MISCELLANEOUS TOPICS

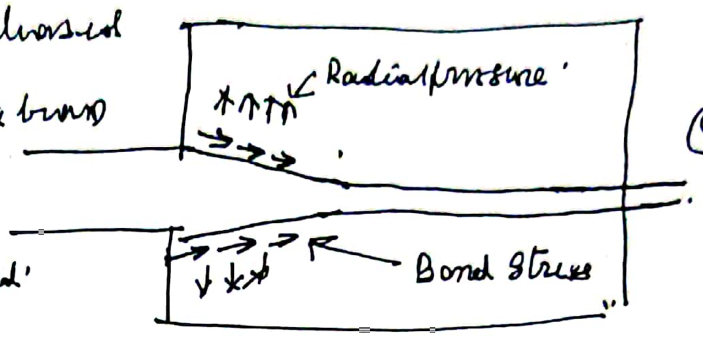
TRANSMISSION LENGTH IN PSC BEAMS KERN AND KERN DISTANCES.

PARTIAL PRESTRESSING.

TRANSMISSION OF PRESTRESS IN PRETENSIONED MEMBERS.

a) HOYER EFFECT

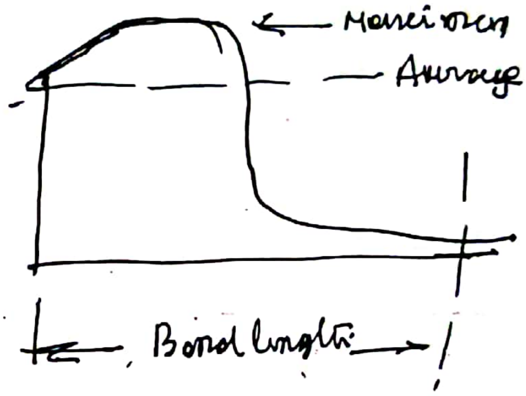
When the ends are released the cross section of the beam bulges out. Radial pressures are produced.



(a)

b) BOND STRESS BETWEEN WIRE AND CONCRETE

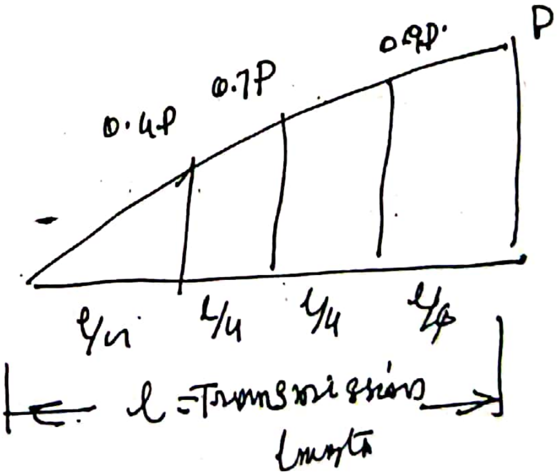
Bond stress varies from maximum value and reaches almost zero value as shown in fig (b) over a length known as 'Bond length'.



(b)

c) Stress in steel

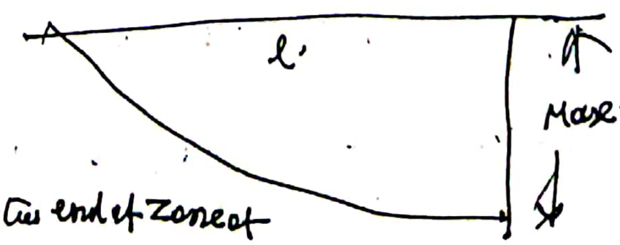
The prestressing force "P" is fully transferred to concrete over a length equal to bond length (l) as shown in fig.



(c)

d) Stress in Concrete

Max. stresses are reached at the end of zone of compression. Transmission length



(d) Stress in Concrete

TRANSMISSION LENGTH (HOYER'S FORMULA)

Definition: "It is the length required at the ends of a pretensioned member for the buildup of stress in Concrete so as to resist the applied S.F and BM. This is particularly important in short pretensioned members"

The transmission length depends mainly on the diameter and the Surface Characteristics of the Wire, elastic properties of steel and Concrete and the Coeff. of friction between steel and Concrete.

Hoyer's expression for transmission length is

$$\boxed{L_t = \frac{\phi}{2\mu}\left(1+\nu_c\right)\left(\frac{\alpha_e}{\nu_s} - \frac{f_{pi}}{E_c}\right)\left(\frac{f_{pe}}{2f_{pi} - f_{pe}}\right)}$$

Where ϕ : Wire diameter,

μ : Coeff. of friction between steel and Concrete.

ν_c and ν_s : Poisson's ratios of Concrete and Steel.

α_e : Modular ratio $= \frac{E_s}{E_c}$, E_s and E_c are the Young's Modulii

f_{pi} and f_{pe} are the initial and effective stresses in ~~Concrete~~ steel.

L_t : Normally varies from 80ϕ to 100ϕ

(Cont'd)

Usually, under normal conditions, the transmission length lies between 80ϕ to 160ϕ, where 'ϕ' is bar diameter.

MARSHALL AND KRISHNA MURTHY'S EMPIRICAL FORMULA.

The transmission length is given by $L_t = \sqrt{\dfrac{\sqrt{f_{cu}} \times 10^3}{\beta}}$

Where f_{cu} = cube strength of concrete at transfer.

β = It is a coefficient, which depends on the type of wires or strand

Table of β values

Diameter of wire or strand.	β
2 mm	0.144
5 mm	0.0235
7 mm	0.0174
10 mm	0.14
7 wire strand	
12.5 mm dia. 7 wire strand	0.055

EXAMPLE: Calculate the transmission length at the end of a pretensioned beam as per Hoyer's method.

Compare the answer, with Marshall and Krishna Murthy's formula.

DATA: Span of the beam = 50 m, Diameter of the wire = 7 mm

$\mu = 0.1$, $\nu_c = 0.15$, $\nu_s = 0.30$.

$E_s = 210\ kN/mm^2$, $E_c = \cancel{300}\ 30\ kN/mm^2$

Ultimate tensile strength of steel wire $= f_{pu} = 1500\ N/mm^2$

Initial stress provided in steel $= 0.7 f_{pu} = 1050\ N/mm^2$

Effective stress in steel after losses $= 0.6 f_{pu} = 900\ N/mm^2$.

Cube strength of concrete at transfer $= f_{cu} = 42\ N/mm^2$.

SOLUTION:

a) Hoyer's expression is

$$L_t = \frac{\phi}{2\mu}(1+\nu_c)\left[\frac{d_e}{\nu_s} - \frac{f_{pi}}{E_c}\right]\left(\frac{f_{pe}}{2f_{pi} - f_{pe}}\right)$$

Substituting, $$L_t = \frac{\phi}{2\times0.1}(1+0.15)\left[\frac{7}{0.3} - \frac{1050}{30\times10^{3}}\right]\left(\frac{900}{2\times1050-900}\right)$$

Hence $L_t = 90\phi = 90\times7 = 630\ mm$; Provide 650 mm. Hence the beam should be projected out wards by 650 mm from centre of support

Hence, the overall length of the beam $= 50 + 2\times0.65 = 51.3\ m$.

BOND STRESS (FOR PRETENSIONED MEMBERS)

At the ends of a pretensioned member, stress transfer from steel to concrete takes place. Some kind of bulging takes place. The bond stress builds up rapidly and becomes maximum over a short length. At a distance equal to the transmission length, the bond stress is almost zero while stresses in steel and concrete reach their maximum values.

τ_{bp} is the bond stress.

$(\tau_{bp})_x$ bond stress at a distance of 'x'

'ϕ' : is the diameter of the wire.

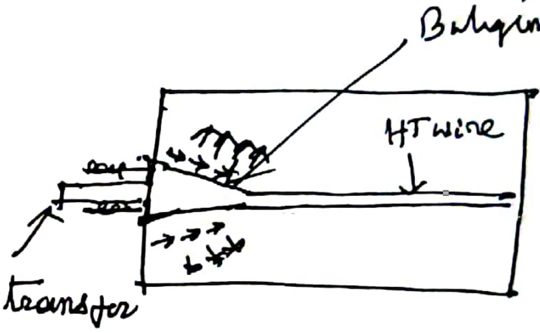


Fig. (a)

Marshall has given that

$$(\tau_{bp})_x = (\tau_{bp\,(Max)}) \cdot e^{-4 k x}$$

'k' : a constant equal to the ratio of change in bond stress to steel stress.

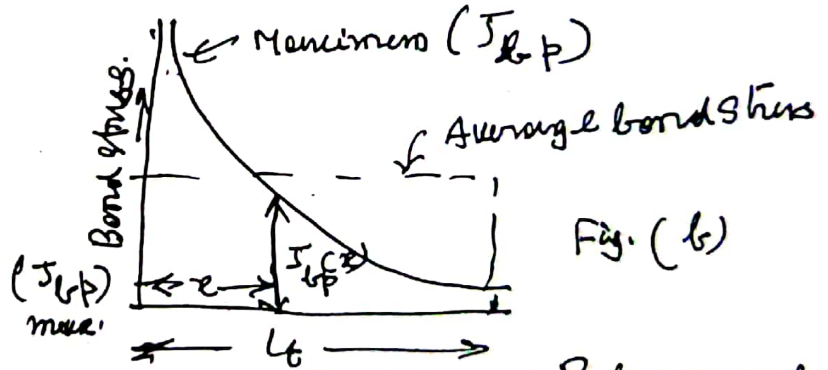


Fig. (b)

Bond stress in Pretensioned Beams

Also, $f_y = f_{se}\left(1 - e^{-4kx/\phi}\right)$ where 'f_y' is the steel stress and 'f_{se}' is steel stress at 'x' from the end.

According to Ross the anchorage bond stress in steel wires (1.5 to 5 mm dia) varies from 3.25 to 1.0 N/mm²

According to Marshall (at Leeds University)

$\tau_{bp}(\text{Max.}) = 7.42\,N/mm^2$ and $\psi = 0.00725$

EXAMPLE : In the case of a pretensioned beam,

ϕ (Bar diameter) = 5mm, f_{pu} (Ultimate) = $1600\,N/mm^2$

Initial prestress = 80% of f_{pu} and f_{ci} at transfer (concrete) = $30\,N/mm^2$

Calculate a) Transmission length

b) Compute the bond stress at $\frac{1}{4}$ and $\frac{1}{2}$ of the transmission length.

c) Calculate the overall average bond stress.

SOLUTION : $L_t = \sqrt{\dfrac{\sqrt{f_{ci}} \times 10^3}{\beta}}$, $\beta = 0.0235$, $f_{ci} = 30$.

Hence $L_t = 485$ mm.

$$(\tau_{bp})_x = \left[\tau_{bp(\text{Max})}\, e^{-4\psi x/\phi}\right] = 7.42 \times e^{-\frac{4 \times 0.00725\, x}{5.0}}$$

$x = \frac{485}{4}$ and $\frac{485}{2}$ are substituted in the above. We get

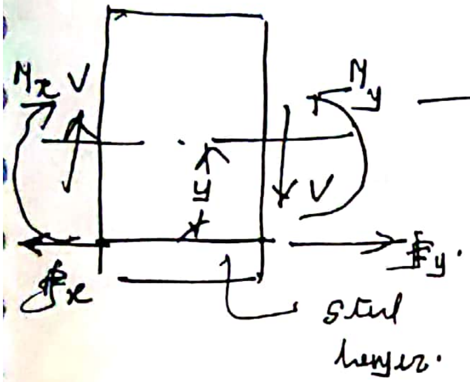
$(\tau_{bp})_{\frac{L_t}{4}} = 3.7\,N/mm^2$, $(\tau_{bp})_{\frac{L_t}{2}} = 1.84\,N/mm^2$
(121.25 mm) (242.5 mm).

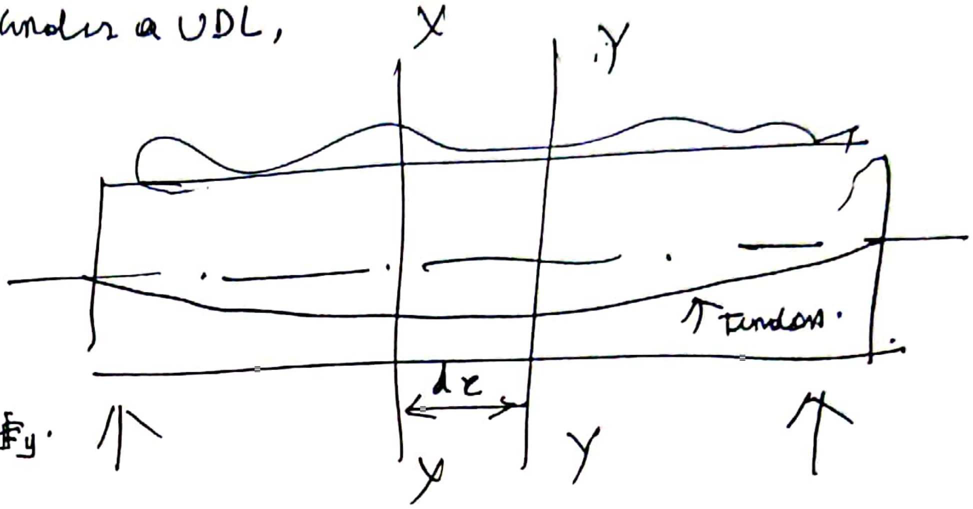
overall average bond stress $= \tau_{bp(\text{ave})} = \dfrac{\frac{\pi}{4}\phi^2 f_{pe}}{\pi \phi L_t}$.

Hence, $\tau_{bp(\text{ave})} = \dfrac{\frac{\pi}{4} \times 5^2 \times 0.8 \times 1600}{\pi \times 5 \times 485} = 3.35\,N/mm^2$

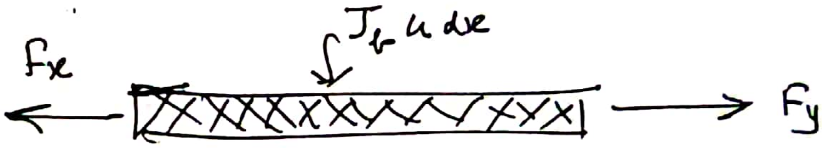
FLEXURAL BOND STRESS

Considering any element between the Sections XX' and YY' of PSC beam under a UDL,



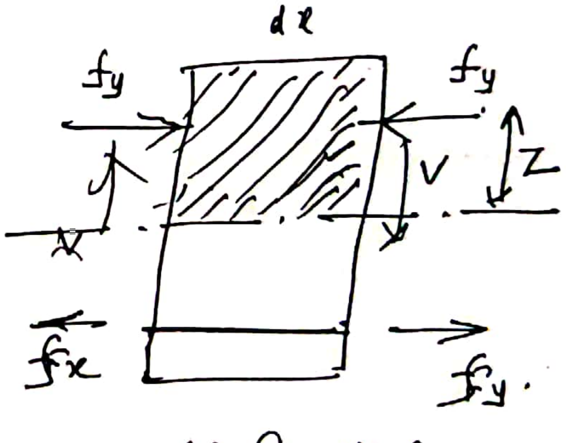


a) Uncracked



Σu : Total perimeter of the tendons

f_x and f_y are the bending stresses at the level of steel in Sections XX' and YY'



b) Cracked.

Considering the equilibrium of the element,

$$(M_y - M_x) = V dx = \left[\frac{f_y I}{y} - \frac{f_x I}{y}\right]$$

Multiplying and dividing by $d_e A_s$, We get

$$V dx = \frac{I}{d_e A_s}\left[d_e A_s f_y - d_e A_s f_x\right] = \frac{I}{d_e A_s}\left[F_y - F_x\right]$$

Since $V dx = \dfrac{I}{d_e A_s\, y}\left(\tau_b\, \Sigma u\, dx\right)$

Hence, $\tau_b = \dfrac{d_e A_s}{I}\,\dfrac{y.V}{\Sigma u}$, $\dfrac{A_s}{\Sigma u} = \phi/4$ (149)

Hence, $\boxed{\tau_b = \dfrac{V y \alpha_e \phi}{4I}}$

In the Case of Cracked Case,

by linear Theory of cracked RCC Sections,

$$\tau_b = \frac{V}{Z \sum_0} \quad \text{where } 'Z' \text{ is the lever arm}$$

EXAMPLE. A post tensioned beam with a size of 240×500 mm has, steel: 3 cables each having 12 nos of 7 mm dia wires

Metallic Hose diameter = 30 mm, Eff ~~Spacer~~ Cover = 50 mm

$E_s = 210\ kN/mm^2$, $E_c = 35\ kN/mm^2$ ~~and~~ Span $l = 10$ m.

There are two loads each of 250 kN at one third points

Compute the Unit bond stress

a) Between each wire and grout b) the hose and Concrete

SOLUTION

$V = 250\ kN$, $I = \dfrac{240 \times 500^3}{12} = \cancel{25}\ 25 \times 10^8\ mm^4$.

$\alpha_e = \dfrac{210}{35} = 6.0$, $y = \dfrac{500}{2} - 50 = 200$ mm.

Bond stress between wire and grout $= \dfrac{V y \alpha_e \phi}{4I} = \dfrac{250 \times 10^3 \times 200 \times 6 \times 7}{4 \times 25 \times 10^8} = 0.21\ N/mm^2$

A_s (in one hose) $= 12 \times 38.5 = 462\ mm^2$, Hole dia = 30 mm

Hose circumference $= \pi \times 30 = 94$ mm.

Hence, bond stress between hose and concrete $= \dfrac{V \alpha_e A_s y}{\Sigma_0 I}$

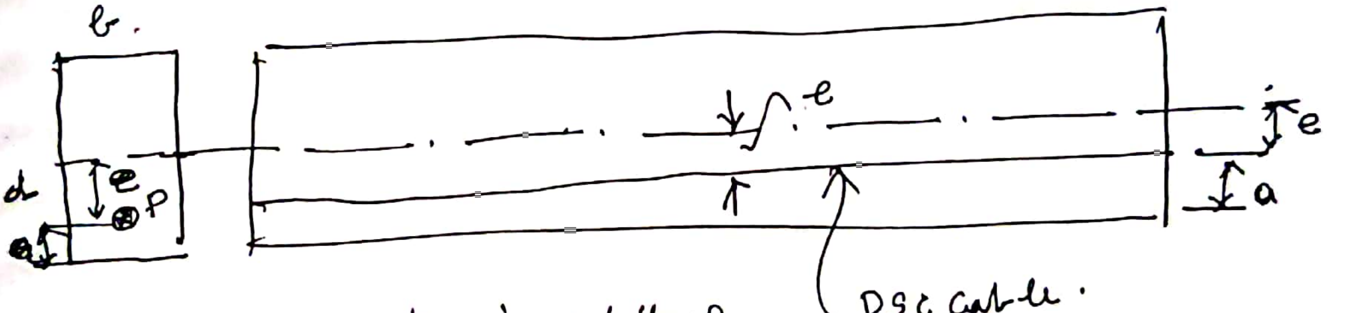
$= \dfrac{250 \times 10^3 \times 6 \times 462 \times 200}{94 \times 25 \times 10^8} = 0.59\ N/mm^2$.

KERN AND KERN DISTANCE.

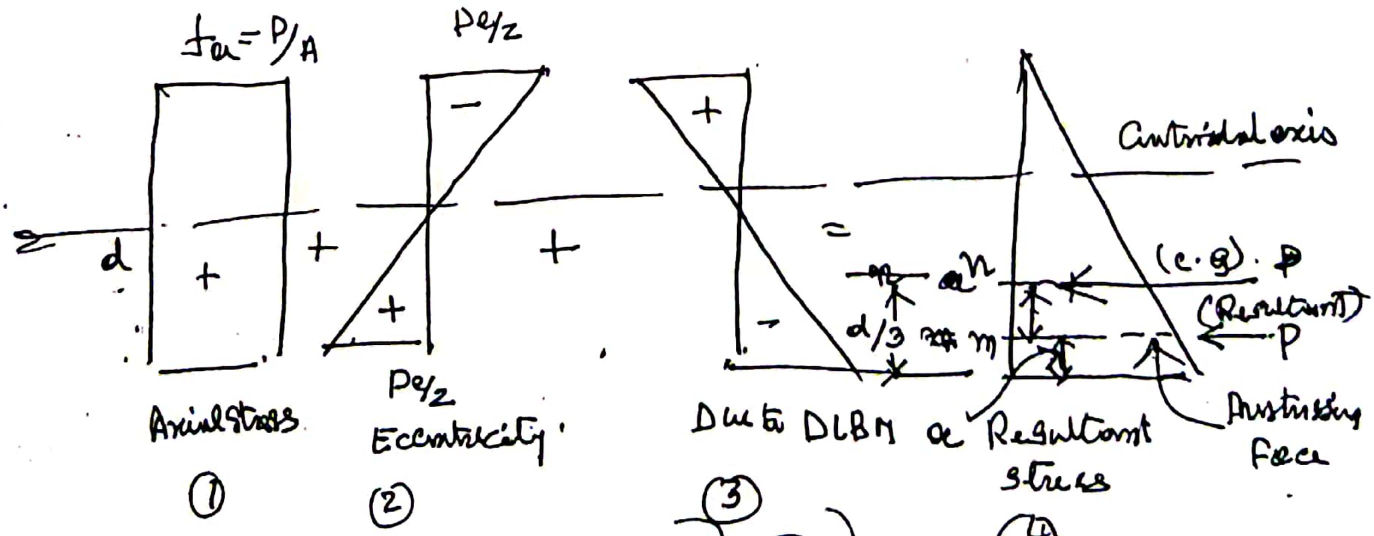
Kern points are important in deciding the limits of stress distributions in a PSC section. By properly locating the kern points and limiting the kern distances, tension in the section can be controlled and a safe section can be evolved.

Consider the following cases of PSC beam.

a) Under Dead load bending moment with prestressing force acting with some eccentricity.



The stress distribution is as follows.



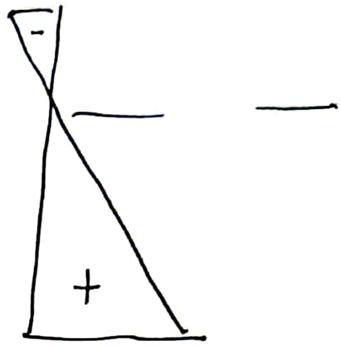
149 (151)

From the above, (fig 4), it can be seen that

$mn = d/3 - a$, Hence $P \times mn = P\left(\frac{d}{3} - a\right) = M_d$ (DLBM)

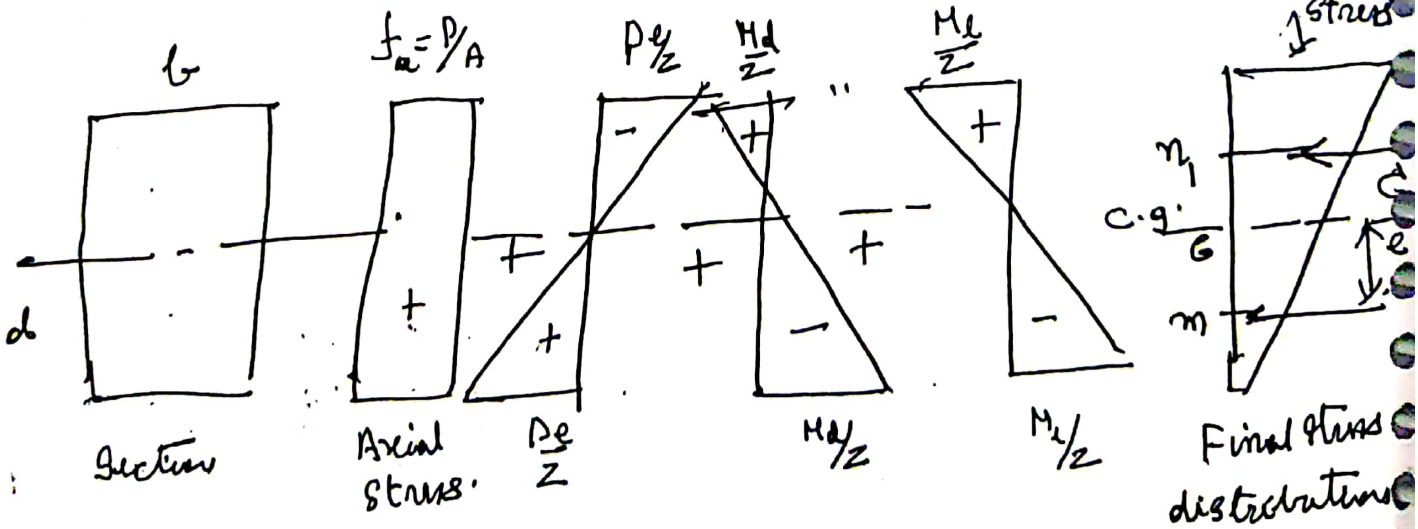
If $\underline{P \times mn < M_d}$.

Hence the moment produced by the prestressing force is less than the DLBM. Hence, there is some tension produced at the top as shown in fig. 5



Hence it is clear that 'mn' should have sufficient length so as to counteract the DLBM to keep the tension at top at the minimum level and the compression at bottom at maximum level. The point n is termed as the lower kern point.

b) When Live load bending moment is applied.



If G is the C.G of the Section, then

$G_n = K_b$ and $G_{n_1} = K_t$. These are 'Kern distances'.

The position of C varies from bottom to top depending upon the loading case. Hence, max. Compressive stress varies from bottom position to top position. If 'n' is higher, then the compressive stress at Top will be maximum.

we have,

$$DLBM = M_d = P(e - k_b)$$

$$M_e = P(K_t + K_b).$$

Adding $(M_d + M_e) = P\ (e + k_t)$

Total B.M $= M_d + M_e = M = f_z$

The effeciency factor of the beam on crack resistance is

$$\rho = \frac{(k_t + k_b)}{d}$$, k_t and k_b are the kern distances.

n and n_1 are the lower and upper kern points.

BONDED AND UNBONDED MEMBERS.

The rate of increase of stress in the tendons of a PSC beam under the loads depends upon the degree of bond between the high tensile steel wires and the surrounding concrete. In the case of bonded members, the stresses in steel can be calculated from those of concrete. In the case of unbonded, the tendons are free to elongate throughout their length under the action of transverse loads. The increase of stress in steel depends upon the average strain in concrete at that level.

a) In the case of Bonded Beams

Stress in steel = Modular ratio x Stress in Concrete $= \alpha_e f$

Where 'f' is the stress in concrete at 'y' from the centroidal axis.

Hence, stress in steel $= \left(\frac{My}{I}\right) \alpha_e$

b) In the case of Unbonded Beams

δL : Total elongation of the cable at 'y' from the centroidal axis.

L : Total length of the cable.

Strain in concrete at the level of steel $= \frac{M_x y}{I E_c}$

Hence, total elongation of concrete fibre $= \delta L = \int_0^L \frac{My}{E_c I} \, dx$.

Average strain $= \frac{1}{L} \int_0^L \left(\frac{M \cdot y}{E_c I}\right) dx = \frac{y}{E_c I L} \int_0^L M \, dx$

Hence, strain in steel $= \left(\frac{E_s}{E_c}\right)\left(\frac{y}{I_L}\right)\int_0^L M\,dx$.

$$\boxed{\text{Hence, strain in steel} = \frac{\alpha_e y}{IL}\int_0^L M\,dx}$$

Let 'A' be the area of the BMD under given loading.

$A = \int_0^L M\,dx$,

Hence, stress in steel $= \frac{\alpha_e y}{IL} \cdot A$

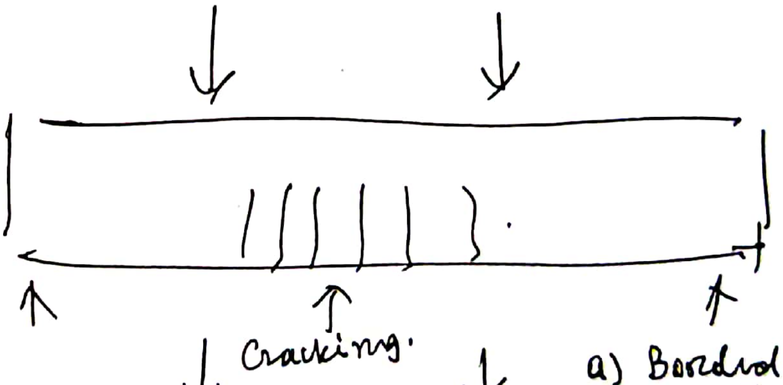
In the case of a UDL on the beam, $A = \int M\,dx = \frac{2}{3} L\, \omega_d \frac{L^2}{8} = \frac{w_d L^3}{12}$

Hence, increase of stress in steel $= \frac{\alpha_e y}{12 I} L^2 \omega_d$

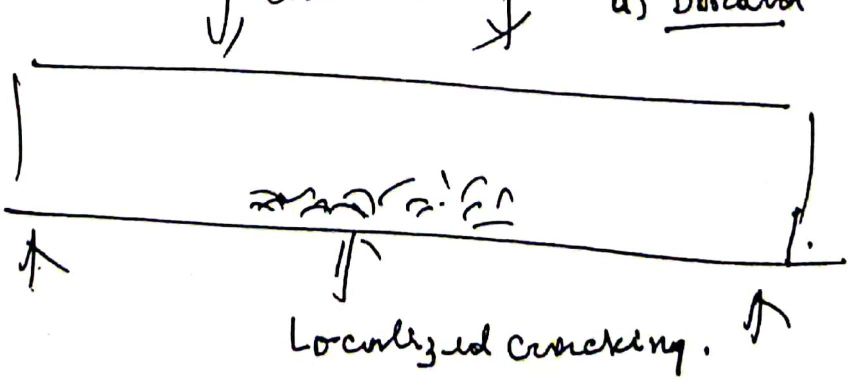
Rate of increase in stress is larger in Bonded Case.

CRACK PATTERNS

The bonded beams have higher flexural strength than unbonded. Hence, they are preferable.



a) Bonded



b) Unbonded.

Scanned with CamScanner

EXAMPLE. A prestressed Concrete beam 200 × 300 mm Supports a live load of 2.56 kN/m over the entire span of 10 m. The tendons in the ducts are located at 100 mm below the centroidal axis. Calculate the increase in steel stress in the following cases.

a) The ducts are grouted so that the strains in Concrete and steel are equal

b) The ducts are ungrouted so that the tendons can move in the ducts without friction

$E_s = 210\ kN/mm^2$, $E_c = 35\ kN/mm^2$,

SOLUTION

Self wt. of the beam $= g = 0.2 \times 0.3 \times 24 = 1.44\ kN/m$.

$LL = q = 2.56\ kN/m$.

Hence, total load on the beam $= 1.44 + 2.56 = 4.0\ kN/m$

$I = \frac{1}{12} \times 200 \times 300^3 = 45 \times 10^7\ mm^4$, $\alpha_e = \frac{E_s}{E_c} = \frac{210}{35} = 6.0$

M_{max} (at centre) $= 4 \times 0.125 \times 10^2 = 50\ kNm$

a) In the case of Bonded beam,

stress in Concrete $= \frac{M}{I} \cdot y = \frac{50 \times 10^6 \times 100}{45 \times 10^7} = 11.1\ N/mm^2$

Hence, stress in steel $= 6 \times 11.1 = 66.6\ N/mm^2$

b) In the case of Unbonded beam,

$$\text{stress in steel} = \frac{\alpha_e\, w_d\, L^2 \cdot y}{12 I} = \frac{6 \times 4 \times 100\ (10 \times 10^3)^2}{12 \times 45 \times 10^7}$$

$$= 44.4\ N/mm^2$$

CRACKING MOMENT

When the transverse loads acting on the beam are increased, the compression below the centroidal axis is neutralised and the stress becomes zero in an ideal case. If the loading is further increased micro cracks occure and further there will be visible cracks of the order of ~~0 to~~ 0.01 to 0.02 mm

In general, the visible tensile cracks in concrete appear when the bending tensile stress equals to modulus of rupture of concrete. ~~The cracks~~ The crack widths ~~are~~ also depend upon the type and distribution of steel reinforcement and the quality of concrete.

EXAMPLE For a PSC beam 120 x 300 mm, the effective prestressing force in a straight cable is 180 kN at 50 mm eccentricity. The imposed load is 3.14 kN/m, span = 6 m, Modulus of rupture of concrete = 5.0 N/mm². Evaluate the load factor against cracking.

SOLUTION Area = 120 x 300 mm², Self wt. = 0.12 x 0.3 x 24 = 0.86 kN/m (g)

Total load = $g+q$ = 0.86 + 3.14 = 4.0 kN/m.

Due to prestress, axial stress = $\frac{P}{A} = \frac{180\times10^3}{120\times300} = 5\,N/mm^2$

Bending stress due to eccentricity = $\pm\frac{Pe}{Z} = \frac{180\times10^3\times50\times6}{120\times300^2} = 5\,N/mm^2$

Due to the loads, Max. B.M = $0.125\times4\times6^2 = 18$ kNm.

$\frac{M}{Z} = \frac{18\times10^6}{18\times10^5} = 10\,N/mm^2$, stress at the bottom fibre under working load = $(5+5-10) = 0.0\,N/mm^2$, stress corresponding to cracking at the bottom fibre = $5\,N/mm^2$, Extra moment required to create this stress = $5\times18\times10^5$ = 9 kNm.

Hence, cracking moment = 18 + 9 = 27 kNm.

Load factor against cracking = $\frac{\text{Cracking Moment}}{\text{Working Moment}} = \frac{27}{18} = 1.5$

PARTIALLY PRESTRESSED MEMBERS.

In the case of partially prestressed members, Untensioned reinforcement is also kept to resist differential shrinkage, temperature effects, handling stresses etc. Hence, the reinforcement (untensioned) helps in the control of cracking which is a serviceability requirement. In addition, this also shares part of the ultimate moment in the limit state of collapse. As a result the area of H.T. steel gets considerably reduced and hence the design becomes economical. Partially prestressed beams have more ductility than the fully stressed beams. In the case of partially prestressed beams, the upward deflections are less. For untensioned reinforcement HYSD bars can be used.

CIRCULAR PRESTRESSING.

Circular prestressing is adopted in the case of circular pipes, tanks, pressure vessels, silos etc. By circumferential prestressing, hoop compression is developed in concrete which counteracts hoop tension produced by internal fluid pressure. Circular prestressing eliminates cracks and the materials are economically used. Even shrinkage cracks are eliminated and this is necessary to have a crack free liquid retaining structure.

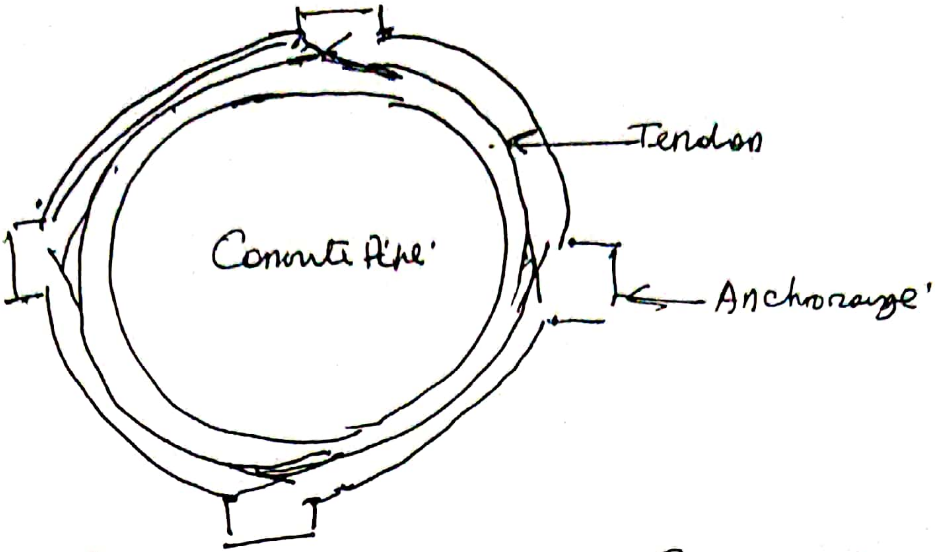


Fig: Prestressing of a Circular Concrete Pipe.