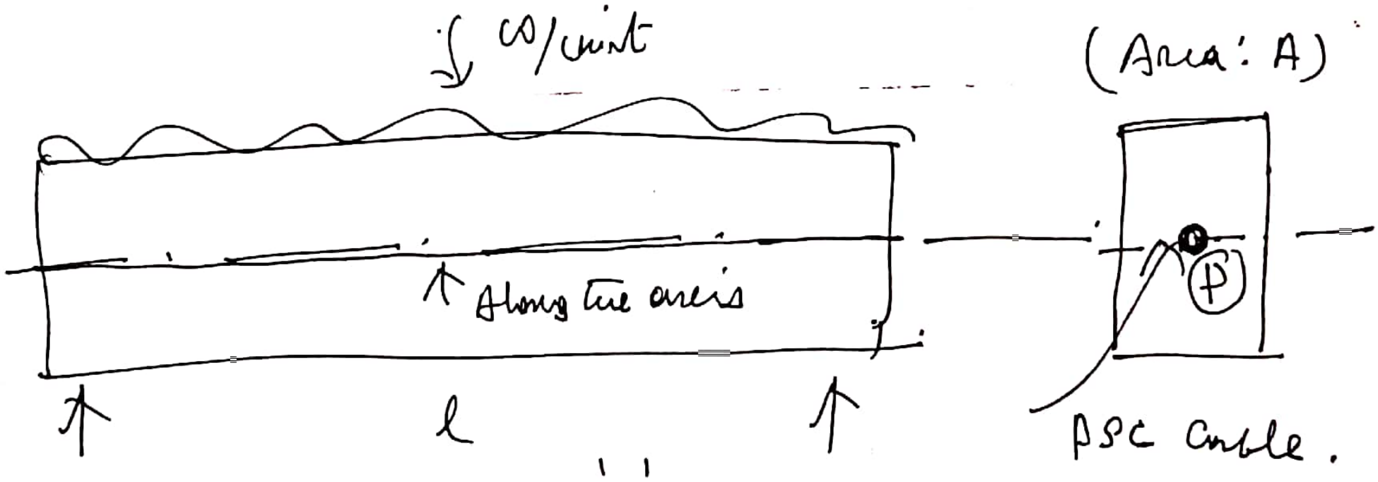
UNITS – II & III

ANALYSIS – DESIGN BY WORKING STRESS, LIMIT STATE DESIGN, COMPOSITE CONSTRUCTION

ANALYSIS OF P.S.C. SECTIONS.

Analysis of PSC Sections includes the Computation of Stresses in the cross section of a PSC beam of given span, section under given loading. The stresses in the cross sections vary depending upon the position of the PSC cables and their profile.

CASE I. Tendons bent along the long. centroidal axis



Due to prestressing force 'P', axial comp. stress = $f_a = +\frac{P}{A}$.

Due to the loading, $M_{max} = \frac{wl^2}{8}$, Bending stress = $\pm \frac{M}{Z}$

Hence, net resultant stresses,

$$f_{(top)} = \frac{P}{A} + \frac{M}{Z}.$$

$$f_{(Bottom)} = \frac{P}{A} - \frac{M}{Z}$$

EX :1 A PSC Beam (400 x 600 mm) has a span of 6 m. It is subjected to a UDL of 16 kN/m inclusive of its self wt. The prestressing tendons which are located at the longitudinal centroidal axis provide an effective prestressing force of 960 kN. Determine the extreme fibre stresses in concrete at the mid span section.

SOLUTION :

$$P = 960\ kN.$$

$$A = 400 \times 600 = 2400 \times 10^2\ mm^2.$$

$$Z = \frac{1}{6} \times 400 \times 600^2 = 24000 \times 10^3\ mm^3$$

$$M = \frac{wl^2}{8} = \frac{16 \times 6^2}{8} = 72\ kNm = 72 \times 10^6\ Nmm.$$

$$f_a = \frac{P}{A} = \frac{960 \times 10^3}{2400 \times 10^2} = 4.0\ N/mm^2.$$

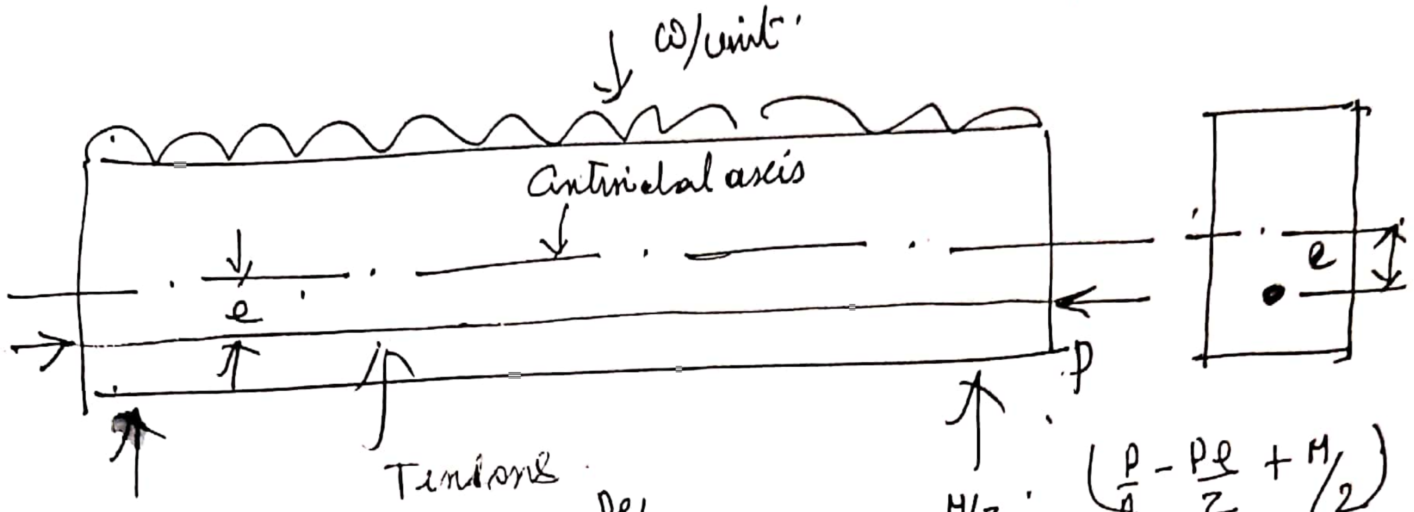
$$f_b = \pm \frac{M}{Z} = \frac{72 \times 10^6}{24000 \times 10^3} = \pm 3.0\ N/mm^2.$$

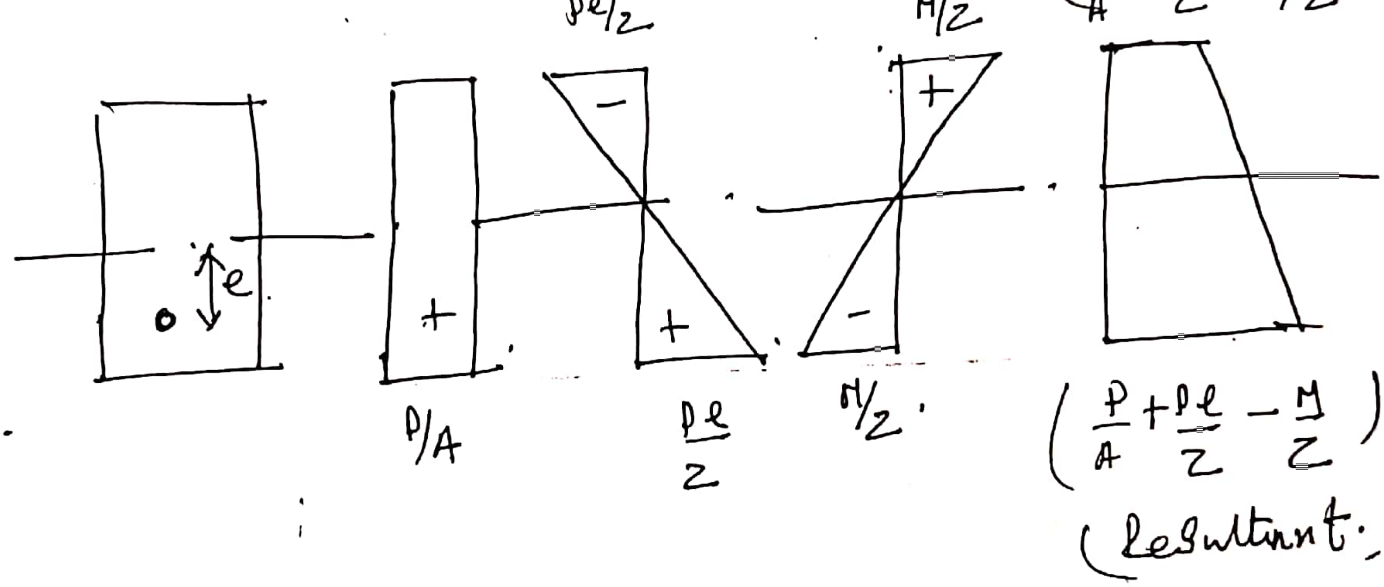
Hence, f_{max} (at top) $= 4.0 + 3.0 = 7.0\ N/mm^2$.

f_{min} (at bottom) $= 4.0 - 3.0 = 1.0\ N/mm^2$.

Both f_{max} and f_{min} are compressive in nature.

CASE II : Tendons placed at an eccentricity with the centre.





In this case there is an additional moment produced due to eccentricity of the prestressing force.

Final stresses

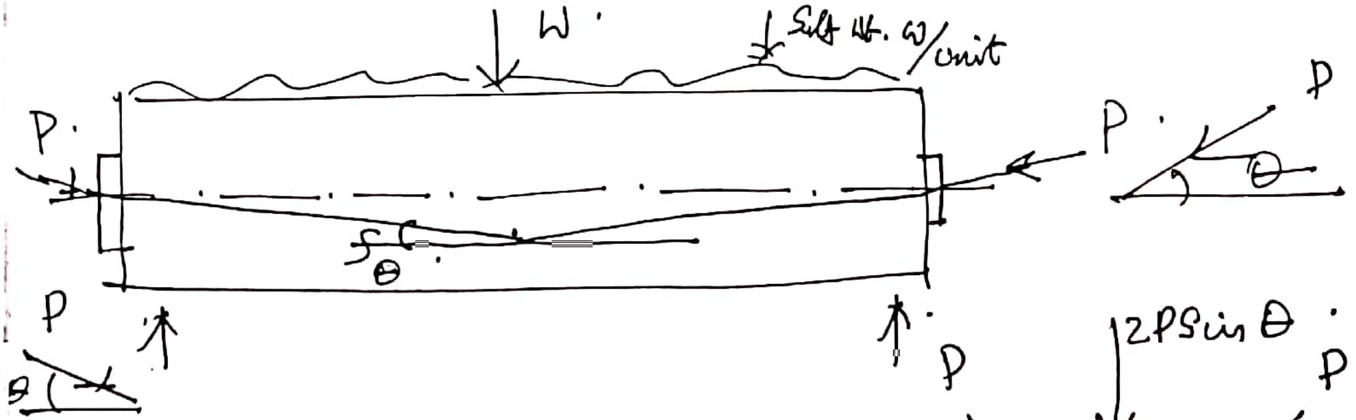
Stress at the extreme fibre at top $=\left(\frac{P}{A}-\frac{Pe}{Z}+\frac{M}{Z}\right)$

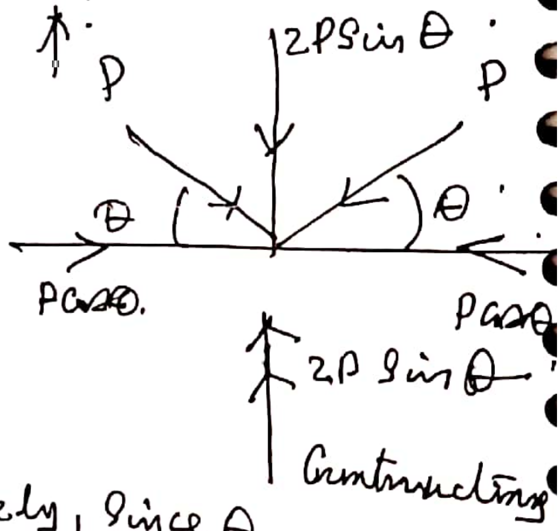
" at bottom $=\left(\frac{P}{A}+\frac{Pe}{Z}-\frac{M}{Z}\right)$

Hence, due to the eccentricity of the cable, the stresses due to external loads are counteracted.

NOTE: Hence it is desirable to have the tendons to have a profile coinciding with the BMD (like parabolic) so that the external B.M is effectively and completely counteracted.

CASE III: Bent down Tendons.





At centre, the counteracting force

$= 2P\sin\theta$.

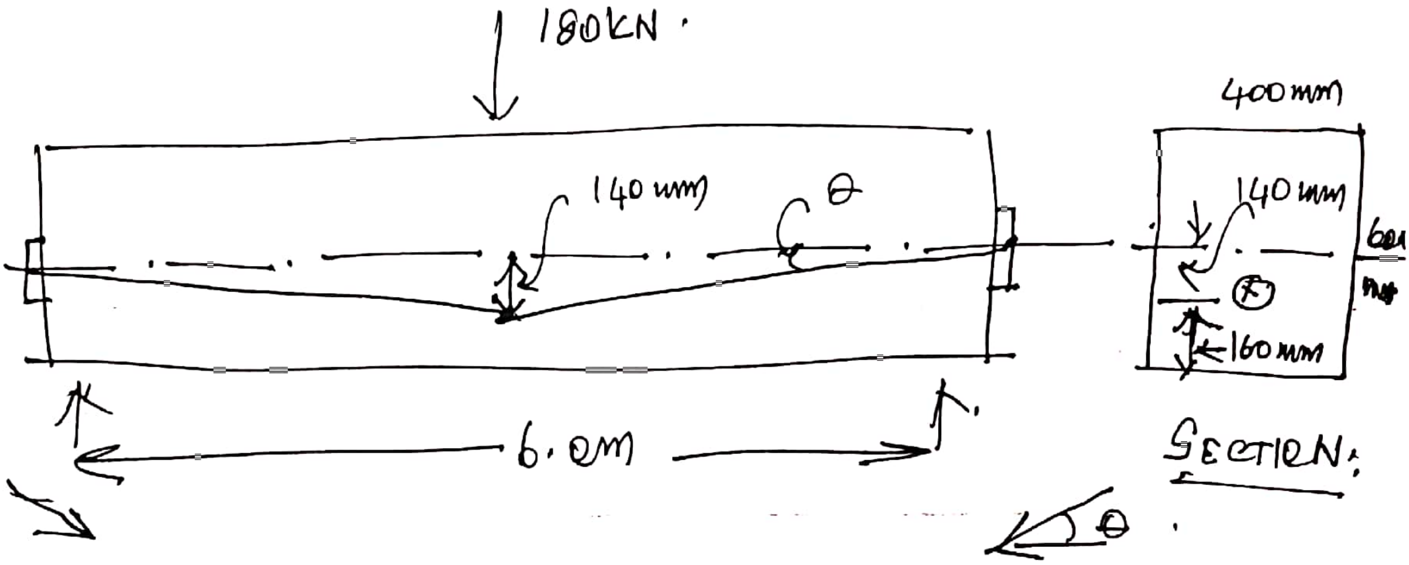
Hence the net load $= (W - 2P\sin\theta)$

Axial force $= P\cos\theta = P$ (Nearly, since θ is very small)

Hence, Direct stress $= P/A$, Net B.M (M) $= (W - 2P\sin\theta)\,l/4 + \frac{wl^2}{8}$ (Self wt)

Hence extreme fibre stresses are $= \frac{P}{A} \pm M/Z$.

Ex. 2. A P.S.C. Beam (400 x 600 mm) has a span of 6 m. The beam is prestressed by a bent tendon as shown. There is a concentrated load of 180 kN at midspan. If the eff. prestressing force is 1200 kN calculate the extreme stresses for the mid span section of the beam.



$\sin\theta = \tan\theta$ (Nearly) $= \frac{140}{3000} = \frac{14}{300}$

Net vertical load at midspan $= W - 2P\sin\theta$

$= 180 - 2 \times 1200 \times \frac{14}{300} = 68$ kN

$B.M = \frac{68 \times 6}{4} = 102 \times 10^6$ Nmm

Self wt. $= 0.4 \times 0.6 \times 24 = 5.76$ kN/m

B.M due to self wt. $= \frac{5.76 \times 6^2}{8} = 25.92$ kNm $= \cancel{25 \times 9}$ 25.92×10^6 Nmm

Hence total B.M at centre $= 102 + 25.92 = 127.92 \times 10^6$ Nmm.

Z (of the Section) $= \frac{1}{6} \times 400 \times 600^2 = 24 \times 10^6 \; mm^3$.

Direct stress, $f_a = \frac{P}{A} = \frac{1200 \times 10^3}{400 \times 600} = +5.0 \; N/mm^2$.

Stress due to B.M $= \pm \frac{M}{Z} = \pm \frac{122.92 \times 10^6}{24 \times 10^6} = \pm 5.33 \; N/mm^2$.

Hence Resultant stresses are,

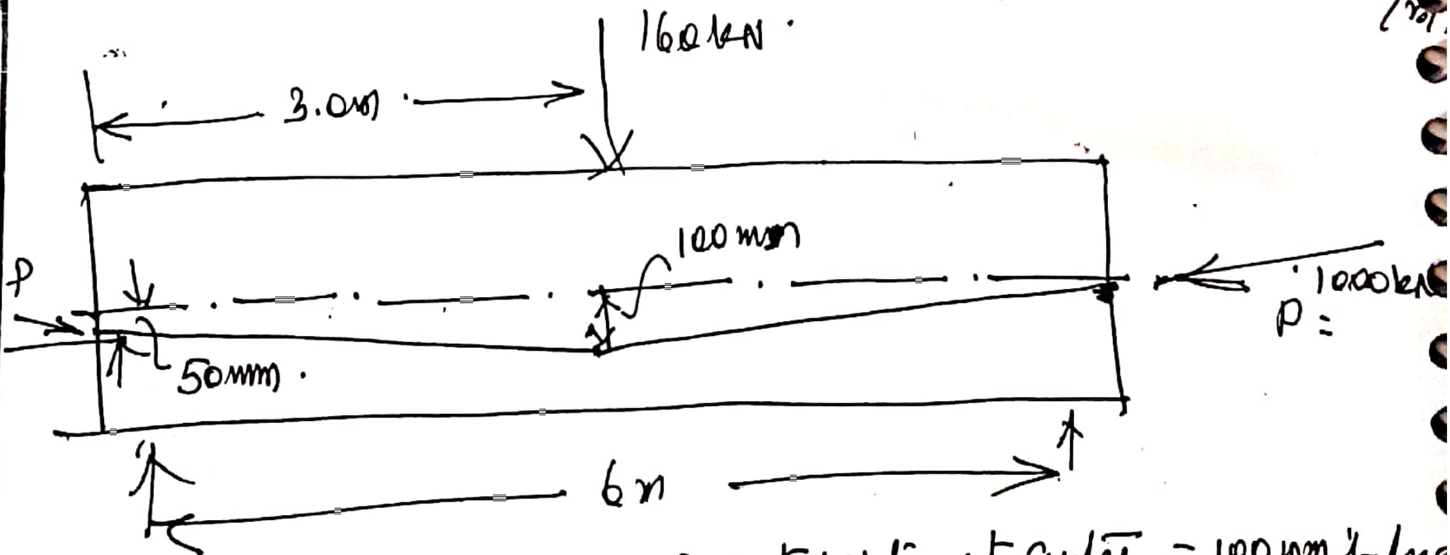
at top $= 5.0 + 5.33 = 10.33 \; N/mm^2$

at bottom $= 5.0 - 5.33 = -0.33 \; N/mm^2$

Case IV   Bent tendon with eccentricity at the ends

Ex: 3   A P.S.C beam is provided with a bent tendon as shown in the fig. The beam carries a point load of 160 kN at centre. Determine the stress distribution for the end and the mid sections of the beam.

The Self Wt. of the beam = 5.76 kN/m



Eccentricity at centre = 100 mm below

at end = 50 mm below

Net eccentricity = 100 − 50 = 50 mm.

SOLUTION:

At the End Section

$A = 400 \times 600 = 24 \times 10^4 \; mm^2$.

$Z = \frac{1}{6} \times 400 \times 600^2 = 24 \times 10^6 \; mm^3$.

Inclination of the tendon is $\sin\theta = \tan\theta = \frac{50}{3000} = \frac{1}{60}$.

(Pe)

Extreme stresses are $\frac{P}{A} \pm \frac{M}{Z} = \frac{1000 \times 10^3}{24 \times 10^4} \pm \frac{1000 \times 10^3 \times 50}{24 \times 10^6}$.

At top $= 4.17 - 2.08 = 2.09 \; N/mm^2$.

At bottom $= 4.17 + 2.08 = 6.25 \; N/mm^2$

Both Comp.

(without external loading

At Mid Section.

Net external load $= W - 2P\sin\theta = 160 - 2 \times 1000 . \frac{1}{60} = 126.67$ kN

(Downward)

B.M due to the load $= \frac{126.67 \times 6}{4} = 190 \; kNm = 190 \times 10^6 \; Nmm$.

Self Wt. $= 5.76$ kN/m, B.M due to Self Wt. $= \frac{5.76 \times 6^2}{8} = 25.92$ kNm

$= 25.92 \times 10^6$ Nmm.

Due to end eccentricity, B.M $= (1000 \times 10^3) 50 = 5000 \times 10^4$

$= 50 \times 10^6$ Nmm.

Hence, net B.M $= (190 + 25.92 - 50) 10^6 = 165.92 \times 10^6$ Nmm.

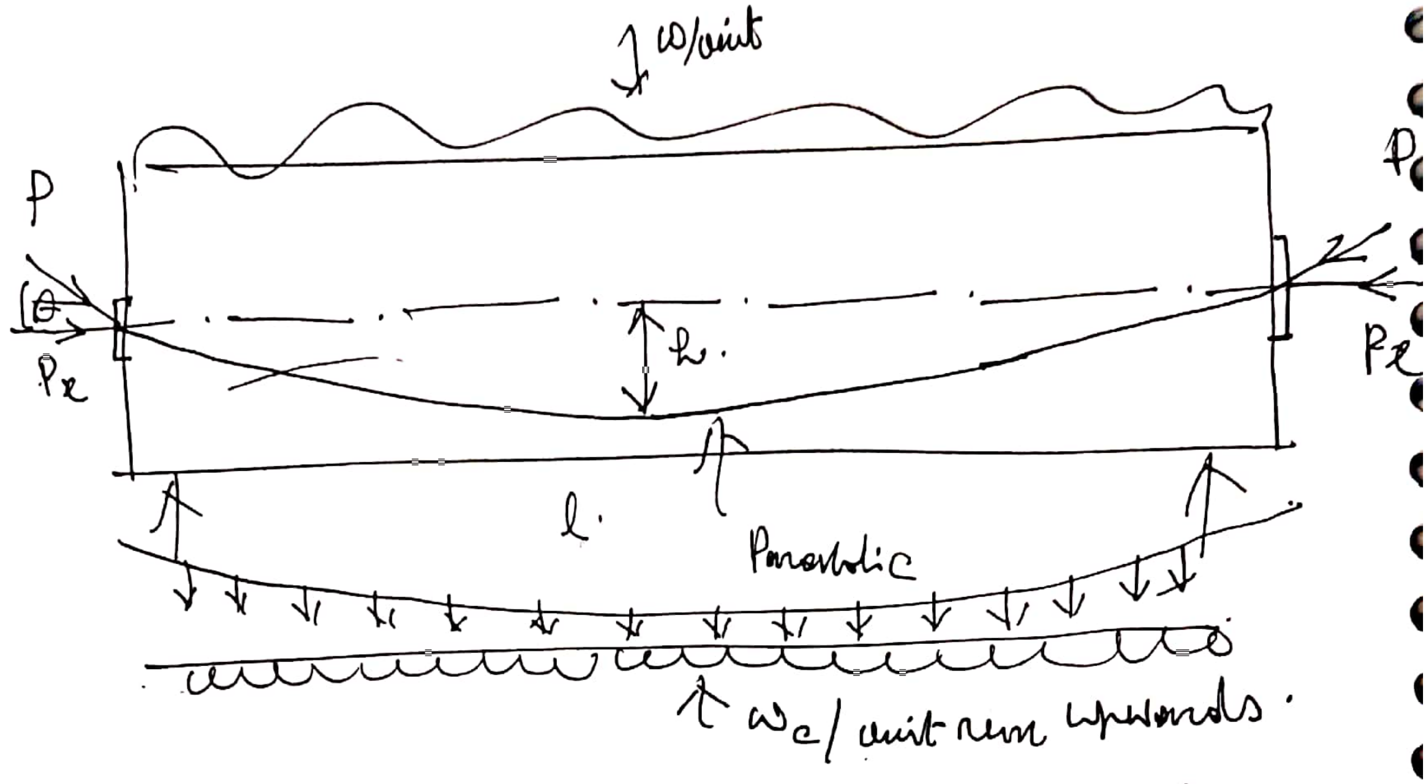
Bending stress $= \pm 165.92 \times 10^6 \times \frac{1}{24 \times 10^6} = \pm 6.91$

By adding the axial direct stress, the resultant stresses are,

At top $= 4.17 + 6.91 = +11.08 \; N/mm^2$ (Comp.)

At bottom $= 4.17 - 6.91 = -2.74 \; N/mm^2$ (Tensile)

Case VI : Tendon with Parabolic Profile.



The horiz. thrust in the curved cable $= P_x = \dfrac{w_e l^2}{8h}$

Where w_e : upward UDL. It is the pressure exerted by the cable in the upward direction.

$P_x = P\cos\theta = P$ (Approximately)

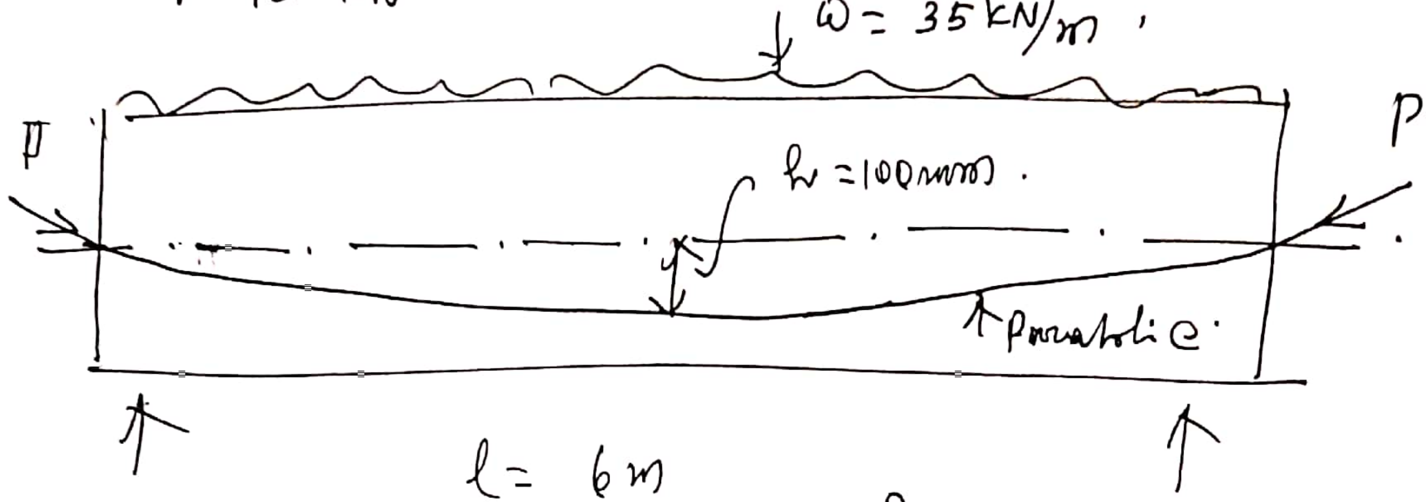
Hence $P_x = P = \dfrac{w_e l^2}{8h}$ or $w_e = 8hP/l^2$.

Hence, net downward load $= (w - w_e)$

Ex: 4,

For the PSC beam shown in the fig. calculate the extreme intensities of stresses at mid span section.

$P = 1000\,kN$.



$P = 1000\,kN$

SOLUTION:

$$A = 400 \times 600 = 24 \times 10^4\ mm^2.$$

$$Z = \frac{1}{6} \times 400 \times 600^2 = 24 \times 10^6\ mm^3.$$

$$l = 6m, \quad h = 100mm = 0.1m.$$

Hence, $\omega_e = \frac{8hP}{l^2} = \frac{8 \times 0.1 \times 1000}{6^2} = 22.22\,kN/m$,

Hence, net downward load $= 35 - 22.22 = 12.78\,kN/m$.

$$M_{max} = \frac{12.78 \times 6^2}{8} = 57.5\,kNm = 57.5 \times 10^6\,Nmm.$$

Extreme stresses at mid-span section $= \frac{P}{A} \pm \frac{M}{Z}$.

$$= \frac{1000 \times 10^3}{24 \times 10^4} \pm \frac{57.5 \times 10^6}{24 \times 10^6} = 4.17 \pm 2.39.$$

Hence, resultant stresses are, at top $= 4.17 + 2.39 = 6.56$ N/mm^2 Comp.

at bottom $= 4.17 - 2.39 = 1.78$

(49)

LOAD BALANCING METHOD.

The cable profile is arranged in such a way that the imposed load is completely neutralised by the upward UDL produced as a result of the curved cable profile. This is known as Load Balancing Method.

Ex.5. Determine the profile of a load balancing cable for a beam of span 6 m, carrying an all inclusive load of 40 kN/m. The prestressing force in the tendons is 1200 kN. The beam section is 400×600 mm.

SOLUTION: Upward pressure $w_c = \frac{8Ph}{l^2} = \frac{8 \times 1200 \times h}{6^2} = \frac{800h}{3}$ kN/m.

For complete load balancing, $\frac{800h}{3} = 40$.

Hence $h = \frac{3 \times 40}{800} = \frac{3}{20} = 0.15\,m$

$= 150\,mm$.

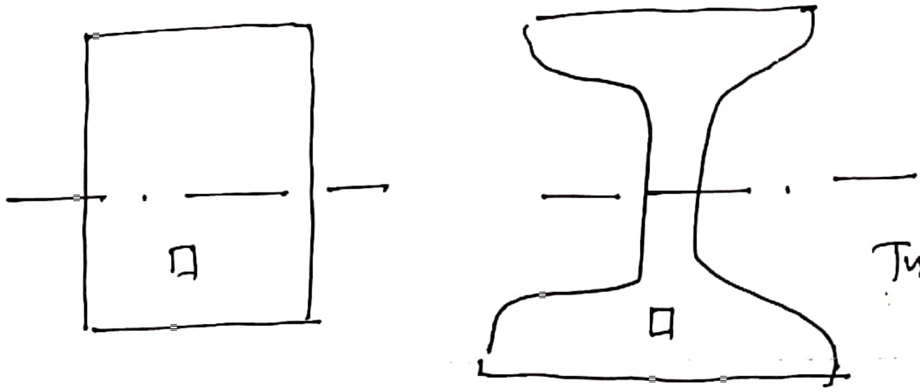
Uniform ~~Sections~~ Stress in the section due to the prestressing force

$$= \frac{P}{A} = \frac{1200 \times 10^3}{24 \times 10^4} = 5.0\ N/mm^2\ (\text{Comp.})$$

DESIGN OF PSC BEAMS: (UNIT II Cont'd).

Arrangement:

Generally in the PSC beam Section (Rectangular or I-Section), the prestressing cables are kept at some eccentricity below the centroidal axis.



Typical Sections:

I st. Stage: Stresses in the cross Section due to Self Wt. (Wo/unit)

Axial stress $f = \frac{P}{A}$, P: Prestressing force

Due to eccentricity, $B \cdot M = Pe$, Bending stress $= \pm \frac{Pe}{Z}$

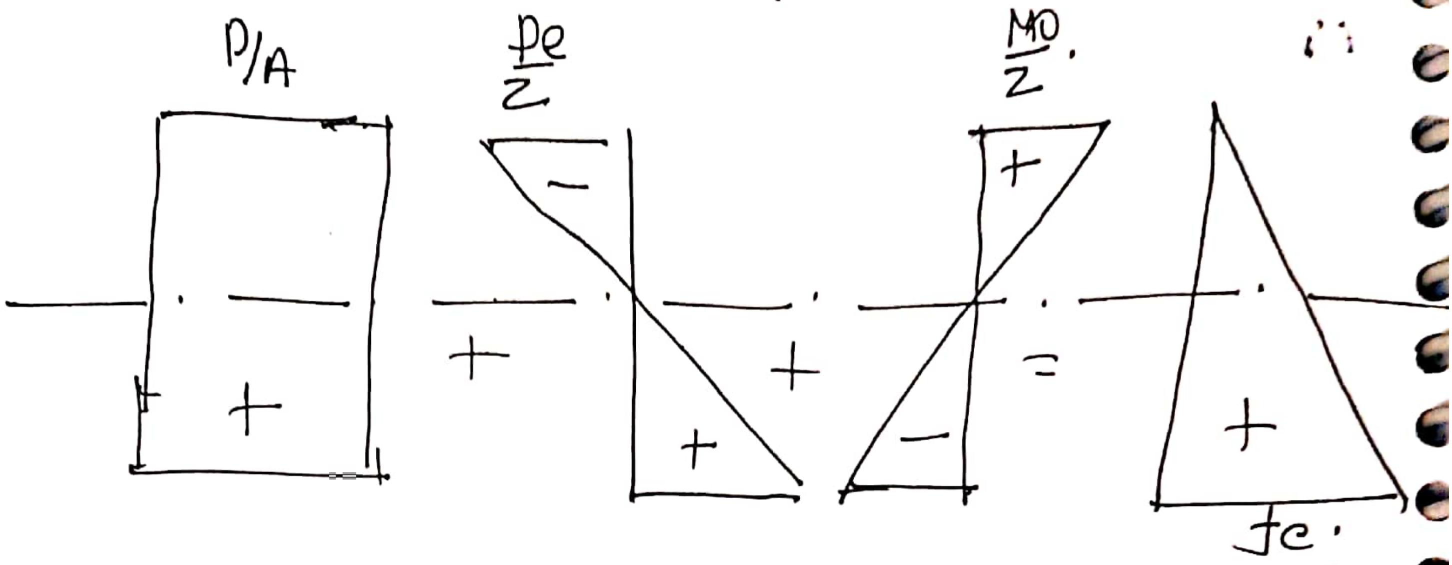
B.M. due to self wt. $= \frac{w_0 l^2}{8} = M_0$

Bending stress $= \pm M_0/Z$

Hence, resultant stress, at top $= \frac{P}{A} - \frac{Pe}{Z} + \frac{M_0}{Z}$

at bottom $= \frac{P}{A} + \frac{Pe}{Z} - \frac{M_0}{Z}$

The stress distribution is as follows.



Hence, as per the required conditions, (Required conditions) the beam should be so designed that due to only self wt. and when the external load is not applied the combination of axial stress, bending stress due to eccentricity of the prestressing force, bending stress due to the self wt. of the beam should produce a ~~result~~ resultant stress having 'zero value at top and equal to f_c' at bottom. Hence, we obtain

Total comp. force $= \frac{1}{2} f_c \, bd = P$.

At top, $\frac{P}{A} - \frac{Pe}{Z} + \frac{Mo}{Z} = 0$.

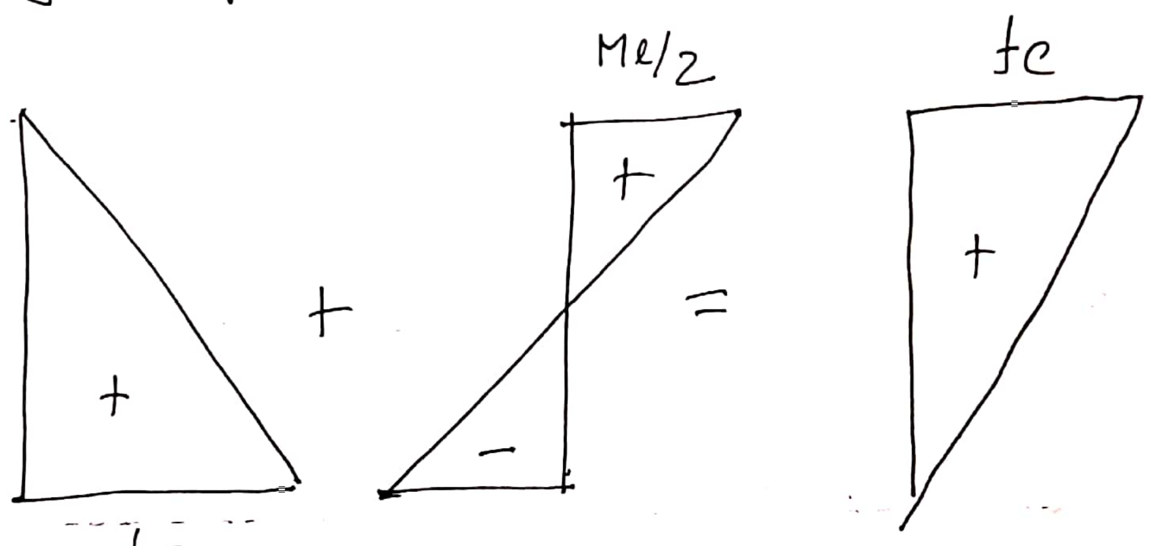
or $\frac{P}{A} + \frac{Mo}{Z} = \frac{Pe}{Z}$ (1).

At bottom, $\frac{P}{A} + \frac{Pe}{Z} - \frac{M_0}{Z} = f_c$. (2).

~~Stage~~ <u>II Stage</u> : When the live load is applied,

Let the L.L.B.M = M_e; The bending stress = $\pm \frac{M_1}{Z}$.

Adding to the previous ~~the~~ stress diagram.



(Required condition)

~~Hence~~ We get the required condition, in which the net stress at top is 'f_c' and the net stress at bottom is zero.

To satisfy this condition, we should have $\frac{M_e}{Z} = f_c$. (3).

Hence, we can write from (2),

$$\frac{P}{A} + \frac{Pe}{Z} - \frac{M_0}{Z} = f_c = \frac{M_1}{Z}.$$

$$\text{or } \frac{Pe}{Z} = \frac{(M_0 + M_e)}{Z} - \frac{P}{A}. \quad (4).$$

Adding eq. (1) and (4), we get.

$$\frac{2Pe}{2} = \frac{(M_0+M_\ell)}{2} + \frac{M_0}{2} = \frac{(2M_0+M_\ell)}{Z} \quad (5)$$

EXAMPLE :. A PSC beam of uniform rectangular section and span 15m supports a total UDL of 272 kN excluding the self wt. Determine suitable dimensions for the beam and calculate the area of the tendons and their position. The permissible stresses are 14 N/mm² for concrete and 1050 N/mm² for HT steel.

SOLUTION: M_{max} due to imposed load $= \frac{272 \times 15}{8}$

Hence, $M_\ell = 510\ kNm = 510 \times 10^6\ Nmm$.

Section modulus (z) Required $= \frac{M_\ell}{f_e} = \frac{510 \times 10^6}{14} = 36.43 \times 10^6\ mm^3$

Assuming the depth of the beam as $\frac{1}{20}$ to $\frac{1}{25}$ of the span,

Taking $\frac{1}{20} \times \ell$, $d = \frac{15 \times 10^3}{20} = 750\ mm$.

Let the width be 'b', Then $\frac{b \times 750^2}{6} = 36.43 \times 10^6$.

Hence, $b = 390\ mm$.

Hence provide a rectangular section of 390×750 mm.

(54)

From the D.L. Stress diagram, we get the prestressing force as, $P =$ Area of the diagram $= \frac{1}{2} f_c \cdot A$.
Avg. × Stress. $f_c = 14.0\ N/mm^2$.

Hence $P = \frac{f_c \cdot bd}{2} = \frac{14 \times 390 \times 750}{2 \times 10^3} = 2047.5$ kN.

Hence, area required for tendons $= \frac{P}{\text{Safe stress}}$

$= \frac{2047.5 \times 10^3}{1050} = 1950\ mm^2$.

Area of 6 mm bar $= 28\ mm^2$, Hence, NO. $= \frac{1950}{28} = 70$ NOS.

B.M. due to Self wt. $= \frac{(0.39 \times 0.75 \times 24)\ 15^2}{8} = 197.4$ kNm.

$= 197.4 \times 10^6$ Nmm.

(∵ eq. 5)

We have derived that $\frac{2Pe}{Z} = \frac{2M_0 + M_l}{Z}$.

or $e = \frac{M_0}{P} + \frac{M_l}{2P} = \left[\frac{197.4 \times 10^6}{2047.5 \times 10^3} + \frac{510 \times 10^6}{2 \times 2047.5 \times 10^3}\right]$

Hence $e = 221$ mm.

Hence, the tendons are kept at an eccentricity of 221 mm below the centre of the beam.

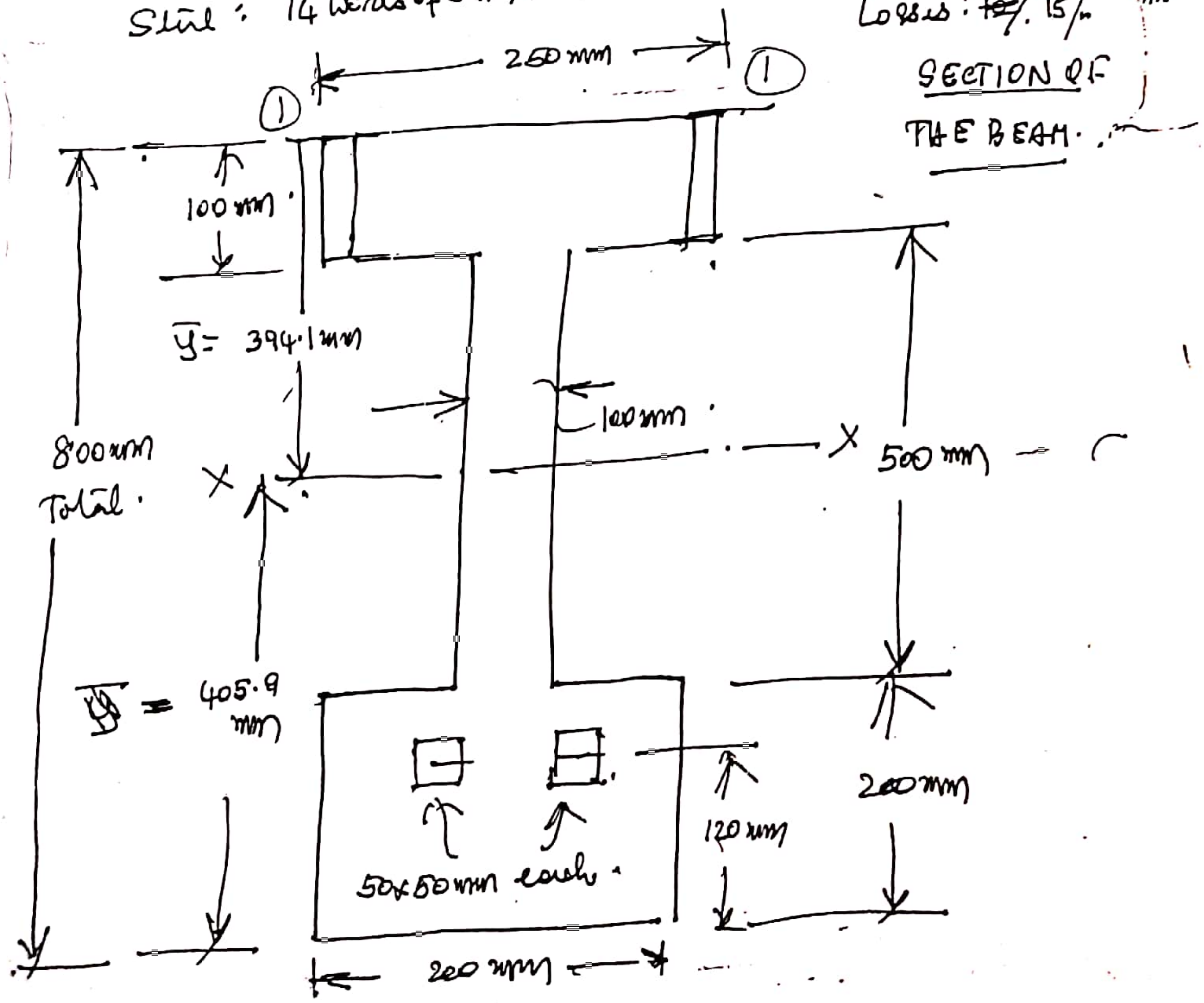
Scanned with CamScanner

EXAMPLE : 2 : Figure shows the section of a PSC beam of span 9 m. The beam has to carry a super load of 20 kN/m. The prestressing force is transmitted by tendons in 2 cables. In each cable the tendons consist of 14 wires of 5 mm dia. subjected to an initial prestress of 1000 N/mm². Analyse the beam section at centre for the stresses induced before and after the application of the super load. Allow ~~10%~~ 15% loss of prestress.

DATA: Span of the beam : ~~9m~~ 9m, Super load = 20 kN/m

Steel : 14 wires of 5 mm Dia. in each cable, Initial prestress : 1000 N/mm²

Losses : ~~10%~~ 15%



SOLUTION. The computations of area, Position of ~~the~~ ~~C~~ centroidal axis, M.I are shown in the table.

Component.	Area 'a' mm^2	Distance of its centroid from 1-1 y mm	ay	ay^2	I_{self} (About its own axis) mm^4
Top flange	250×10^2	50	1250×10^3	6250×10^4	$\frac{250 \times 100^3}{12} = 2083 \times 10^4$
Web	500×10^2	350	$17,500 \times 10^3$	612500×10^4	$100 \times 500^3/12 = 104,167 \times 10^4$
Bottom flange	400×10^2	700	$28,000 \times 10^3$	$1960,000 \times 10^4$	$200 \times 200^3/12 = 13,333 \times 10^4$
Gross Total.	1150×10^2	~~46750×10³~~	~~2578750 ×10⁴~~ $46,750 \times 10^3$	2578750×10^4	119583×10^4
Deduction for cable ducts. $(2 \times 50 \times 50)$	50×10^2	680	3400×10^3	$2,31,200 \times 10^4$	$\frac{2 \times 5 \times 5^3}{12} = 104 \times 10^4$
Net Total	~~1007~~ 1100×10^2	—	$43,350 \times 10^3$	2347550×10^4	119479×10^4

Dist. of the c.g, $\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{43,350 \times 10^3}{1100 \times 10^2} = 394.1\,mm$

About section 1-1, $M.I = I_{1-1} = \Sigma I_{self} + \Sigma ay^2$

Hence. $I_{1-1} = (119479 + 2347550)10^4 = 2467029 \times 10^4\,mm^4$.

Now about the centroidal axis $I_{xx} = I_{11} - a\bar{y}^2$.

Hence, $I_{xy} = [2467029 \times 10^4 - 1100 \times 10^2 \times 394.1^2]$

$I_{xy} = 7,58,570 \times 10^4\,mm^4$

Hence $Z_{Top} = \frac{7,58,570 \times 10^4}{394.1} = 19,250 \times 10^3\,mm^3$, $Z_{Bottom} = \frac{7,58,570 \times 10^4}{405.9} = 18,680 \times 10^3\,mm^3$

$Z_t = \left(\frac{I_{xx}}{y}\right)$ $\qquad Z_b$

D.L. of the girder $= \left(\frac{1100 \times 10^2}{10^6}\right)_{m^2} \times \underset{(m)}{1} \times \underset{(kN)}{\frac{2400}{10^2}} = 2.64\,kN/m$.

$D.L.B.M = \frac{2.64 \times 9^2}{8} = 26.73\,kNm$

Area of 5 mm dia $= \frac{\pi}{4} \times 5^2 = 20\,mm^2$

Initial prestressing force 'F' $= (2 \times 14 \times 20)\,1000 = 5,60,000\,N$

$= 560\,kN$.

eccentricity from the centroid 'e' $= 405.9 - 120 = 285.9\,mm$

Direct comp. stress due to 'F' $= \frac{F}{A} = \frac{5,60,000}{1100 \times 10^2} = 5.091\,N/mm^2$

Due to eccentricity, $f_{(Top)} = -\frac{5,60,000 \times 285.9}{19,250 \times 10^3} = -8.318\,N/mm^2$,

$f_{(Bottom)} = +\frac{5,60,000 \times 285.9}{16,680 \times 10^3} = +8.57\,N/mm^2$

Scanned with CamScanner

Due to D.L.B.M, $f_{(Top)} = +\frac{26.73\times10^6}{19,250\times10^3} = +1.389\ N/mm^2$ (Comp.)

$f_{(Bottom)} = -\frac{26.73\times10^6}{18,680\times10^3} = -1.431\ N/mm^2$ (Tension)

Hence in the first stage, before the L.L is applied, the net total stresses are,

At top, $f = +5.091 - 8.318 + 1.389 = -1.838\ N/mm^2$

At Bottom $f = +5.091 + 8.57 - 1.431 = +12.232\ N/mm^2$

After the application of the L.L.

After allowing for 15% losses, final prestressing force

$= 0.85 \times 5,60,000 = 4,76,000\ N.$

$L.L.B.M = \frac{20\times9^2}{8} = 202.5\ kNm = 202.5\times10^6\ Nmm$

Direct stress $= \frac{4,76,000}{1100\times10^2} = +4.327\ N/mm^2$

Due to eccentricity $f_{(Top)} = -\frac{4,76,000\times285.9}{19,250\times10^3} = -7.07\ N/mm^2$

$f_{(Bottom)} = +\frac{4,76,000\times285.9}{18,680\times10^3} = +7.37\ N/mm^2$

Due to L.L.B.M, $f_{(Top)} = +\frac{202.5\times10^6}{19,250\times10^3} = +10.5\ N/mm^2$

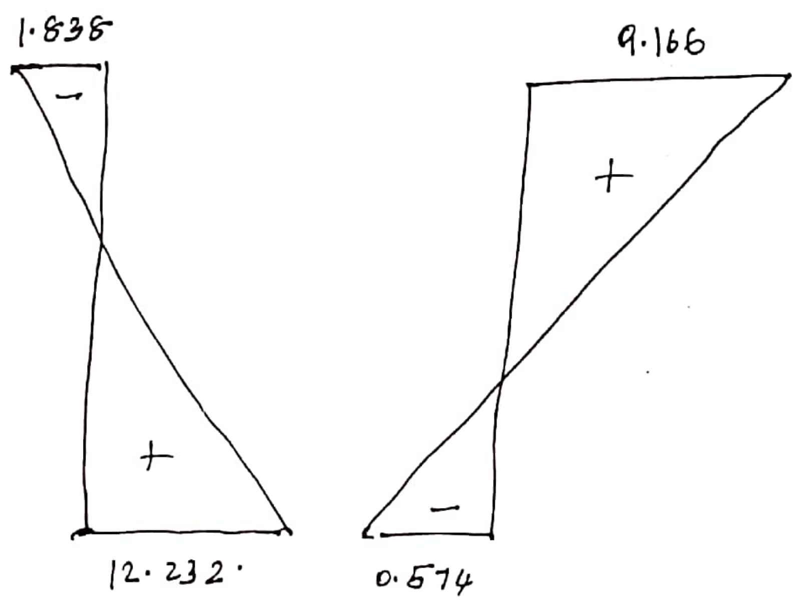
$f_{(Bottom)} = -\frac{202.5\times10^6}{18,680\times10^3} = -10.84\ N/mm^2$

Stresses due to D.L.B.M have already been computed,

FINAL STRESSES :

	At top N/mm^2	At Bottom N/mm^2
1. Direct stress.	$+4.327\ N/mm^2$	$+4.327$.
2. Due to eccentricity	-7.07	$+7.37$.
3. Due to DLBM.	$+1.389$.	-1.431.
4. Due to LLBM.	$+10.52$	-10.84
Net total.	$+9.166\ N/mm^2$	$-0.574\ N/mm^2$.

Stresses.



a) Before LL was applied

b) After the LL was applied.

Indian Standard Code of Practice IS 1343-1980 for Prestressed Concrete.

Recommendations for Design

Concrete:

1) Min. grade is M-30 (Post tensioning) and M-40 for (pretensioning)

2) In addition to all the data, minimum cement content to satisfy durability requirements should be maintained.

3) Design mix concrete should be used.
Max. cement content ≯ 530kg/m³ should be preferably maintained.

Reinforcement:

Hard drawn H.T. steel wires with dia from 1.5mm to 8mm may be used. The Ultimate strength varies from 2350 to 1500N/mm².

Prestressing Equipment.

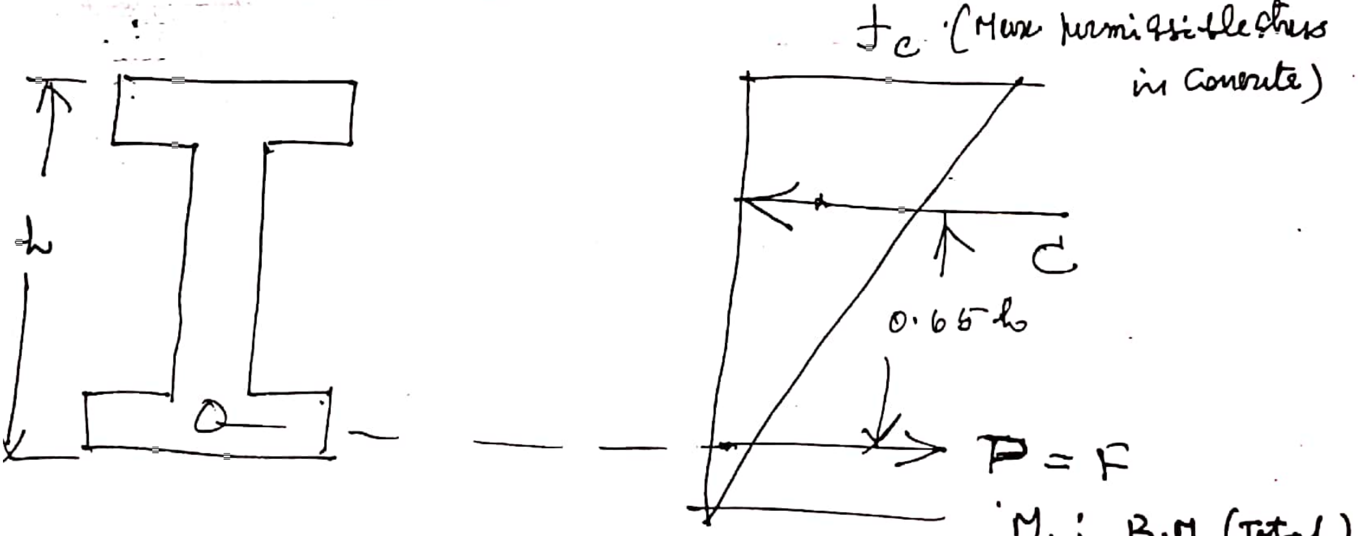
Standard equipment should be used for prestressing. When the tendon is subjected to a load which is midway between the initial prestressing load and the ult. strength of the prestressing tendon, then the anchorage shall be capable of holding without more than nominal slips.

Other Requirements

Other design requirements for cover, spacing etc. should be as per IS:1343.

Design: In calculating areas and M.I's allowance for tendons is not necessary in the case of pretensioning, whereas it is necessary in the case of post tensioning.

Preliminary Design



The lever arm corresponding M.R. produced is approximately $= 0.65\,h$, h can be taken $= \frac{l}{12}$ to $\frac{l}{15}$

Hence $C = F = \frac{M_t}{0.65h} = P$, $A_t = \frac{P}{f_s} = \frac{M_t \text{ (Approx.)}}{0.65\,h\,f_s}$

Where f_s : Permissible stress in tendons.

Total Compression $= \frac{f_c}{2} A_c = 0.5 f_c A_c$, A_c : Area of Concrete.

or $A_c = \frac{C}{0.5 f_c} = A_t f_s / 0.5 f_c$

DESIGN EXAMPLE: Design a P.S.C beam for the following requirements.

1. Span : 15 m
2. Super load : 34 kN/m
3. 28 d. cube strength of concrete $f_c = 35 N/mm^2$ (Characteristic strength)
4. Safe stress at transfer $= 0.5 f_c$ (17.5 N/mm²)
5. Due to final prestress $= 0.4 f_c$ (14.0 N/mm²)
6. Total loss of prestress $= 20\%$.
7. Allowable tensile stress in concrete ~~1000~~ 1.0 N/mm².
8. Ult. stress in steel : 1500 N/mm².
9. Safe stress in steel : 60% of Ult. stress = 900 N/mm²

SOLUTION.

Stresses.

Permissible stress in concrete at transfer $= f_t = 0.5 f_c$

$= 17.5 N/mm^2$.

" due to final prestress $= 0.4 f_c = 14.0 N/mm^2$.

" tensile stress in concrete $= 1.0 N/mm^2$

" in steel $= 0.6 \times 1500 = 900 N/mm^2$.

$M_l = \frac{wl^2}{8} = \frac{34 \times 15^2}{8} = 956.3$ kNm.

Approximate D.L (at 20%) $= \frac{34}{5} = 6.8$ kN/m

Approximate D.L.B.M (Assume) $= 20\%$ of $M_l = \frac{1}{5} \times 956.3$
$= 191.3$ kNm.

$M_t = M_l + M_d = 956.3 + 191.3 = 1147.6$ kNm.

(114.76 tm).

Using the empirical formula, $h = d_{overall} = k\sqrt{M_t}$

Where M_t : t-m and h is in cm; k : 11 to 14

For the present design, $h = 11\sqrt{114.76} = 117.9$ cm.

(or take $\frac{l}{12}$ to $\frac{l}{15}$ approximately) $= 1179$ mm

Provide $h = 1200$ mm ✓

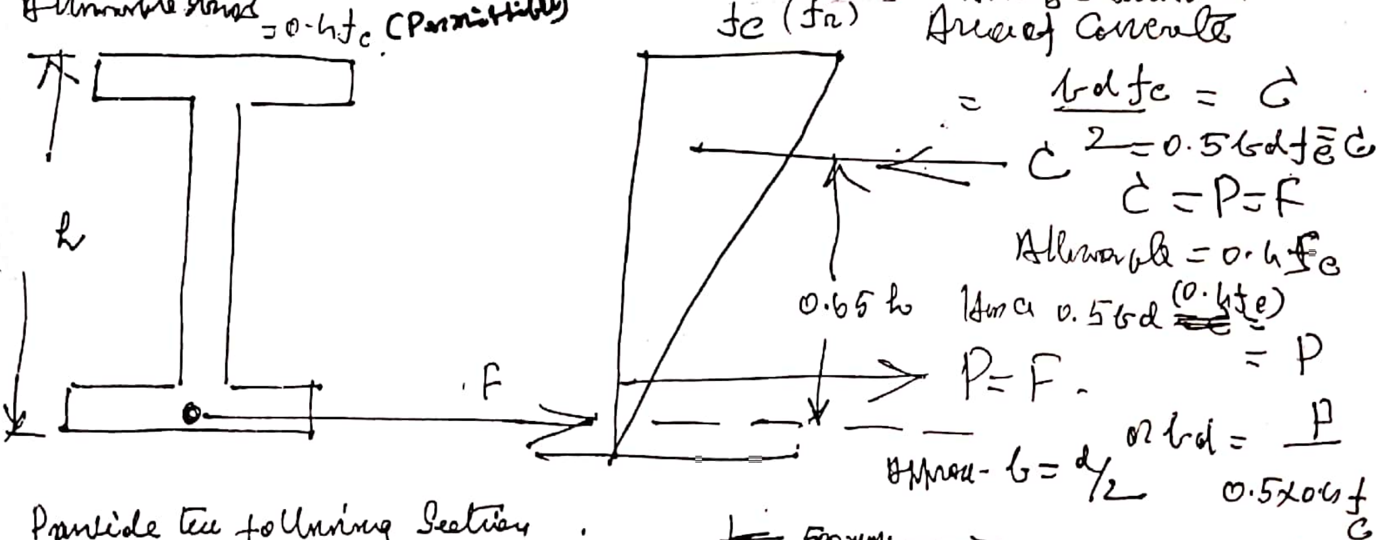
Prestressing force, $P = \frac{M_t}{0.65h}$ (0.65 h is the lever arm, approximate)

Hence, $P = \frac{\cancel{114.76}\ 1147.6 \times 10^6}{0.65 \times 1200} = 14{,}72{,}000$ N.

Area of Concrete required $= \frac{P}{0.5 f_c (0.4 f_c)} = \frac{14{,}72{,}000}{0.5 \times 14} = 2103 \times 10^2$ mm²

$= \frac{P}{0.5 f_c}$

Allowable stress $= 0.4 f_c$ (Permissible)



Provide the following Section.

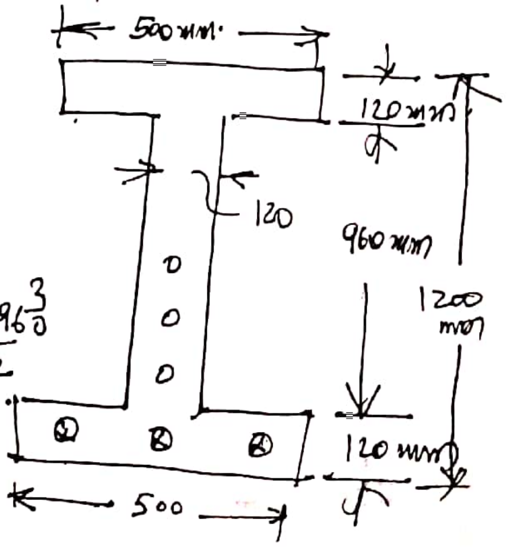
For the Section provided,

$A = 2 \times 500 \times 120 + 120 \times 960$

$= 2352 \times 10^2$ mm²

$I = \cancel{\frac{50 \times 120^3}{12}} \; \frac{500 \times 1200^3}{12} - \frac{(500-120)\,960^3}{12}$

$= 4398000 \times 10^4$ mm⁴



$Z = \dfrac{I}{d/2} = \dfrac{I}{600 \text{ (mm)}} = \dfrac{43{,}98{,}000 \times 10^4}{600} = 73{,}300 \times 10^3 \text{ mm}^3.$

Actual $D.L/m = \dfrac{2362 \times 10^2}{10^6} \times 1.0 \times 24 = 5.645 \text{ kN/m}$

(less than the assumed. Hence it is on the safe side).

Actual $D.L B M = \dfrac{5.645 \times 15^2}{8} \text{ kNm} = 158.8 \text{ kNm}.$

Providing 6mm tendons, area of one wire $= 28 \text{ mm}^2$.

assuming the same A_t (as it is on the safer side)

the No. of wires required $= \dfrac{P}{\text{Safe stress in steel}} \times \dfrac{1}{\text{Area of one wire}}$

$= \dfrac{14{,}72{,}000}{900} \times \dfrac{1}{28} = 59 \text{ Nos say } 60 \text{ Nos}.$

Hence, provide ~~10 Nos~~ 6 Nos cables with ~~6~~ 10 Nos wires in each.

$A_t \text{ (actual)} = 60 \times 28 = 16.8 \times 10^2 \text{ mm}^2.$

Hence, final prestressing force that can be safely taken

$= 900 \times 16.8 \times 10^2 = \boxed{15{,}12{,}000 \text{ N}}$ (final)

Allowing for 20% losses, Initial prestressing force required (More) $= \dfrac{15{,}12{,}000}{0.8} = \boxed{18{,}90{,}000 \text{ N.}}$ ✓

(Initial)

Scanned with CamScanner

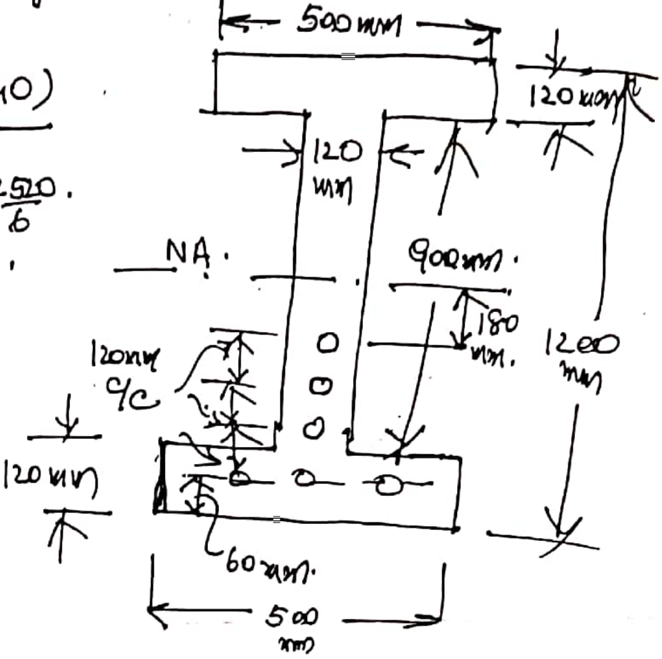
Check for stresses

The prestressing cables are arranged as shown in the fig.

c.g of the cables is given by

$$\frac{A(180+300+420+\overset{3\times}{540})}{6A} = \frac{2520}{6}$$

$= \frac{420}{\cancel{240}}$ mm from the N.A.



Hence $e = 420$ mm.

Comp. (+ve)
Tension (−ve).

Initial

$$f_t(\text{Top}) = \frac{18,90,000}{2352\times10^2} - \frac{18,90,000\times420}{73,300\times10^3} + \frac{158.8\times10^6}{73,300\times10^3}$$

$$\frac{P}{A} - \frac{Pe}{Z} + \frac{M_d}{Z}$$

$= -0.599\ N/mm^2$

$$f_b(\text{Bottom}) = \frac{P}{A} + \frac{Pe}{Z} - \frac{M_d}{Z} = +16.707\ N/mm^2.$$

Final.

$$f_t = \frac{15,12,000}{2352\times10^2} - \frac{15,12,000\times420}{73,300\times10^3} + \frac{158.8\times10^6}{73,300\times10^3} + \frac{956.3\times10^6}{73,300\times10^3}$$

Act. (D.LBM + LLBM)

$$\frac{P}{A} - \frac{Pe}{Z} + \frac{DLBM}{Z} + \frac{LLBM}{Z}.$$

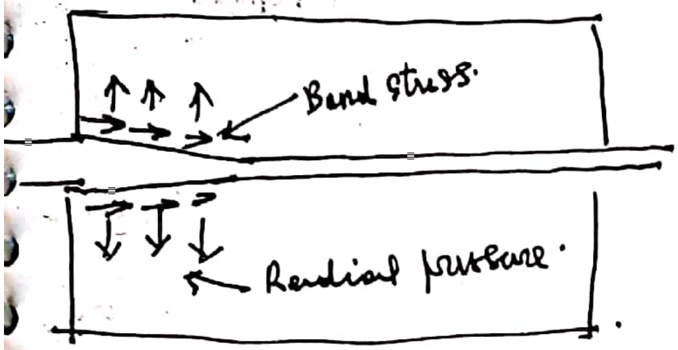
$= 12.831\ N/mm^2.$

The stress distribution can be plotted.

$$f_b = \frac{P}{A} + \frac{Pe}{Z} - \frac{DLBM}{Z} - \frac{LLBM}{Z} = -0.125\ N/mm^2$$

Hence, the stresses are within the safe limits. The section may be adopted.

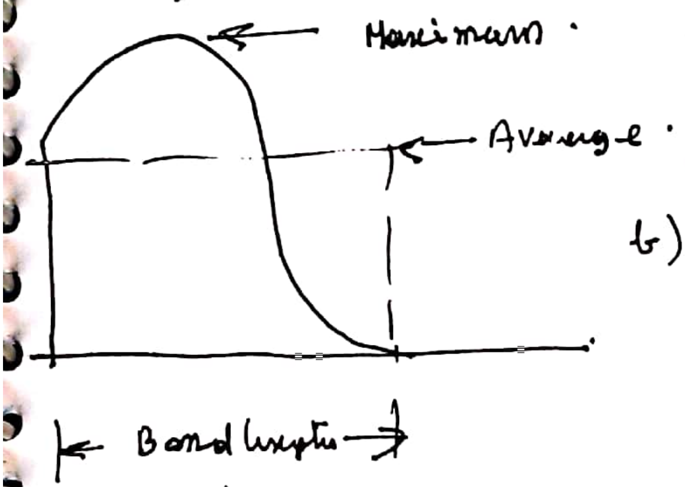
TRANSMISSION OF PRESTRESS IN PRETENSIONED MEMBERS.



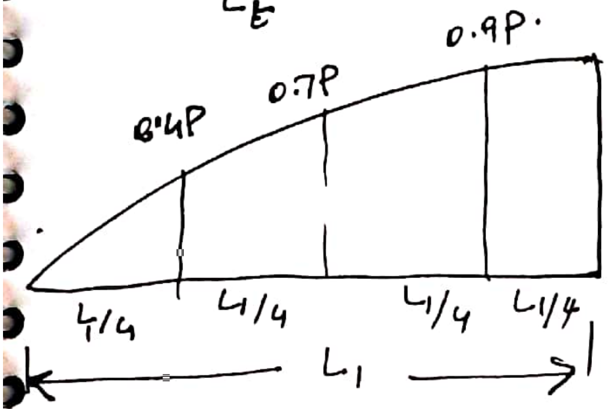
a).

a) Hoyer Effect:

After the member is ready, the end of the reinforcement is released. The cross section bulges out and radial pressures are produced.

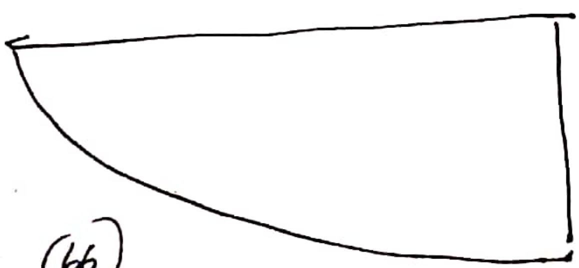


b) Bond stress between wire and Concrete.



P (Prestressing force)

c) Variation of stress in steel.



d) Stress in Concrete.

The max. values of stresses in steel and concrete are reached at the end of zone of compression after a length known as the 'TRANSMISSION LENGTH'

TRANSMISSION LENGTH

is the length required at the ends of a pretensioned member for the build up of stress in Concrete, so as to resist the applied S.F and B.M. This is particularly important in short pretensioned members.

The transmission length depends mainly on the diameter and the surface characteristics of the wire, elastic properties of steel and Concrete and the Coefficient of friction between steel and Concrete.

LIMIT STATE DESIGN OF PSC MEMBERS

(As per IS 1343-2012)

Design loads.

They are obtained as the products of characteristic loads (working) and partial safety factors.

Partial Safety factors

They are same as given in IS 456-2000.

D.L + L.L = 1.5 , DL + LL + WL or E.L = 1.2.

Partial Safety factors for materials.

1.5 for concrete and 1.15 for steel.

Types of Limit States

1) Limit state of collapse.

2) Limit state of serviceability.

Classes of structures (For serviceability considerations)

Class 1 : In these structures no tension is permitted in concrete

Class 2 : Limited tension is permitted but not exceeding the modulus of rupture of concrete.

Class 3 : Tension even exceeding the modulus of rupture under service loads of concrete is permitted. Visible cracking within the permissible crack width is permitted. This case arises in the case of partially prestressed members with conventional reinforcement and limited prestressing steel.

As per IS 1343 - 2012 (pp. 30, 31 and 32 of the Code) (2)

Simplified Code procedure for LSD

It is based on rectangular and parabolic stress block.

$$\left(\frac{0.67 f_{ck}}{1.5}\right) = 0.45 f_{ck}$$

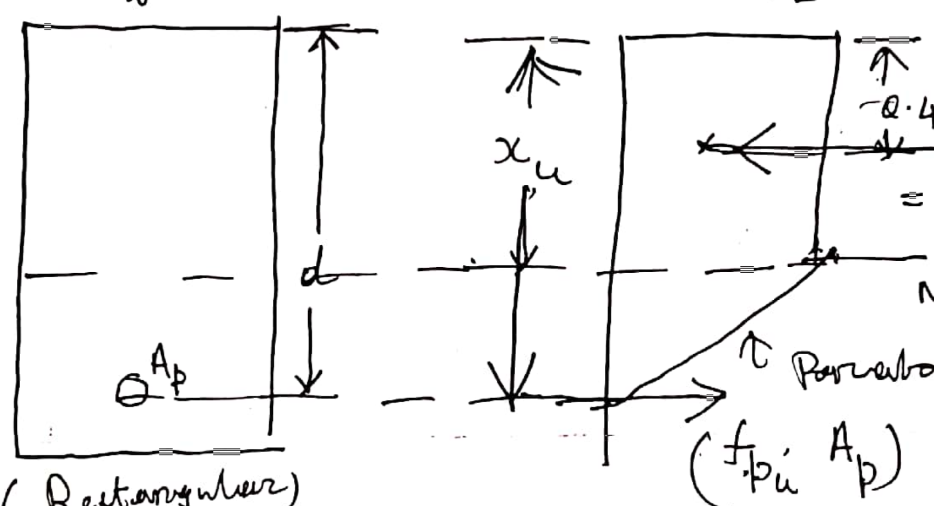


Fig. 1

Moment of resistance of the Section $= f_{pu} A_p (d - 0.42 x_u) = M_u$ (Ultimate)

x_u: Neutral axis depth; M_u: Ultimate M.R.

A_p: Area of prestressing steel

f_{pu}: Stress in steel at the time of failure.

f_p, f_{pk}: Characteristic tensile stress in prestressing steel,

f_{pe}: Stress in tensile steel after all the losses (Effective)

A_p: Area of prestressing steel.

b, d are the width and effective depth of the Section.

The values are given in the table. The tables are given assuming fully bonded sections (between concrete and cables).

Table 1. Conditions for both pretensioned and post tensioned rectangular beams (fully bonded)

Sl-NO.	$\frac{A_p f_p}{b\,d\,f_{ck}}$	Stress in tendons as a proportion of the designed strength $f_{pu}/0.87 f_{p}$		Ratio of x_u/d	
		Pretensioning	Post tensioning	Pretensioning	Post tensioning
1	0.025	1.0	1.0	0.054	0.054
2	0.50	1.0	1.0	0.109	0.109
3	0.10	1.0	1.0	0.217	0.217
4	0.15	1.0	1.0	0.326	0.326
5	0.20	1.0	0.95	0.435	0.414
6	0.25	1.0	~~0.85~~ 0.90	0.542	0.488
7	0.30	1.0	0.85	0.655	0.558
8	0.4	0.90	0.75	0.783	0.653.

NOTE:

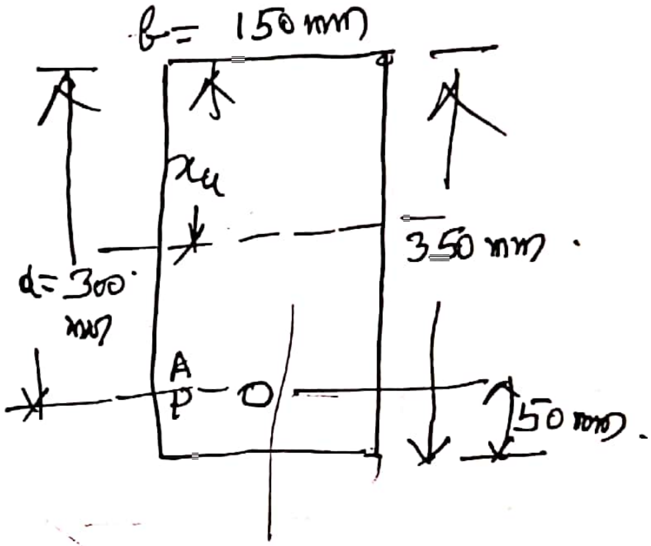
1) The effective prestress after all losses should not be less than $0.45 f_p$ $(f_{pe} \nless 0.45 f_p)$

2) For flanged sections the stress block should be taken as shown in the fig. 2

4

Ex. 1 A prestressed and pretensioned beam 150×350 mm section, has an eff. cover of 50 mm. If $f_{ck} = 40\,N/mm^2$ and $f_p = 1600\,N/mm^2$ and area of prestressing steel $A_p = 461\,mm^2$, calculate the ultimate flexural strength of the section (M_u) as per IS1343.

SOLUTION



The reinf. ratio

$$\frac{f_p A_p}{f_{ck}\,b\,d} = \frac{1600\times461}{40\times150\times300} = 0.40$$

From table (1) we get

$$\frac{f_{pu}}{0.87 f_p} = 0.9 \text{ and } \frac{x_u}{d} = 0.783$$

Hence, $f_{pu} = 0.9 \times 0.87 \times 1600 = 1253\,N/mm^2$, $x_u = 0.783\times300 = 234.90\,mm$

Hence $M_u = f_{pu} A_p (d - 0.42 x_u) = 1253\times461(300 - 0.42\times234.9)$

$M_u = 116\times10^6\,Nmm = 116\,kNm$.

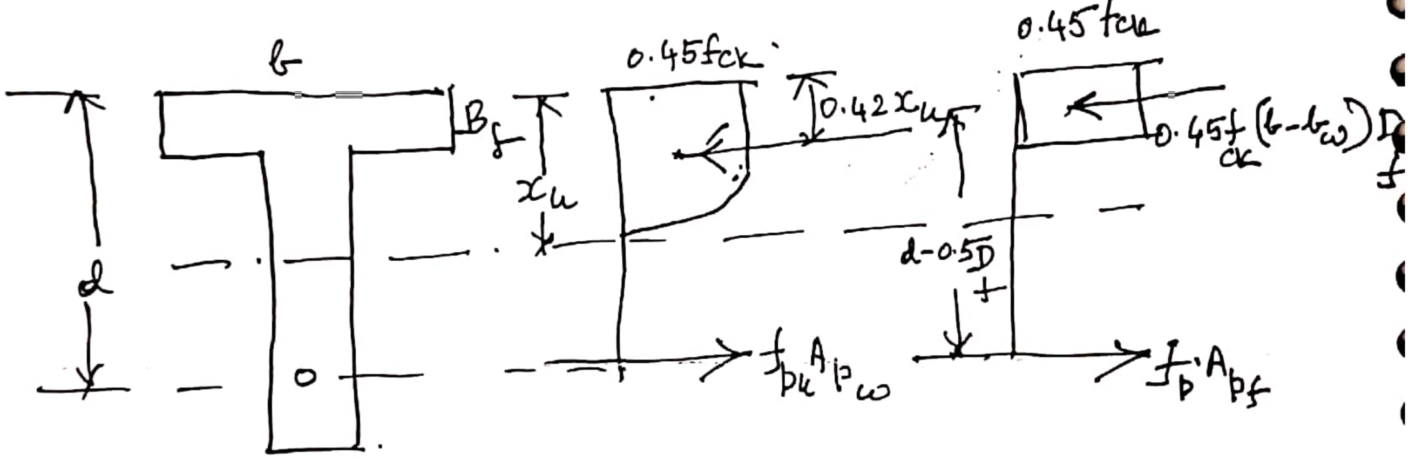
Ex. 2 In a pretensioned T-beam, the flange is 300 mm wide and 200 mm thick. The rib is 150 mm thick and 350 mm deep. The effective depth of the cross section is 500 mm, $A_p = 200\,mm^2$, $f_{ck} = 50\,N/mm^2$ and $f_p = 1600\,N/mm^2$. Estimate the ultimate moment capacity of the T-section as per IS 1343.

FLANGED SECTIONS

1. If the N.A is passing through the flange, then M_u is calculated as in the case of rectangular section.

$$M_u = f_{pu} A_p (d - 0.42 x_u) \quad (1)$$

2: If the N.A is passing through the web,



A_{pf} and A_{pw} are the areas of prestressing steels in flange and web respectively.

$$A_p (\text{total}) = A_{pf} + A_{pw}$$

$$A_{pf} = 0.45 f_{ck} (b - b_w) \frac{D_f}{f_p} \cdot (\text{Due to flange})$$

Hence. $A_{pw} = (A_p - A_{pf})$

For the effective reinforcement ratio of $\left[\frac{A_{pw} \cdot f_p}{b_w d f_{ck}} \right]$

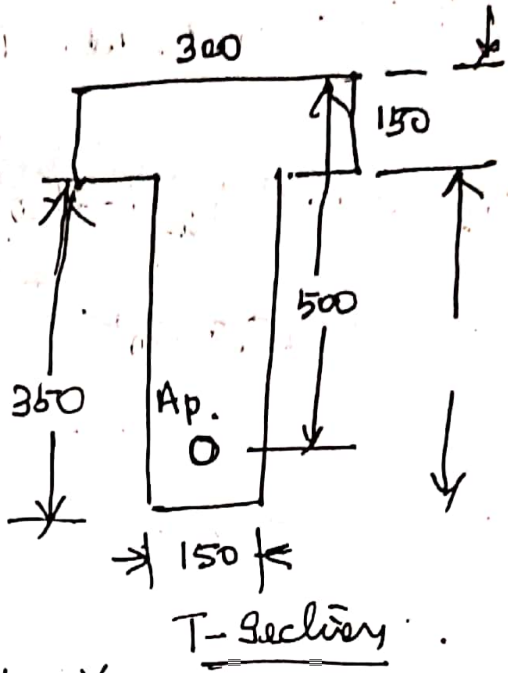
For the above ratio the values of

$f_{pw} / 0.87 f_p$ and x_u/d are obtained from the table 1.

Hence, $M_u = f_{pw} A_{pw} (d - 0.42 x_u) + 0.45 f_{ck} (b - b_w) D_f (D - 0.5 D_f)$

SOLUTION

1) Assuming the N.A is passing through the flange of the Section having $b = 300$ mm, We get



$$\frac{f_p A_p}{f_{ck} b d} = \frac{1600 \times 200}{50 \times 300 \times 500} = 0.04$$

Hence, from the table, we get

$$\frac{f_{pu}}{0.87 f_p} = 1.0 \;, \quad \frac{x_u}{d} = 0.09$$

Hence, $f_{pu} = 0.87 \times 1600 = 1392$ N/mm²

$x_u = 0.09 \times 500 = 45$ mm

Hence, the N.A falls within the flange.

Hence M_u (ultimate) $= f_{pu} A_p (d - 0.42 x_u) = 1392 \times 200 (500 - 0.42 \times 45)$

Hence $M_u = 134 \times 10^6$ N mm $= 134$ kNm.

EX. 3 Calculate the flexural strength of the T-beam for the following data.

Flange: 1200 mm wide and 150 mm thick (b_f)

Web; 300×1500 mm (b)

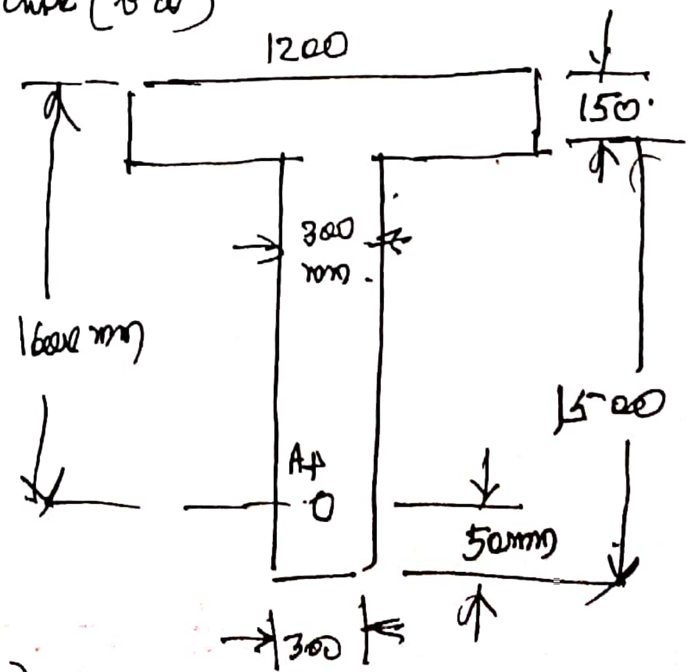
$A_p = 4700$ mm²

$d_{eff} = 1600$ mm, Eff. Cover = 50 mm

$f_{ck} = 40$, $f_p = 1600$ N/mm²

Overall depth $D = 1500 + 150 = 1650$ mm.

D_f (flange) $= 150$ mm.



SOLUTION

We have, $A_p = (A_{pw} + A_{pf})$

$$A_{pf} = 0.45 f_{ck}(b - b_w)\frac{D_f}{f_p} = 0.45 \times 40\,(1200-300)\,\frac{150}{1600}$$

$$= 1518 \text{ mm}^2$$

Hence $A_{pw} = (4700 - 1518) = 3182 \text{ mm}^2$

Reinforcement ratio for the web portion is,

$$\frac{A_{pw} \cdot f_p}{b_w d\, f_{ck}} = \frac{3182 \times 1600}{500 \times 1600 \times 40} = 0.265,$$

From table 1, $\dfrac{f_{pu}}{0.87 f_p} = 1.00$, $f_{pu} = 0.87 \times 1600 = 1392 \text{ N/mm}^2$

$\dfrac{x_u}{d} = 0.56$, $x_u = 0.56 \times 1600 = 896$ mm.

It is falling within the web.)

Hence, $M_u = f_{pu} A_{pw}(d - 0.42 x_u) + 0.45 f_{ck}(b - b_w) D_f (D - 0.5 D_f)$

(Taken as 'd')

Hence M_u (Total) $= 1392 \times 3182\,(1600 - 0.42 \times 896) + 0.45 \times 40 \times (1200-300) \times (150)\left(1600 - \frac{150}{2}\right)$

Hence $M_u = (5420 + 3705)10^6 = 9125 \times 10^6$ Nmm $= 9125$ kNm

NOTE: In all the above cases, fully bonded sections are considered. For unbonded tendons, we have to refer to table 2 given in IS1343-2012.

COMPOSITE CONSTRUCTION.

Types of Composite Construction

1) <u>Unpropped:</u>

The precast beams are not supported. Hence, the precast units have to carry their own self wt. and also the dead load effect of the in-situ slab cast over it. in addition the effect of prestressing force and its eccentricity. After the entire roof is ready the L.L is applied and its effect is taken by the beam and slab unit (T-Section)

2) <u>Propped.</u>

In this case, the precast units are supported (propped) while the slab is being cast, then the precast unit carries the effects of Prestressing force, its eccentricity and its self wt.

The whole Composite unit carries the effect of the self wt. of slab, and the L.L effect.

9 - Section

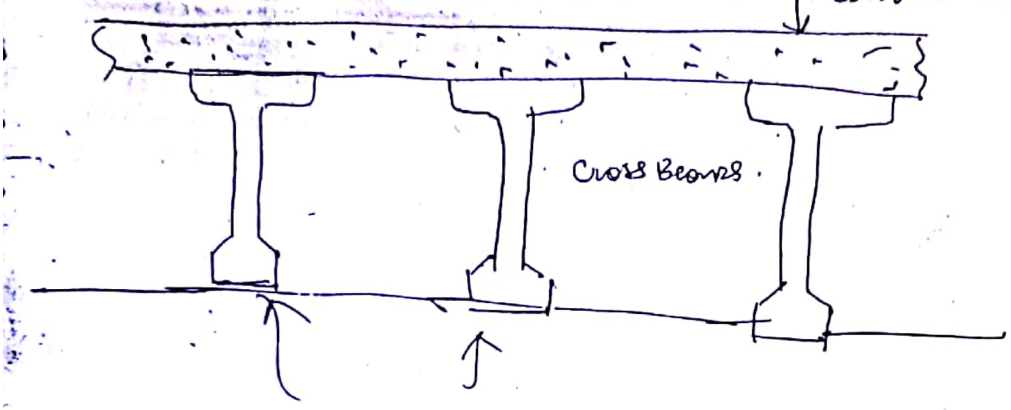
COMPOSITE CONSTRUCTION.

Prestressed and precast + cast in situ

Beams + slab.

Connection provided between them.

Have several advantages. well suited for bridges.



Typical construction of COMPOSITE FLOOR.

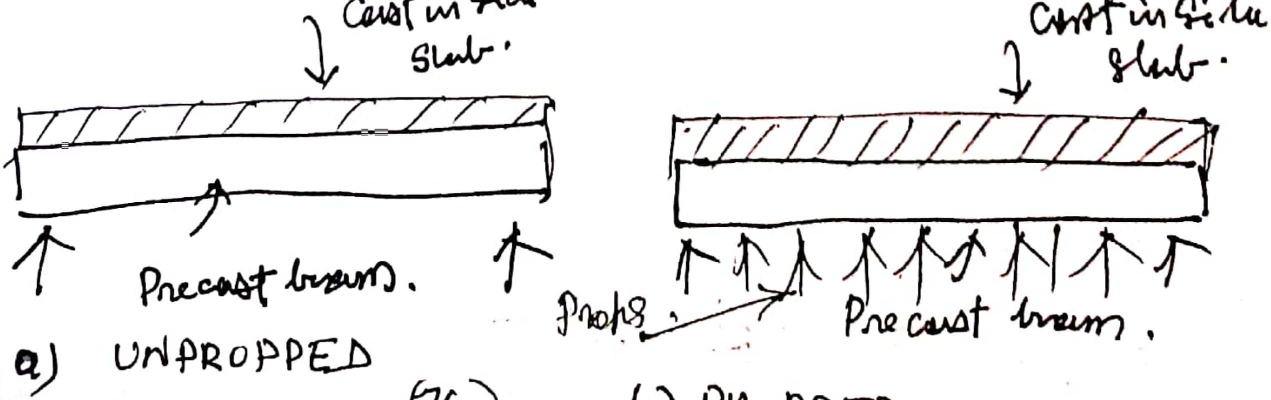
NOTE: Similarly inverted T's, I-Sections and box sections can be used.

Analysis

Depending upon the type of construction,

We can have propped or unpropped Precast beams.

Analysis for stresses has to be done accordingly.



Ex. A precast pretensioned beam has 100×200 mm section. Eff. span : 5m. The centroid of the tendons coincides with the bottom kern. Initial force in the tendons is 150 kN. Losses : 15%. Top flange (cast in situ) is breadth 400 mm, thickness 40 mm. L.L : 8 kN/m². Calculate the stresses. Consider the cases as

a) Unpropped b) Propped.

Assume same Young's Modulus.

SOLUTION. Unpropped.

a) For the pretensioned Beam $A = 100 \times 200 = 20,000\ mm^2$,

$P_{eff} = 0.85P$ (15% losses) $Z = 100 \times 200^2 / 6 = 667 \times 10^3\ mm^3$.

Stress at bottom due to ~~Eff prestress~~ eff. prestressing force $= \frac{2 \times 0.85 \times 150 \times 10^3}{20,000} = \left(\frac{2P}{A}\right)$. (Pretensioned.)

$= 12.8\ N/mm^2$ (at bottom)

Top stress at ~~bottom~~ $= 0$.

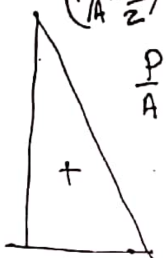
(Pretensioned

$\leftarrow P \qquad P \rightarrow$

~~= 2P.~~

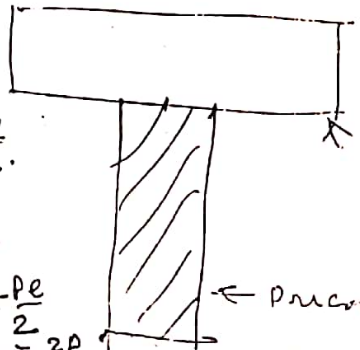
$\left(P/A - \frac{Pe}{Z}\right) = 0$.

$\frac{P}{A} = \frac{Pe}{Z}$.



$\frac{P}{A} + \frac{Pe}{Z} = \frac{2P}{A}$

(Due to prestress)



Self wt. of precast beam $= 0.1 \times 0.2 \times 24 \times 10^3 = 480\ N/m$.

Moment $= \frac{480 \times 5^2}{8} = \cancel{1200}\ 1500\ Nm$.

Stress due to self wt. $= \pm \frac{M}{Z} = \pm \frac{\cancel{1200}\ 1500 \times 10^3}{667 \times 10^3}$

$= \pm\ 2.25\ N/mm^2$. (a)

(− at bottom + at top.)

(b) (b)

I. Due to insitu slab,

Self wt. = $0.04 \times 0.4 \times 24 \times 10^3 = 384\ N/m$

$$\text{Moment} = \frac{384 \times 5^2}{8} = 1200\ N\cdot m$$

$$\text{Stresses in the precast beam} = \pm \frac{1200 \times 10^3}{667 \times 10^3} = \pm 1.8\ N/mm^2 \quad (b)$$

(−ve at bottom +ve at top)

c) For the composite section

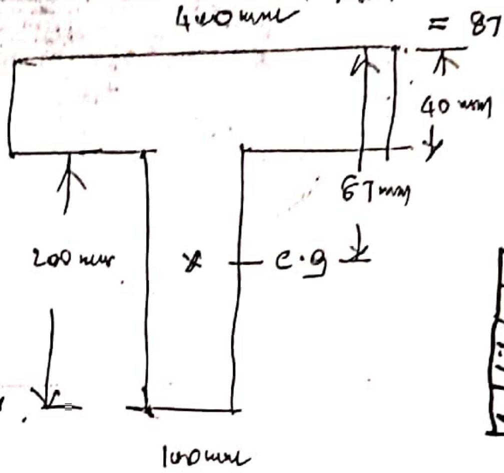
$$\text{Dist. of c.g from top} = \frac{(400 \times 40 \times 20 + 100 \times 200 \times 140)}{400 \times 40 + 200 \times 100} = 87\ mm \text{ from top.}$$

Dist. of c.g from top = 67 mm

$I = 1946 \times 10^5\ mm^4$

$Z_t = 225 \times 10^4\ mm^3$

$Z_b = 128 \times 10^4\ mm^3$



$L.L = 1 \times 0.4 \times 8 \times 10^3 = 3200\ N/m$

$$L.L\ \text{Moment} = \frac{3200 \times 5^2}{8} = 10{,}000\ N\cdot m$$

$$\text{Stress (due to L.L) at Top} = \frac{10{,}000 \times 10^3}{225 \times 10^4} = 4.45\ N/mm^2\ (+\ \text{Comp.})$$

$$\text{at Bottom} = \frac{10{,}000 \times 10^3}{128 \times 10^4} = 7.85\ N/mm^2\ (-\ \text{tense})$$

Combining (a) + (b) + (c) = Resultant final stresses of the unpropped beams (C)

At top = $(0 + 2.25 + 1.8 + 4.45) =$

At bottom = $(+12.8 - 2.25 - 1.8 - 7.85) =$

(b) PROPPED.

In this case the self wt. of slab also is to be considered for the entire composite beam

Hence due to this stress in the composite beams,

$$\text{at top} = \frac{1200 \times 10^3}{225 \times 10^4} = 0.53\ N/mm^2$$

$$\text{at bottom} = \frac{1200 \times 10^3}{128 \times 10^4} = 0.94\ N/mm^2$$

Stress distribution can be shown.

Differential Shrinkage.

Precast. beam → Almost all the shrinkage has taken place.

Cast in situ slab → shrinkage occurs.

Hence there is differential shrinkage. As per B.S. code 100 micro strains

Procedure: A net tensile force is produced in the cast in situ slab. This is counteracted by a comp. force applied in the precast unit together with the B.M. Hence the resultant stresses are computed.

EX. For a composite T-Beam, the pretensioned rib : 100 × 200 mm.

Slab : 400 mm × 40 mm thick.

$E_c = 28\,kN/mm^2 = 28 \times 10^3\,N/mm^2$

Determine the shrinkage stresses if diff. shrinkage $= 100 \times 10^{-6}$

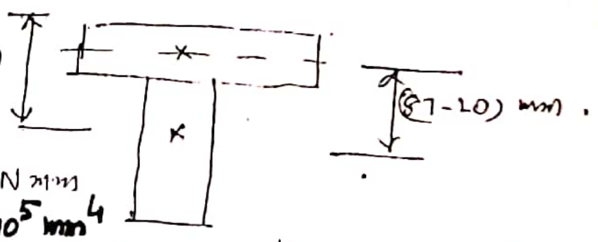
SOLUTION: Area of in situ concrete $A_c = 400 \times 40 = 16000\,mm^2 = 16 \times 10^3\,mm^2$

Uniform tensile stress induced $= 28 \times 100 \times 10^{-6} \times 10^3 = 2.8\,N/mm^2$

Force $=$ ~~2.8 × 16 × 10~~ $2.8 \times 16 \times 10^3 = 44.8 \times 10^3\,N$.

C.G of the composite section $= 87\,mm$ from top.

Eccentricity of the comp. force w.r.t the C.G $= 87 - 20 = 67\,mm$.



Hence moment $= 44.8 \times 10^3 \times 67 = 3 \times 10^6\,N\,mm$

I of the composite section $= 1948 \times 10^5\,mm^4$

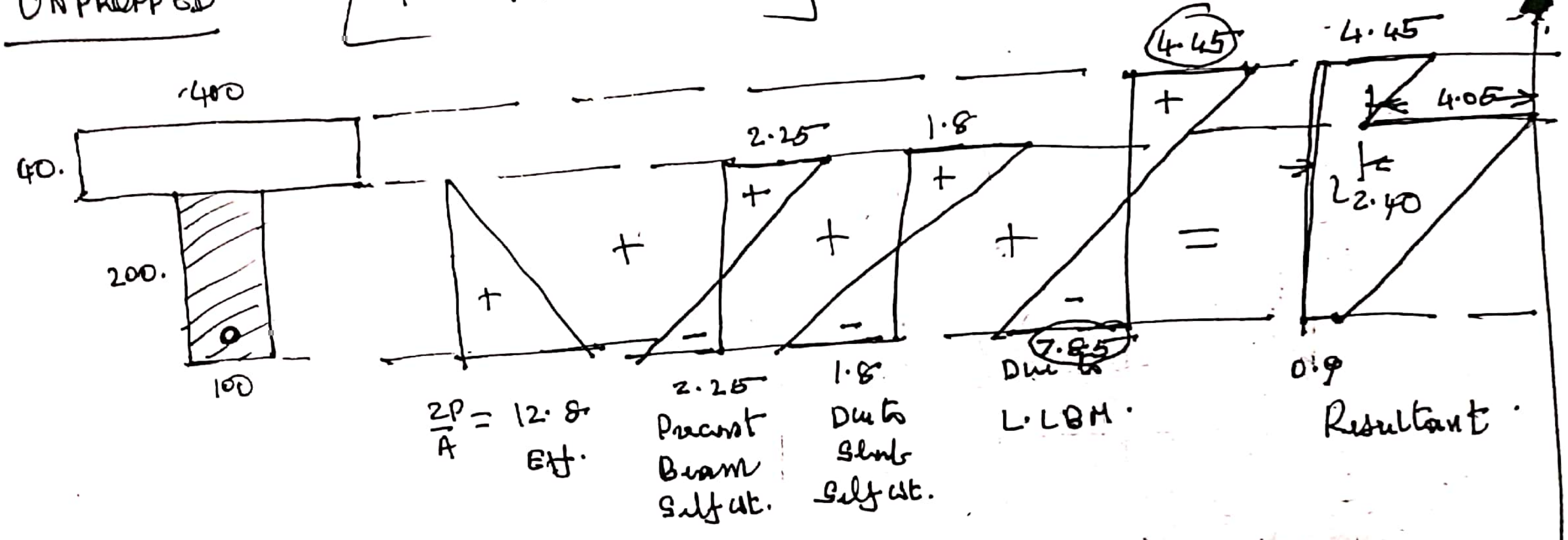
Section moduli of the composite section are $Z_t = 225 \times 10^4$, $Z_b = 128 \times 10^4$

At the junction $Z_j = 414 \times 10^4\,mm^3$.

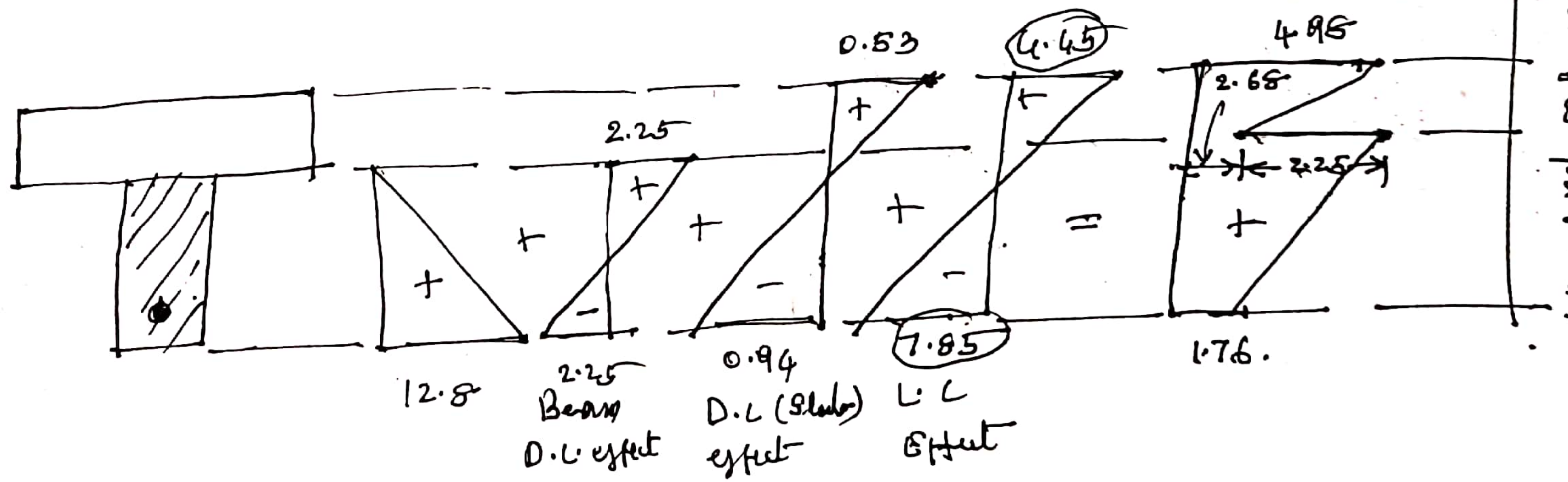
Direct comp. stress $= \dfrac{44.8 \times 10^3}{(16 \times 10^3 + 100 \times 200)} = \dfrac{44.8 \times 10^3}{36 \times 10^3} = 1.24\,N/mm^2$

Stress Distribution in the Designs problem.

a) UNPROPPED .. [FOR THE EXAMPLE]

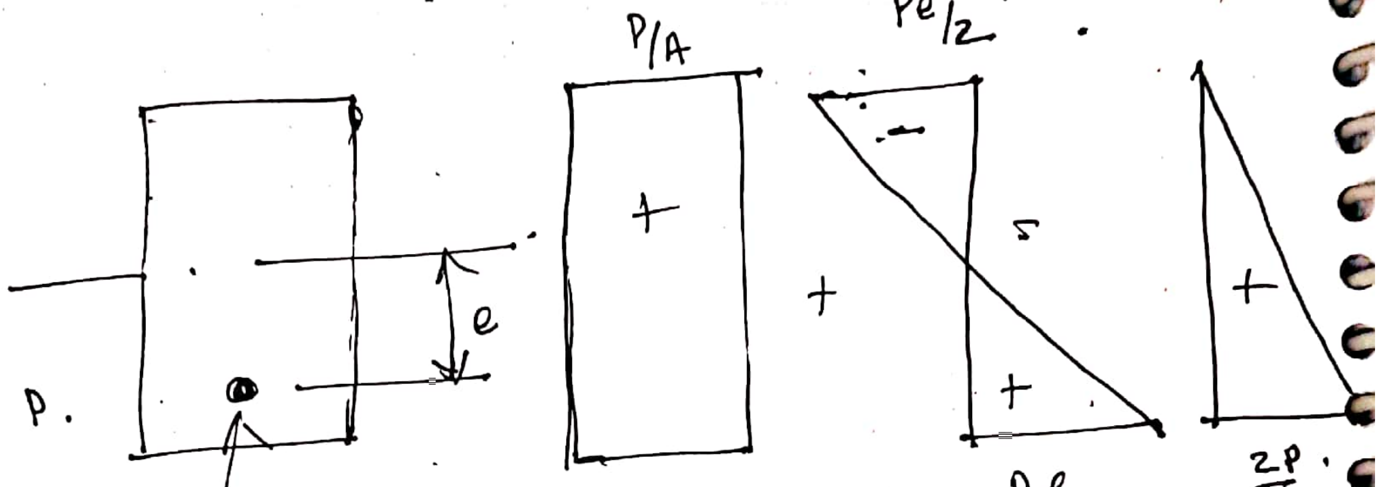


b) PROPPED ..



(80)

In the case of a member, where prestressing steel is k[ept] at the position of bottom kern.



Bottom kern position

Direct = P/A + $\frac{Pe}{Z}$

Lower kern point is ~~the~~ provided in such a way that ~~the kern distance between the prestressing~~ the stress produced due to eccentricity of the prestressing force is equal to the direct stress at the extreme fibre so that the net stress at the top is equal zero and hence $\frac{P}{A} = \frac{Pe}{Z}$ and at bottom the total compression $= 2P/A$.

$$\left(\frac{1}{2} \times f_c\, h \cdot b = P\right)$$

$$\frac{f_c}{2} A = P$$

$$\left(f_c = \frac{2P}{A}\right)$$

Bending stress at top = $3\times10^6 / 225\times10^4$ = $1.33\ N/mm^2$ (comp.)

at Bottom = $3\times10^6 / 128\times10^4$ = $2.34\ N/mm^2$ (Tension)

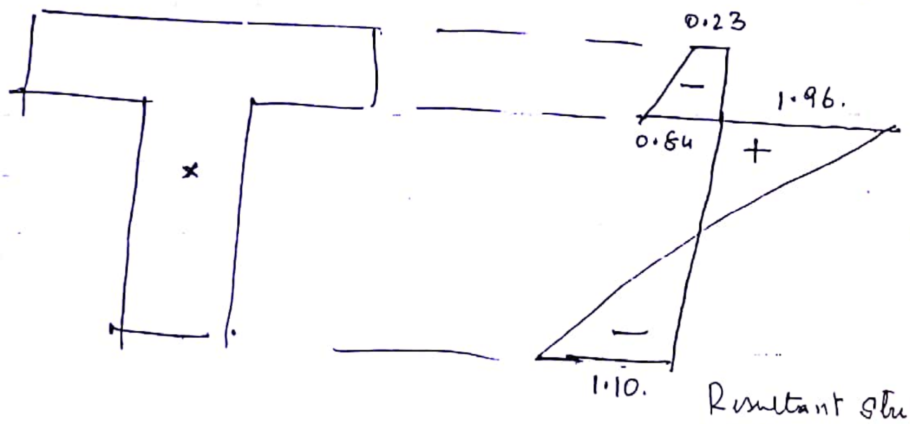
at junction = $3\times10^6 / 414\times10^4$ = $0.72\ N/mm^2$ (comp.)

Hence resultant, at top of beam = $1.24+0.72 = 1.96\ N/mm^2$ (comp.)
(At the junction)

at bottom of beam = $1.24-2.34$ = $-1.10\ N/mm^2$ (Tension)

at top of slab = $1.24+1.33-2.8$ = $-0.23\ N/mm^2$ (Tension).

At bottom of slab (Junction) = $1.24+0.72-2.8$ = $-0.84\ N/mm^2$.



Resultant stress Distribution

DESIGN : The design of the Composite beam and slab construction may be carried out to keep the stresses within the limits at top and bottom. To keep them within the limits the Section moduli may be determined and hence the sectional dimensions.

Scanned with CamScanner