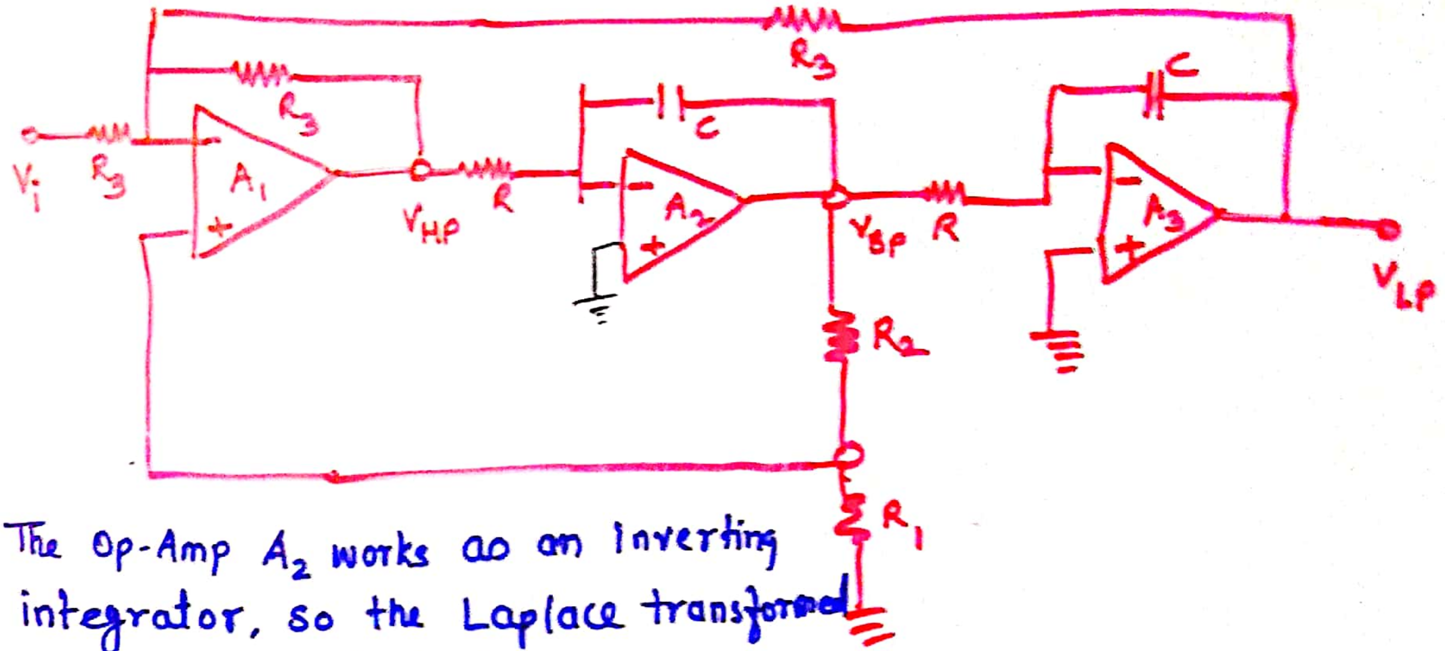
STATE VARIABLE FILTER :-



The Op-Amp A_2 works as an Inverting integrator, so the Laplace transformed o/p V_{BP} is given by,

$$V_{BP} = -\frac{1}{sRC} V_{HP} \quad \text{—— (1)}$$

If $R = 1M\Omega$ and $C = 1\mu F$, so that $RC = 1$, we get,

$$V_{BP} = -\frac{1}{s} V_{HP} \quad \text{—— (2)}$$

Also for inverting integrator A_3, we may write

$$V_{LP} = \frac{1}{s} V_{BP} = \frac{1}{s^2} V_{HP} \quad \text{—— (3)}$$

The Op-Amp A_1 is a three I/p summer. The o/p V_{HP} can be written using superposition theorem. That is,

$$V_{HP} = -\left(\frac{R_3}{R_3}\right)V_i - \left(\frac{R_3}{R_3}\right)V_{LP} + \left(1 + \frac{R_3}{R_3 \| R_3}\right)\left(\frac{R_1}{R_1+R_2}\right)V_{BP}$$

$$= -V_i - V_{LP} + 3\left(\frac{R_1}{R_1+R_2}\right)V_{BP}$$

Put $\alpha = 3\left(\frac{R_1}{R_1+R_2}\right)$

Then $V_{HP} = -V_i - V_{LP} + \alpha V_{BP}$ —— (4)

Eliminating V_{BP} and V_{LP} using eq.^{n} (2) and (3)

$$V_{HP} = -V_i - \frac{V_{HP}}{s^2} - \frac{\alpha}{s} V_{HP}$$

$$V_{HP}\left(1 + \frac{\alpha}{s} + \frac{1}{s^2}\right) = -V_i$$

So, the high pass transfer function H_{HP} is

$$H_{HP} = \frac{V_{HP}}{V_i} = \frac{-s^2}{s^2+\alpha s+1}$$

The damping factor α can be set by R_1 and R_2 for Bessel, Butterworth or Chebyshev response

Comparing to the std. high pass transfer fun.^{n}

$$\frac{A_o s^2}{s^2+\alpha \omega_L s+\omega_L^2}$$

So for high-pass filter of state variable filter,

$$A_o = -1 \text{ and } \omega_L = 1$$

The low pass transfer fun.^{n} is obtained by eliminating V_{HP} and V_{BP} from eq.^{n} (4)

$$H_{LP} = \frac{V_{LP}}{V_i} = \frac{-1}{s^2+\alpha s+1}$$

As in High Pass Filter, the low-pass filter has

$$A_o = -1, \ \omega_H = 1$$

$$\alpha = 3\left(\frac{R_1}{R_1+R_2}\right)$$

Finally the band-pass impulse response is obtained from eq.^{n} (4) by eliminating V_{HP} and V_{LP} as

$$H_{BP} = \frac{V_{BP}}{V_i} = \frac{s}{s^2+\alpha s+1}$$

The std. band pass transfer fun.^{n} is $\dfrac{A_o \alpha \omega_o s}{s^2+\alpha\omega_o s+\omega_o^2}$

Comparing these eq.^{n}

$$A_o \alpha \omega_o = 1$$

$$\omega_o = 1/RC = 1 \quad \text{(RC has been assumed to be 1)}$$

$$A_o = \frac{1}{\alpha} = \frac{R_1+R_2}{3R_1}$$

From the anlaysis, we found that the band-pass response can be generated by integrating the high-pass response and that the low pass is generated by integrating the band-pass.