

$$M = W_T h$$

$$y = t/2$$

$$I = \frac{bt^3}{12}$$

$$\sigma_w = \frac{W_T h \times \frac{t}{2}}{\frac{bt^3}{12}\,6}$$

$$= \frac{W_T h 6}{bt^2} \Rightarrow \boxed{\sigma_w = \frac{W_T h 6}{bt^2}} \; N/mm^2$$

$$W_T = \frac{\sigma_w b t^2}{6h}$$

$t = x p_c \quad h = K p_c$

$$W_T = \sigma_w \frac{b x^2 p_c^{\cancel{2}}}{6 K \cancel{p_c}}$$

$$W_T = \sigma_w b \frac{x^2 p_c}{6K}$$

$$W_T = \sigma_w b Y \quad \therefore Y = \frac{x^2 p_c}{6K}$$

Y is Lewis form factor or tooth form factor.

$$= \frac{t^2}{P_c^2 6} \times \frac{P_c}{h}$$

$$Y = \frac{t^2}{6hP_c}$$

$$Y = 0.124 - \frac{0.684}{T} \rightarrow 14\tfrac{1}{2} \text{ composite \& FDI}$$

$$Y = 0.154 - \frac{0.912}{T} \rightarrow 20^\circ \text{ FDI}$$

$$Y = 0.175 - \frac{0.846}{T} \rightarrow 20^\circ \text{ stub}$$

The Analysis of bending stresses of gear tooth is calculated by lewis eqn it is used to determine the load carrying capacity of tooth gear by considering each tooth has a cantilever beam & it is subjected to tangential load (W_T) at the end of the tooth. the lewis eqn is derived by following assumptions.

1) Neglecting radial load which causes compressive stresses & tooth gear.

2) stress concentration is neglected.

3) Tangential load is uniformly is distributed on entire width of tooth.

4) For analysis the bending stress only one pair of teeth is in contact & takes place total load.

construct a parabolic curve to get exact critical cross-sectional area (the area subjected to max. bending stress). The magnitude of the σ_b is calculated for bending moment eqn.

Permissible working stress for gear teeth in lewis eqn:-

$$\boxed{\sigma_w = \sigma_o C_v}$$

$C_v = \frac{3}{3+v}$ → ordinary cut gear operating at less than or equal to 12.5 m/s

$C_v = \frac{4.5}{4.5+v}$ → for carefully gear cut operating at equal to 12.5 m/s

$C_v = \frac{6}{6+v}$ → for accurately gear cut operating at less than or equal 20 m/s

$C_v = \frac{0.75}{0.75+\sqrt{v}}$ → for precision cut gear operating at equal to 20 m/s.

Dynamic tooth load:

$$\omega_D = \omega_T + \frac{21v\left[bc+\omega_T\right]}{21v+\sqrt{bc+\omega_T}}$$

$$C = \frac{Kc}{\frac{1}{E_p}+\frac{1}{E_G}}$$

Where V = pitch line velocity

C = dynamic load factor

k = factor depends on smooth of tooth gearing.

$k = 0.101$ for $14.5°$ composite full depth involute (tooth gearing) system

$k = 0.111$ for $20°$ full depth involute system.

$k = 0.115$ for $20°$ stub system.

Where E_p = Young's modulus for pinion

E_G = Young's modulus for gear

b = width of tooth in mm

ω_T = tangential load in N

This load is calculated due to following reason.

i) Inaccuracies tooth space

ii) Irregularities of tooth profile

iii) Deflection of teeth due to dynamic load.

Static load:

This static load is also known as 'beam strength (or) endurance strength' of tooth. It is obtained by lewis eqn by substituting the flexural endurance limit (or) elastic limit, in place of permissible working stress.

$$\boxed{\omega_s = \sigma_e b P_c Y}$$

Wear tooth load:

$$\boxed{\therefore \omega_w = b D_p Q K}$$

$$Q = \text{ratio factor} = \frac{2 V.R}{V.R \pm 1}$$

D_p = pitch circle diameter

K = load stress factor

$$K = \frac{\sigma_{es}^2 \sin\phi}{1.4}\left[\frac{1}{E_p} + \frac{1}{E_G}\right]$$

→ causes of gear tooth failure:

1) Bending failure:

Every gear tooth act as a cantilever if the total repeatative dynamic load acting on gear tooth is greater than a beam strength of gear tooth, then the gear tooth is going to failure in bending. In order to avoid such failure the module and face width of gear is adjusted, so that beam strength is greater than dynamic load.

2) Pitting:

It is the surface fatigue failure which occurs due to many reputations of contact stresses, the failure occurs when the surface contact stresses are higher than endurance limit of material. In order to avoid pitting dynamic load b/w gear tooth should be less than wear load.

3) Scoring:

The excessive heat generated when there is excessive surface pressure, high speed, supply of lubrication is fail. It is a stick and slip phenomena in which alternate shearing and welding takes place. This type of failure is avoided by properly designing the parameters such as speed, pressure, proper flow of lubricants so take temp. at rubbing surfaces is within permissible limit.

Design procedure for spur gear:

1. Find the tangential tooth load is obtained by power transmitted by gear & pitch line velocity by using following relation.

$$W_T = \frac{P}{V} C_s$$

Where
P = power in watts
V = pitch line velocity $\frac{\pi D N}{60}$
C_s = service factor (C from the data table by working hrs)

2. Apply the lewis eqn as follows

$$W_T = \sigma_w b P_c y$$

$$\sigma_w = \sigma C y$$

Note:
1) The lewis eqn is applied only to the weaker of the two wheels (either pinion or gear)
2) When both pinion & gear made of same material then pinion is the weaker.
3) When the pinion & gear are made of diff. materials then the product is known as deciding factor. ($\sigma_w y$) or (σy) the lewis eqn is used to that wheel for which deciding factor is less.
4) Face width (b) may be taken as

$3P_c - 4P_c$ (or) $9.5\,M - 12.5$ Module → cut teeth

$2P_c - 3P_c$ (or) $6.5\,M - 9.5\,M$ → cast teeth.

3. calculate dynamic load

$$W_D = \frac{21v\left[bc + w_T\right]}{21v + \sqrt{bc + w_T}} + w_T$$

$$w_T = \frac{P}{v} \text{ (neglect the cs)}$$

4. Find the static load by using the relation.

$$\boxed{w_s = \sigma_e b P_c Y}$$

For safety purpose (against breakage)

w_s should be greater than w_d.

5. Find the wear load by using eq^n.

$$w_w = b D_p Q K$$

$$W_w > w_D.$$

2. Dynamic tooth load for helical gear:

$$W_D = W_T + \frac{21v\left[bc\cos^2\alpha + W_T\right]\cos\alpha}{21v + \sqrt{bc\cos^2\alpha + W_T}}$$

where b face width in mm

c deformation or dynamic factor in N/mm

$$C = \frac{Ke}{\frac{1}{E_P} + \frac{1}{E_G}}$$

Where k is a factor depends on the no. of teeth same as in spur gear

e is tooth error in mm

3. Static load for Helical gear:

$$W_S = \sigma_e b P_c y'$$

4. The max. or limiting wear load:

$$W_W = \frac{b D_P Q K}{\cos^2\alpha}$$

Where Q is a ratio factor.

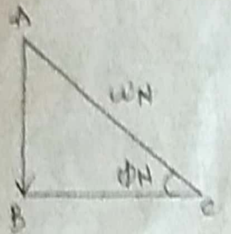
$$Q = \frac{2VR}{VR \pm 1} \qquad VR = \frac{T_G}{T_P} \text{ or } \frac{D_G}{D_P} \text{ or } \frac{N_P}{N_G}$$

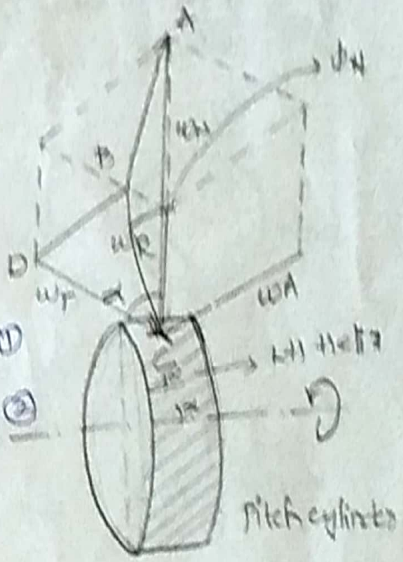
K is load stress factor

$$= \frac{(\sigma_{es})^2 \sin\phi_N}{1.4}\left[\frac{1}{E_G} + \frac{1}{E_P}\right]$$

Where ϕ_N is normal pressure angle.

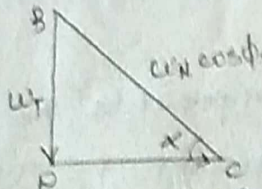
Force Analysis of Helical gear:





$\sin\phi_N = \frac{w_R}{w_N} \Rightarrow w_R = w_N \sin\phi_N \rightarrow (1)$

$\cos\phi_N = \frac{BC}{w_N} \Rightarrow BC = w_N \cos\phi_N \rightarrow (2)$



$\sin\alpha = \frac{w_A}{w_N \cos\phi_N} \Rightarrow w_A = w_N \cos\phi_N \sin\alpha \rightarrow (3)$

$\cos\alpha = \frac{w_T}{w_n \cos\phi} \Rightarrow w_T = w_n \cos\phi_N \cos\alpha \rightarrow (4)$

Divide (3) by (4)

$$\frac{w_A}{w_T} = \frac{\cancel{w\cos\phi_N} \sin\alpha}{\cancel{w\cos\phi_N} \cos\alpha}$$

$$\boxed{w_A = w_T \tan\alpha} \rightarrow (5)$$

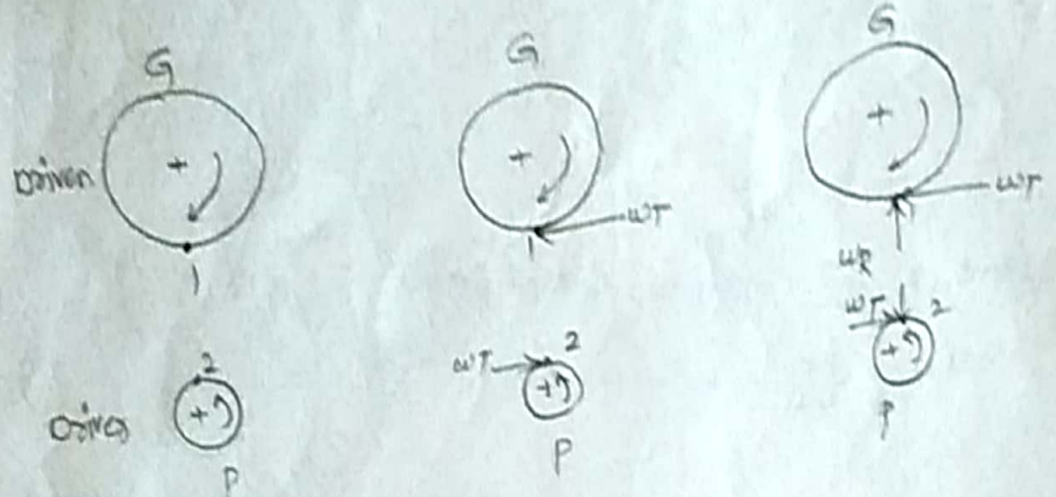
Divide (1) by (4)

$$\frac{w_R}{w_T} = \frac{\cancel{w_N} \sin\phi_N}{\cancel{w_N} \cos\phi \cos\alpha}$$

$$\boxed{w_R = w_T \left[\frac{\tan\phi_N}{\cos\phi}\right]} \rightarrow (6)$$

Scanned with CamScanner

$$P = \frac{2\pi NT}{60}$$



(To know the directions of components) the following points are important to decide the direction of components:-
which element is driver and which element is driven.

1. Which element is driver and which direction (clockwise or Anticlockwise)
2. Driver rotates in which direction
3. What helix are there.

Assume pinion has a driver and gear as a driven. Driver rotates in a clockwise direction and follower rotates in a anticlockwise direction.

Tangential load (W_T) :→

The direction of Tangential for driver (pinion) acts opposite to the rotation of pinion.

For gear (follower) the W_T acts in the same direction of gear.

Radial load (W_R) : [load on bearing]

A radial load for driving gear acts towards the centre of rotation (Pinion)

For driven gear acts towards the centre of rotation.

Axial load or End thrust load :→

1. Select right hand for right hand helix. And
2. Select left hand for LH helix.
3. Keep the (show) fore finger in direction of rotation the thumb direction for driving gear. indicates the direction of axial load for driving gear.

For driven gear opposite direction to the driving gear.

Design of Bevel gear:→

Terms used in Bevel gear:-

1. Pitch cone :→ It is a cone containing the pitch element of the teeth.
2. Cone centre :→ It is the apex of the pitch cone the point where axis of two meeting gears intersect each other.
3. Pitch angle :→ It is the angle made by pitch line with the axis of the shaft. It is denoted by θ_p.
4. Cone distance :→ It is the length of the pitch cone element. It is also called as pitch cone radius. It is denoted by pitch cone radius 'r'.

mathematically

$$r = \frac{\text{Pitch radius}}{\sin \theta_p}$$

5. Addendum angle: It is the angle subtended by the addendum of the tooth with the cone centre. It is denoted by α.

$$\alpha = \tan^{-1}\left(\frac{a}{\text{cone distance}}\right)$$

6. Deddenum angle: It is the angle subtended by deddenum of the tooth at the cone centre. It is denoted by β.

$$\beta = \tan^{-1}\left[\frac{d}{\text{cone distance}}\right]$$

7. ~~Phase~~ Face angle :→ It is the angle subtended by ~~Phase~~ face of the tooth at cone centre. It is denoted by ϕ.

$$\phi = \text{Pitch angle} + \text{Addendum angle}$$

8. Root angle :→ It is the angle subtended by root of the tooth at cone centre. It is denoted by θ_R.

θ_R = Pitch angle − dedendum angle

9. Crown height : It is the distance of the crown point from the cone centre ~~(back of boss)~~ parallel to the axis of the gear. It is denoted by H_e.

10. Outside addendum or Addendum cone diameter :→ It is the max. cone dia. of teeth of gear. It is equal to $D_o = D_p + 2a\cos\theta_p$

D_p = Pitch circ dia.

11. Inside dia or Dedendum cone dia :→ The inside is the minimum dia. of teeth of the gear. It is denoted by D_d.

$D_d = D_p - 2d\cos\theta_p$

Standard proportions of Bevel gear :→

1) $a = 1M$
2) $d = 1.2M$
3) $c = 0.2M$
4) Working depth $= 2M$
5) Thickness of the tooth $= 1.5708M$
6) Slant height $[L] = 27.54m$

Equalent no. of teeth on Bevel gear :→

$$T_E = \frac{2R_B}{m}$$

where $R_B = R\sec\theta_p$

$T_E = T\sec\theta_p$

θ_p = Pitch angle

Pitch angle for Bevel gear:

$$\theta_{P1} = \tan^{-1}\left[\frac{\sin\theta_s}{VR + \cos\theta_s}\right] \quad \text{for pinion}$$

$$\theta_{P2} = \tan^{-1}\left[\frac{\sin\theta}{\frac{1}{VR} + \cos\theta_s}\right] \quad \text{for gear}$$

Where

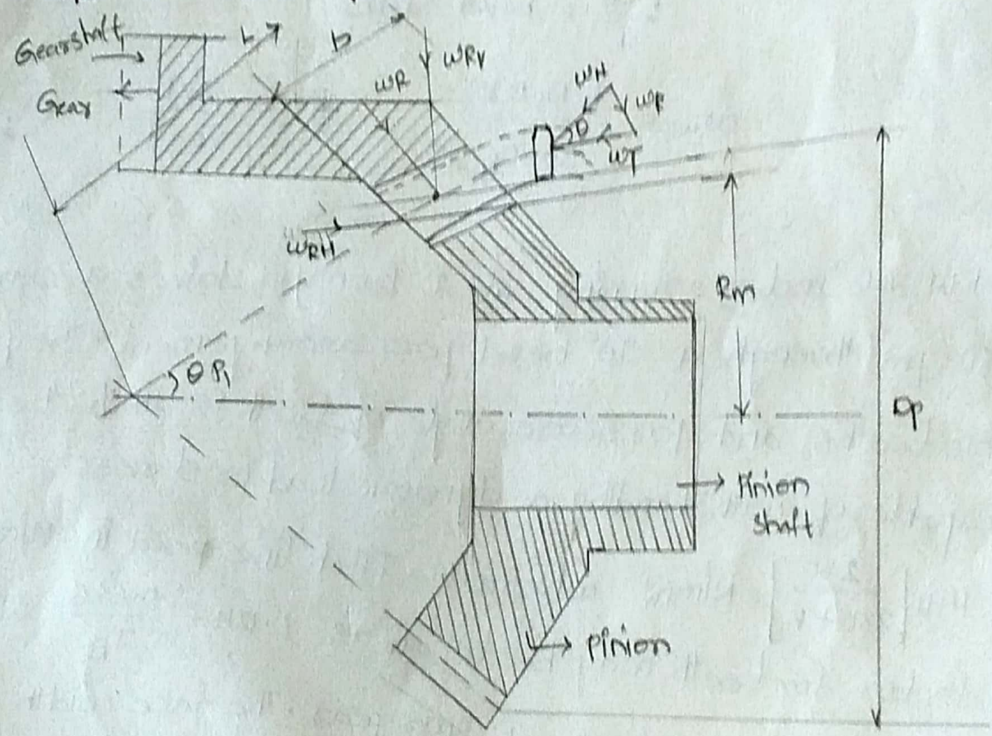
θ_s = angle b/w two shaft axes

Note 1 :→

Axis of shaft 90° to each other $\theta_{P1} = \tan^{-1}\left[\frac{1}{VR}\right]$

$$\theta_{P2} = \tan^{-1}[VR]$$

Force analysis on Bevel gear :→



Forces acting on Bevel gear

$$W_{RH} = W_R \sin\theta_{P_1} = \omega_T \tan\phi \sin\theta_{P_1}$$

$$W_{RV} = W_R \cos\theta_{P_1} = \omega_T \tan\phi \cos\theta_{P_1}$$

Strength of the Bevel gear:

$$\omega_T = \sigma_\omega \, b \, P_c \, Y' \left[\frac{L-b}{L} \right]$$

$$\sigma_\omega = \sigma_b \, C_V$$

$$C_V = \frac{3}{3+V} \rightarrow \text{ordinary cut}$$

$$= \frac{6}{6+V} \rightarrow \text{Precision cut}$$

$$L = \sqrt{\frac{D_g^2}{2} + \frac{D_p^2}{2}}$$

$$\frac{L-b}{L} = \text{bevel factor}$$

$$\omega_\omega = \frac{b \, D_p \, Q \, K}{\cos\theta_{p_1}} \qquad Q = \frac{2VR}{VR \pm 1}$$