UNIT - II

DEFLECTIONS

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IMPORTANCE OF DEFLECTIONS

According to various Codes, it is general practice that Structural Concrete members should be designed to have Adequate Stiffness to limit deflections, which may adversely affect the Strength or Serviceability of the Structure at working loads.

Factors influencing deflections

1. Imposed load & self wt
2. Magnitude of Prestressing force (P)
3. Cable profile
4. M.I of C.S
5. Modulus of Elasticity of Concrete (Ec)
6. Shrinkage, creep and relaxation of stress in steel.
7. Span of the member.
8. Fixity conditions.

The methods of deflection calculations are different in the "pre crack" conditions and "post crack" Cr

In the "precrack" condition the entire section of the member is effective and accordingly deflections are determined by considering the mem's second moment of area of the gross c.s ... tic body.

In "post crack" condition the PSC beam behaves in a similar to that of a RC beam and computation of deflection ~~Comput~~ is made by considering moment of curvature relationship.

Sol The effect of creep and shrinkage of concrete is to increase long term deflection. Such deflections are determined using long term modulus of elasticity or by magnifying short term deflection by a suitable correction factor.

* SHORT TERM DEFLECTION

Bending moment and flexural rigidity are the two fundamental properties by which short term deflections are determined. Variation of tension in tendons has negligible effect on ~~short~~ term deflection caused by the loads provided the beam remains uncracked and the strain increases with stress in concrete & steel.

In the uncracked condition of the beam, the short term deflections are determined by elastic theory. The gross sectional area may be adopted in the calculations.

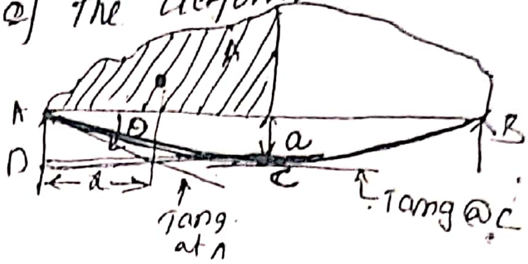
Deflections caused by prestress, self weight and imposed loads may be determined by two approaches, namely (short terms)

i) may regard concrete as a freebody subjected transverse forces transmitted to it by tendon.

... method we consider the moment due the tendon. Mohr's diagram (moment) ...ctions are calculated from it.

Mohr's Theorem

* consider a beam AB subjected to a BM distribution due to Prestressing force of Self wt. of imposed loads
ACB is the centre line of the deformed structure under the System of given loads.



If θ = slope of the elastic curve at A
AA = intercept b/w the tangent at C and the vertical at A
a = deflection at the Centre for symmetrically loaded SSB
A = Area of BMD b/w A & C
x = dist. of Centroid of the BMD b/w A & C from left support
EI = flexural Rigidity of the beam

then according to Mohr's first theorem:

$$\text{Slope} = \frac{\text{area of BMD}}{\text{flexural Rigidity}} \Rightarrow \theta = \frac{A}{EI}$$

from Mohr's Second theorem

$$\text{deflection} = \frac{\text{Moment of the area of BMD}}{\text{flexural Rigidity}}$$

$$\delta = \frac{Ax}{EI}$$

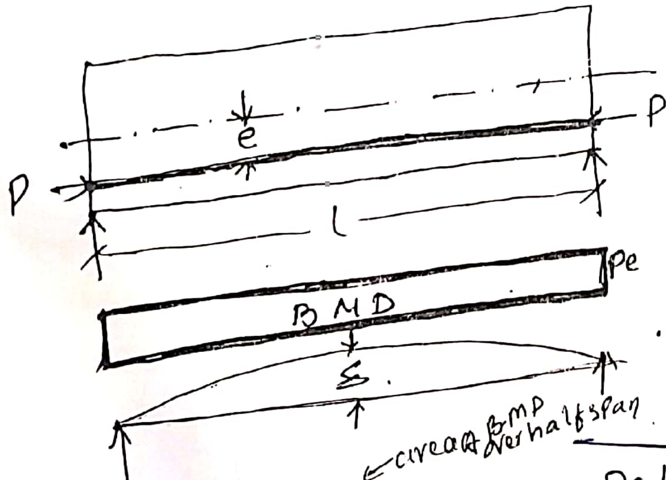
∴ deflections of symmetrically loaded and SSB at the midspan (a) are directly obtained from Mohrs II theorem. More complicated problems involving unsymmetrical loading may be solved by combining both I + II theorems.

Effect of tendon profile on Deflections ①

In most cases of PSBs, tendons are located eccentricities towards the soffit of the beams to counteract the sagging B.M due to transverse loads. Consequently the concrete beams deflect upwards on the application or transfer of prestress. Since the BM at every section is the product of PS force and eccentricity (Pe), the tendon profile itself will represent the shape of BMD.

Method of computing deflections of beams with different cable profile is explained below.

1. Straight Tendon : a st. tendon with uniform eccentricity below the centroidal axis



u/w def is considered as −ve

$$\therefore \delta = -\frac{\left(\frac{Pe\,L}{2} \times \frac{L}{4}\right)}{EI} = \frac{-PeL^2}{8EI}$$

(← area of BMD over half span)

2. Trapezoidal Tendon : considering the BMD, the deflection at the centre of the beam is obtained by taking the moment of area of BMD over one-half of the span.

③

resultant deflection

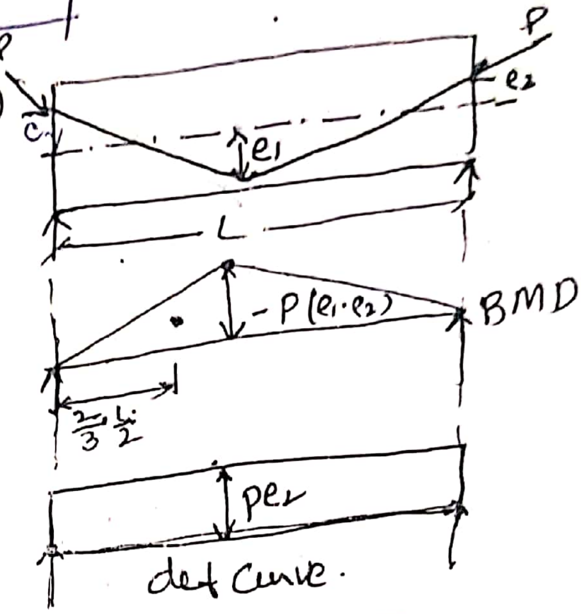
$$\delta = \left[\frac{-5PL^2(e_1+e_2)}{48EI}\right] + \left[\frac{Pe_2 L^2}{8EI}\right]$$

$$\boxed{\delta = \frac{PL^2}{48EI}(-5e_1+e_2)}$$ ★

★ 5. SLOPING TENDONS (Eccentric anchors)

∴ similar to ④

$$★ \delta = \left[\frac{-PL^2(e_1+e_2)}{12EI}\right] + \left[\frac{Pe_2 L^2}{8EI}\right]$$



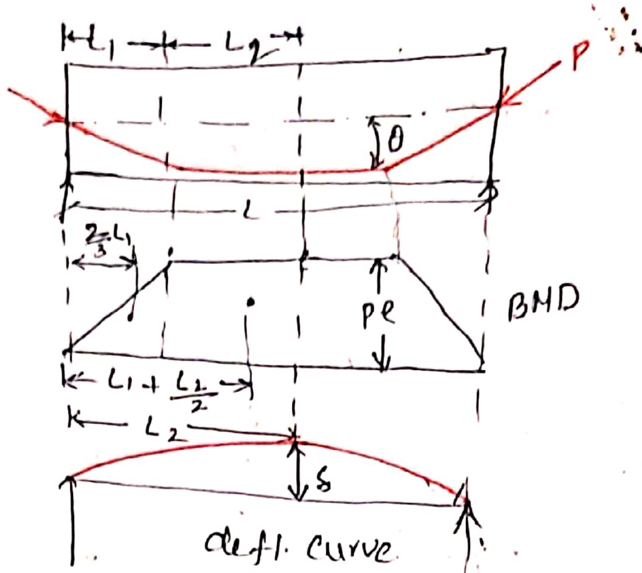
Sloping tendons

DEFLECTIONS DUE TO SELF WEIGHT AND IMPOSED LOAD

At the time of transfer of prestress, the beam hogs up due to the effect of prestressing. At this stage, the self wt of the beam induces d/w deflection, which further increase due to the effect of imposed loads on the beam.

If g = self wt of the beam/m ; q = imposed load/m (udl)

D/w def ★ $$\boxed{\delta = \frac{5(g+q)L^4}{384EI}}$$



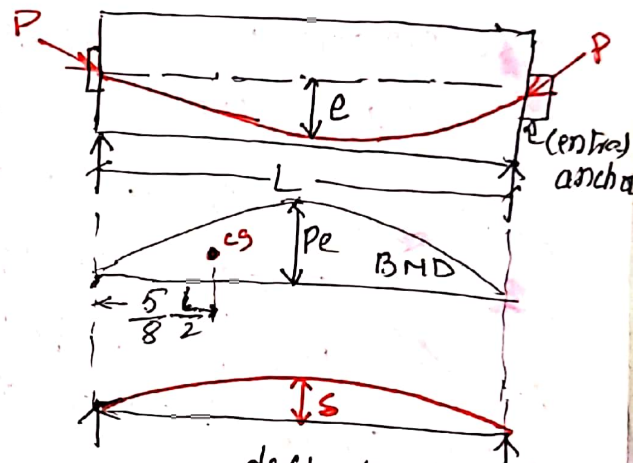
Trapezoidal or Draped tendon

3) PAROBOLIC TENDON (central anchors)

$$\delta = \frac{Pe}{EI}\left[\frac{2}{3}\cdot\frac{L}{2}\times\frac{5}{8}\cdot\frac{L}{2}\right]$$

(A = $\frac{2}{3}\cdot\frac{L}{2}$, $\bar{x}$ = $\frac{5}{8}\cdot\frac{L}{2}$)

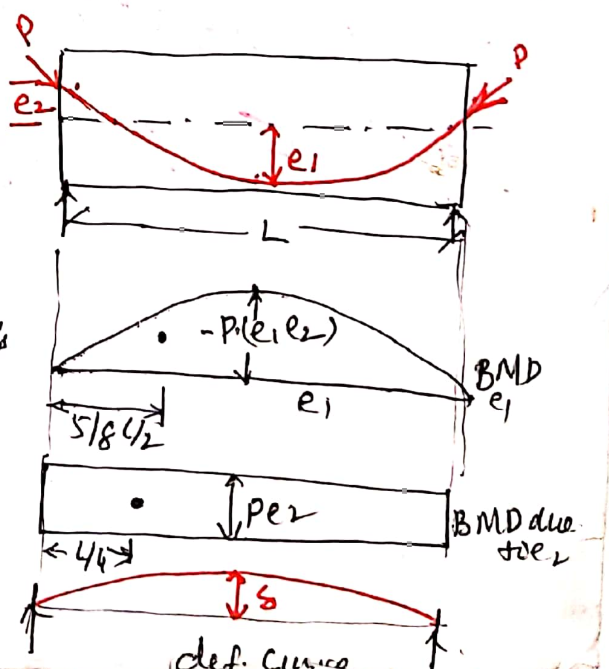
$$\delta = \frac{5PeL^2}{48EI}$$



deflection curve.

4) PARABOLIC TENDONS (Eccentric anchors)

In this case resultant deflection at the centre is obtained as the sum of the u/w deflection of a beam with a parabolic tendon of eccentricity (e_1+e_2) at the centre and zero at the supports and the d/w deflection of a beam subjected to a uniform sagging BM of intensity Pe_2 throughout the length.



of the bea...
area of BMD over one-half of

4. A rectangular conc. beam 300mm x 500mm is prestressed by 2 post tensioned cables of area $600 mm^2$ each, initially prestressed to $1600 N/mm^2$. The cables are located at a constant eccentricity of 10mm throughout the length of the beam having a span of 10m. $E_s = 210 kN/mm^2$ & $E_c = 38 kN/mm^2$

i) Neglecting all losses, find the deflection at the centre of span when it is supporting its own weight.

ii) Allowing for 20 percent loss in prestress, find the final deflection at the centre of span when it carries an imposed load of 18kN/m ; $D_c = 24 kN/m^3$.

Sol (i) Self wt of beam $= (0.3 \times 0.5 \times 24) = g = 0.0036 kN/mm$

$$I = \frac{300 \times 500^3}{12} = 3125 \times 10^4 mm^4$$

Prestress force; $P = 1600 \times 2 \times 600 = 1920 kN$

i) D/W δ due to self wt $= \frac{5gL^4}{384EI} = \frac{5 \times 0.0036 \times (10 \times 1000)^4}{384 \times 38 \times 3125 \times 10^4}$ (E_c)

$$\delta = 3.95 mm \downarrow$$

U/W δ due PS Force $= \left[\frac{P.e.L^2}{8EI}\right] = \left[\frac{1920 \times 10 \times (10 \times 1000)^2}{8 \times 38 \times 3125 \times 10^4}\right]$

$= 20.20 mm$.

∴ Net U/W δ of beam, when it support its own weight $= (20.2 - 3.90) = 16.3 mm$

ii) U/W δ due to PSF after allowing 20 % loss

$= 0.8 \times 20.20 = 16.16 mm$ $\delta(L.L) = \frac{5qL^4}{384EI}$ ↓+ ↑-

∴ Final D/W (δ) of beam due to (self wt + PSF + L.L)

$= 3.95 - 16.16 + \left[\frac{5 \times 18 \times (10 \times 1000)^4}{384 \times 38 \times 3125 \times 10^4}\right] = 3.95 - 16.16 + 19.\ldots$

$\delta = 7.29 mm$

#2 A Concrete beam with CS area of 32×10^3 mm² and radius of gyration of 72mm is prestressed by a parabolic cable carrying an eff. stress of 1000N/mm². The span of beam is 8m. The cable composed of 6 wires of 7mm dia has an eccentricity of 50mm at centre and zero at supports. Neglecting all losses, find the central deflection of the beam for

(a) self wt + prestress (b) self wt + PS + L.L of 2 kN/m ; 2×10^{-3} kN/mm

Sol

$E_c = 38$ kN/mm² & $D_c = 24$ kN/m³ ; $P = (6\times38.5\times1000) = 231$ kN

$A = 32\times10^3$ mm² ; $k = 72$ mm ; $\frac{I}{A} = k \Rightarrow I = 72\times32\times10^3 = 166\times10^6$ mm⁴

$l = 8000$ mm

$e = 50$ mm ; $g = \left(\frac{32\times10^3}{10^6}\times24\right) = 0.77$ kN/m $= 0.00077$ kN/mm

∴ D/W δ due to self wt

$= \left[\frac{5\times0.00077\times8000^4}{384\times38\times166\times10^6}\right]$

$\delta = 6.5$ mm

U/W δ due PSF $= \frac{5Pel^2}{48EI} = \left(\frac{5\times231\times50\times8000^2}{48\times38\times166\times10^6}\right) = 12.2$ mm

D/W δ due to LL $= \left(\frac{5\times2\times8000^4}{1000\times384\times38\times166\times10^6}\right) = 16.9$ mm

(a) Def. due to (self wt + PS)

$= (12.2 - 6.5) = 5.7$ mm ↑

(b) Def. due to (self wt + PS + L.L)

$= (6.5 - 12.2 + 16.9) = 11.2$ mm ↓

Scanned with CamScanner

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#3 A rectangular concrete beam of c/s 150mm X 300mm deep is SS over a span of 8m and is prestressed by means of a symmetric parabolic cable, at a distance of 75mm from the bottom of the beam at mid span and 125mm from the top of the beam at support section. If the force in the cable is 350kN and $E_c = 38\,kN/mm^2$, Calculate

i) deflection at midspan when the beam is supporting its own weight and

ii) The concentrated load which must be applied at midspan to restore it to the level of support

$P = 350kN$; $E_c = 38\,kN/mm^2$; $I = 3375 \times 10^4\,mm^4$; $e_1 = 75mm$

$e_2 = 125mm$

Net deflection due to the PS Force $= \frac{PL^2}{48EI}(-5e_1 + e_2)$

$$= \left[\frac{350 \times 8000^2}{48 \times 38 \times 3375 \times 10^4}(-5 \times 75 + 125)\right] = 12.7mm \uparrow$$

Self wt of the beam, $g = (0.15 \times 0.3 \times 24) = 1.08\,kN/m$
$= 0.00108\,kN/mm$

D/W deflection due to self wt

$$= \frac{5gL^4}{384EI} = \left[\frac{5 \times 0.00108 \times 8000^4}{384 \times 38 \times 3375 \times 10^4}\right] = 4.5mm$$

Def. due to (Ps + self wt) $= (-12.7 + 4.5) = -8.2mm \uparrow$

If Q = Conc. load reqd at the centre of span

Then $\frac{QL^3}{48EI} = 8.2 \Rightarrow Q = \frac{8.2 \times 48 \times 38 \times 3375 \times 10^4}{8000^3} = 9.9kN$

II LONG TIME DEFLECTION

Creep + shrinkage of concrete, and ~~shrinkage~~ stress relaxation of steel produce changes in the deflection of beams.

The two opposing effects producing deformations ~~namely~~ are prestress effect and transverse load effect.

The net curvature at any section at any stage is given by

$$\phi_t = \phi_{mt} + \phi_{pt}$$

where ϕ_{mt} = change of curvature caused by transverse loads

ϕ_{pt} = change of curvature caused by prestress.

The creep strain stram due to transverse loads is directly computed as a function of the creep coeff. (ϕ) so that the change of curvature (i.e deformation) can be estimated from the expression,

$$\phi_{mt} = (1+\phi)\phi_i$$

where ϕ = creep coeff.

ϕ_i = initial curvature caused by the application of transverse ~~loads~~ load.

Different approaches have been developed to determine ϕ_{pt} (i.e curvature produced by prestress). ϕ_{pt} depends on the combined effect of creep & shrinkage of concrete and relaxation of stress in steel.

According to A.M.Neville, the creep curvature produced by prestress is determined, assuming that the creep is produced based on the average prestress which acts over the given time.

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Let P_i = initial prestress

P_t = prestress after a time 't'

$\Delta P_i = P_i - P_t$ i.e loss of prestress due to creep, shrinkage and stress relaxation.

The curvature (def) due to prestress at time 't' is given by

$$\phi_{pt} = \frac{P_i e}{EI}\left\{1 - \frac{\Delta P_i}{P_i} + \left[1 - \frac{\Delta P_i}{2P_i}\right]\phi\right\}$$

Let δ_{il} = initial deflection due to transverse load

δ_{ip} = initial deflection due to prestress

Now the total long time deflection after time 't' is given by

$$\delta_t = \delta_{il}(1+\phi) - \delta_{ip}\left\{\left[1 - \frac{\Delta P_i}{P_i}\right] + \left[1 - \frac{\Delta P_i}{2P_i}\right]\phi\right\}$$

[-ve sign for u/w δ]

A much simplified and approximate procedure is suggested by LIN for computing long time deflection

Total long term deflection is given by

$$\delta_t = \left[\delta_{il} - \delta_{ip}\cdot\frac{P_t}{P_i}\right](1+\phi)$$

1) A PSC beam of span 10m is rectangular section, 120mm wide & 300mm deep and is prestressed by a parabolic cable, the initial ps force being 280kN. The eccentricity of the cable @ the centre is 50mm and the cable is concentric at the ends. The beam carries a liveload of 2.20kN/m. Calculate the short time deflection at the centre of span. Take $E_c = 40kN/mm^2$ and creep coeff $\phi = 2.0$. loss of prestress = 18%

the initial prestress after a duration of 6 months. Find the long time deflection at the Centre. Assume that the beam is subjected to DL & LL simultaneously when the prestress is applied.

sol

$P_i = 280\,kN$; $I = \frac{120 \times 300^3}{12} = 2.7 \times 10^8\,mm^4$; $e = 50\,mm$

$D.L = 0.12 \times 0.30 \times 24 = 0.864\,kN/m$

$L.L = 2.20\,kN/m$

Total load $= 2.2 + 0.864 = 3.064\,kN/m = 0.003064\,kN/mm$

Loss of Prestress $\Delta P_i = 0.2 P_i$.

i) SHORT-TIME DEFLECTION

upward deflection due to prestress $= \frac{5 \cdot P_i e l^2}{48 EI}$

$= \frac{5}{48} \times \frac{280 \times 50 \times 10000^2}{40 \times 2.7 \times 10^8}$ (↑)

$=$ ~~36.94 mm (↑)~~ 13.50 mm

d/w deflection due to DL & LL $= \frac{5 w l^4}{384 EI}$

$= \frac{5}{384} \times \frac{0.003064 \times 10{,}000^4}{40 \times 2.7 \times 10^8} =$ ~~13.50 mm~~ 36.94

∴ Net deflection $= 36.94 - 13.50 = 23.44\,mm$ (↓)

(ii) Long time deflection

δ_{il} = initial deflection due to transverse load = ~~13.5 mm~~ 36.94 mm

δ_{ip} = initial deflection due to prestress = ~~36.94 mm~~ 13.50 mm

using Neville's formula

Long time deflection $\delta_i = \delta_{il}(1+\phi) - \delta_{ip}\left\{1 - \frac{\Delta P_i}{P_i} + \left[1 - \frac{\Delta P_i}{2P_i}\right]\phi\right\}$

$\delta_i = 36.94(1+2) - 13.50\left\{1 - \frac{0.2P_i}{P_i} + \left[1 - \frac{0.2P_i}{2P_i}\right]2\right\}$

$=$ ~~40.5 − 36.94 (0.82) = 10.2 mm~~ ↓

$= 110.82 - 13.50(1 - 0.2 + 1.8)$

$= 75.72\,mm$ (↓)

... simplified Lin's formula

$$\delta_i = \left(\delta_{il} - \delta_{ip} \cdot \frac{P_t}{P_i}\right)(1+\theta)$$

$$= ~~[13.5 - 36.94 \times 0.8](1+2)~~$$

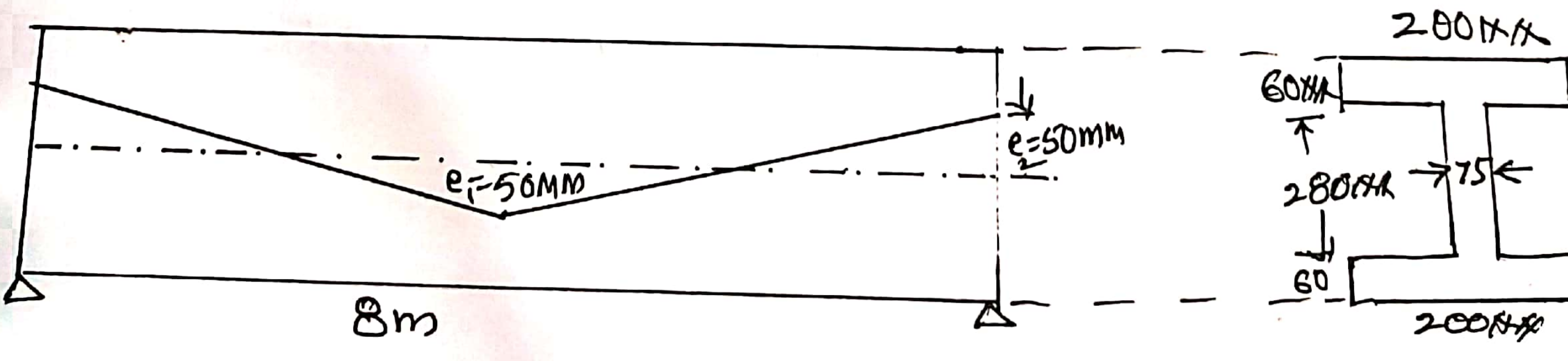
$$\delta_i = (36.94 - 13.5 \times 0.8)(1+2) = 78.42\ mm \downarrow$$

Permissible deflection as per I.S code

→ $E_c = 5700\sqrt{f_{ck}}$.

Load condition	allowable def.
Total def. including cree + shrinkage →	$\frac{1}{250} \times$ span
Total def including all effects after placing permanent loadi →	$\frac{1}{350}$ span or 200mm (whichever less)
Max. u/w deflection (camber) →	$\frac{1}{300} \times$ span

#2 The following fig. shows a prestressed concrete beam of I-section of span 8m. The cable carries a prestressing force of 980kN. Calculate the initial deflection at midspan due to prestress and the DL of the beam. Modulus of Elasticit for concrete may be taken as 38kN/mm². State also whether this deflection is within the permissible limit. density of conc may be taken as 24kN/m³



Sol

Area of beam section $=(2\times200\times60)+(75\times280)$

$= 45000 mm^2$

DL of the beam $= \frac{45000}{10^6}\times 24 = 1.08 kN/m = 0.00108 kN/mm$

$e_1 = 50mm$; $e_2 = 50mm$; $l = 8000mm$

$$I = \frac{200\times400^3}{12} - \frac{125\times280^3}{12} = 8.38\times10^4 mm^4$$

u/w deflection due to Prestress

$$= \frac{-PL^2(e_1+e_2)}{12EI} + \frac{PL^2 e_2}{8EI}$$

$$= \frac{-980\times8000^2(50+50)}{12\times38\times8.3\times10^8} + \frac{980\times50\times8000^2}{8\times38\times8.3\times10^8}$$

$= -16.57 + 12.48$

$= 4.10 mm$ (↑)

D/W deflection due to D.L of the beam.

$$= \frac{5w_d l^4}{384EI}$$

$$= \frac{5}{384}\times\frac{0.00108\times8000^4}{38\times8.3\times10^8}$$

$= 1.82mm$ (↓)

Net u/w deflection $= 4.10-1.82 = 2.28mm$

∴ per. u/w deflection $= \frac{1}{300}\times span = \frac{8000}{300} = 26.7mm$.

Hence the deflection is within permissible limit