LOSSES OF PRESTRESS

UNIT – I (Cont'd).

The effective prestress in Concrete gradually undergoes reduction with time from the stage of transfer. This is generally referred to as loss of prestress.

Types of Losses

In Pretensioning	In Post tensioning
1. Elastic deformation of Concrete.	1. This occurs if only the wires are stretched successively one after the other.
2. Relaxation of stress in steel.	2. Relaxation of stress in steel.
3. Shrinkage of concrete.	3. Shrinkage of Concrete.
4. Creep of concrete.	4. Creep of Concrete.
	5. Friction
	6. Anchorage Slip.

1. Loss due to elastic deformation of Concrete ~~and loss of stress in steel due to it.~~

f_e : prestress in concrete at the level of steel.

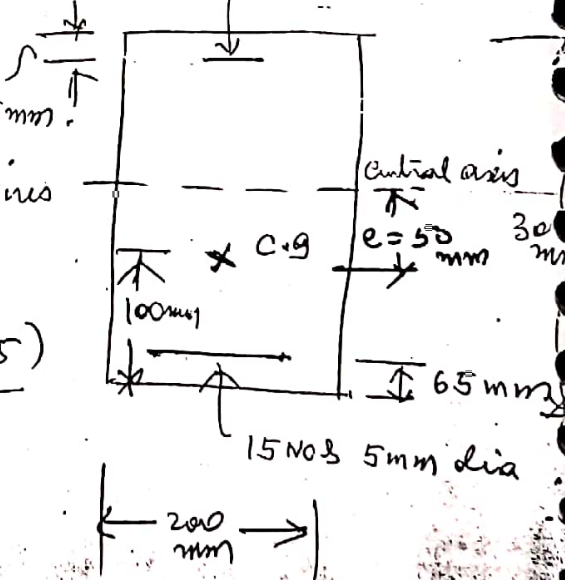
strain in concrete at the level of steel $= f_e / E_c =$ strain in steel ~~stress~~ = at the same level, hence the stress in steel $= \frac{f_e}{E_c} \cdot E_s$

$= f_e \, \alpha_e$

where α_e = modular ratio

The above stress is lost by the steel due to elastic deformation of concrete

EX: A ~~RCC~~ beam 200×300 mm is prestressed by means of 15 Nos 5mm dia wires at 65mm from bottom and 3 Nos 5mm wires at 25mm from top. If the wires are initially tensioned to a stress of 840 N/mm² calculate the percentage loss of prestress due to elastic deformation of concrete only. $E_s = 210\ kN/mm^2$, $E_c = 31.5\ kN/mm^2$.



SOLUTION

The position of the c.g. of the wires is given by,

$$y = \frac{(15\times 65 + 3\times 275)}{15+3}$$

(from bottom) = 100mm.

$e = 150 - 100 = 50$ mm towards bottom

Area of each wire $= \frac{\pi}{4}\times 5^2 = 19.7\ mm^2$

$A = 200 \times 300 = 6 \times 10^4 \text{ mm}^2$

(of the beam)

$I = \dfrac{200 \times 300^3}{12} = 4.5 \times 10^7 \text{ mm}^4$

$P = 840 \times 18 \times 19.7 = 3 \times 10^5 \text{ N} = 300 \text{ kN}$

Stresses in Concrete,

At the level of the top wires $= \dfrac{P}{A} - \dfrac{Pe}{I} \times y_t$

$= \dfrac{300 \times 10^3}{6 \times 10^4} - \dfrac{300 \times 10^3 \times 50 \times 12}{4.5 \times 10^7}$

$= 0.83 \text{ N/mm}^2.$

At the level of bottom wires $= \dfrac{P}{A} + \dfrac{Pe}{I} \cdot y_b$

$= \dfrac{300 \times 10^{3}}{6 \times 10^4} + \dfrac{300 \times 10^3 \times 50}{6 \times 10^4} \times (150 - 65)$

$= 7.85 \text{ N/mm}^2.$

Modular ratio $\alpha_e = \dfrac{E_s}{E_c} = \dfrac{210}{31.5} = 6.68.$

Loss of stress in wires at top $= 6.68 \times 0.83 = 5.55 \text{ N/m}$

at bottom $= 6.68 \times 7.85 = 52.5 \text{ N/m}$

Percentage loss of stress

for wires at top $= \dfrac{5.55}{840} \times 100 = 0.66\%$

" at bottom $= \dfrac{52.5}{840} \times 100 = 6.25\%$

EX.

A post tensioned concrete beam 100 x 300 mm is prestressed by three cables each with a cross section i.e. area of 50 mm² and with an initial prestress of 1200 N/mm². All the cables are straight and located at 100 mm from the bottom. If the modular ratio is 6, calculate the loss of stress in the three cables due to elastic deformation of concrete due to successive tensioning of the cables one at a time.

SOLUTION.

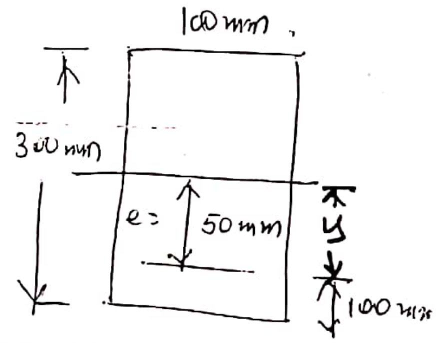
Force in each cable = $50 \times 1200 = 60000\,N$

$= 60\,kN$.

$A = 100 \times 300 = 3 \times 10^4\,mm^2$.

$e = 50\,mm$, $I = 225 \times 10^6\,mm^4$, $y = 50\,mm$

Stress in concrete at the level of steel



$$= \frac{P}{A} + \frac{Pe \cdot y}{I}$$

$$= \frac{60 \times 10^3}{3 \times 10^4} + \frac{60 \times 10^3 \times 50 \times 50}{225 \times 10^6}$$

$= 2.7\,N/mm^2$.

∴ When the cables are successively tensioned

Cable 1 : Tensioned and anchored. Hence no loss of prestress.

Cable 2 When tensioned and anchored it will not undergo any loss, but there will be loss in Cable (1)

$= \alpha_e \times 2.7 = 6 \times 2.7 = 16.2\,N/mm^2$. (24)

Cable 3 : No loss. But loss in (1) and (2) 16.2 N/mm² in both

Total loss in (1) = $2 \times 16.2 = 32.4\,N/mm^2$

" (2) = $1 \times 16.2 = 16.2\,N/mm^2$, Cable (3) = 0

NOTE:

NOTE: Average = $\frac{32.4 + 16.2}{3} = 16.2\,N/mm^2$

LOSS DUE TO SHRINKAGE of concrete

~~Loss~~ Total residual strain in concrete in Pre ~~post~~ tensioning $= 300 \times 10^{-6} = e_{cs}$

For pretensioning post tensioning (Strain.) $= \dfrac{200 \times 10^{-6}}{\log_{10}(t+2)} = e_{cs}$

t: Age of concrete at transfer in days.

Hence loss of stress in steel due to shrinkage $= e_{cs} \cdot E_s$.

LOSS DUE TO CREEP of concrete

1) Ultimate creep strain Method.

Loss of stress in steel due to creep of concrete $= e_{cc} \cdot f_c \cdot E_s$

e_{cc} ultimate creep strain for a sustained unit stress

f_c: Comp. stress in concrete at the level of steel.

2) Creep Coefficient Method

creep coeff. $= \phi = \dfrac{e_c}{e_e}$
(creep strain / Elastic strain)

e_c: creep strain, e_e: Elastic strain.

$$e_c = \phi \cdot e_e = \phi \cdot \frac{f_c}{E_c}$$

Hence loss of stress in steel $= e_c \cdot E_s$

$$= \phi \cdot \frac{f_c}{E_c} \cdot E_s = \phi \cdot f_c \cdot \alpha_e$$

~~Loss due to~~

LOSS DUE TO RELAXATION OF STEEL

As per IS 1343

Initial stress	Relaxation loss in N/mm^2
$0.5 f_{pu}$	0
$0.6 f_{pu}$	35
$0.7 f_{pu}$	70
$0.8 f_{pu}$	90

f_{pu} : ultimate tensile stress

Symbols:

e_c : creep strain, e_e : Elastic strain

f_c : stress in concrete, ϕ : creep coeff.

α_e : Modular ratio

EX: For the following data of a PSC beam calculate the loss of prestress due to shrinkage of concrete only.

Initial prestressing force = 300 kN.

(Area of steel) $A_s = 300\ mm^2$.

$E_s = 210\ kN/mm^2$.

The beam is a) Pretensioned b) Post tensioned.

Age = 8 days

SOLUTION:

a) For pretensioned, Initial stress in steel $= \frac{300 \times 10^3}{300}$

$= 1000\ N/mm^2$

Total residual shrinkage strain $= 300 \times 10^{-6}$.

Loss of stress in steel $= 300 \times 10^{-6} \times 210 \times 10^3$ (E_s)

$= 63\ N/mm^2$.

Percentage loss $= \frac{63}{1000} \times 100 = 6.3\%$

b) Post-Tensioned

As per IS code, Loss (strain) $= \frac{200 \times 10^{-6}}{\log_{10}(8+2)} = 200 \times 10^{-6}$.

Loss of stress $= 200 \times 10^{-6} \times 210 \times 10^3$ (E_s) $= 42\ N/mm^2$.

Hence, Percentage loss $= \frac{42}{1000} \times 100 = 4.2\%$

Data

Ex:

Beam size : 100 mm x 300 mm

Steel : 5 Nos 7 mm ϕ

e : 50 mm ; Initial stress = 1200 N/mm²

Estimate the loss due to creep by both methods.

~~Solution~~.

$E_s = 210\ kN/mm^2$, $I = 225 \times 10^6\ mm^4$

$E_c = 35\ kN/mm^2$

I ; e_{cc} Ult. creep strain

$= 41 \times 10^{-6}$ mm/mm $\quad A = 3 \times 10^4\ mm^4$

Creep coeff. $\phi = 1.6$.

II SOLUTION..

$$P = 1200 \times 5 \times \frac{\pi}{4} \times 7^2 = 23 \times 10^4\ N.$$

$$\alpha_e = \frac{E_s}{E_c} = \frac{210}{35} = 6$$

Stress in concrete at the level of steel.

$$f_c = \frac{P}{A} + \frac{Pe}{I} \cdot y = \frac{23 \times 10^4}{3 \times 10^4} + \frac{23 \times 10^4 \times 50}{225 \times 10^6} \times 50 = \cancel{10.2\ N/mm^2} = 10.2\ N/mm^2.$$

$e_{cc} f_c E_s$ = Loss of stress in steel

$$= 41 \times 10^{-6} \times 10.2 \times 210 \times 10^3 = 88\ N/mm^2$$

~~Creep Coeff. Method~~

Ultimate creep strain Method

Loss of stress in steel = $e_c f_c E_s$.

$$= 41 \times 10^{-6} \times \cancel{10.2}^{\,10.2} \times 210 \times 10^3 = 88\ N/mm^2.$$

Creep Coeff. Method

Loss of stress in steel = $\phi f_c \alpha_e = 1.6 \times 10.2 \times 6$

$= 97.92\ N/mm^2$

Loss of Stress due to Friction

In the case of post tensioned members when straight or moderately curved cables are tensioned, the friction against the wall of the duct or the spacing grills results in a loss of stress which increases with the distance from the jack.

Loss due to anchorage slip.

Due to slipping of anchorages or grips. This depends upon the type of anchorages used.

Last.

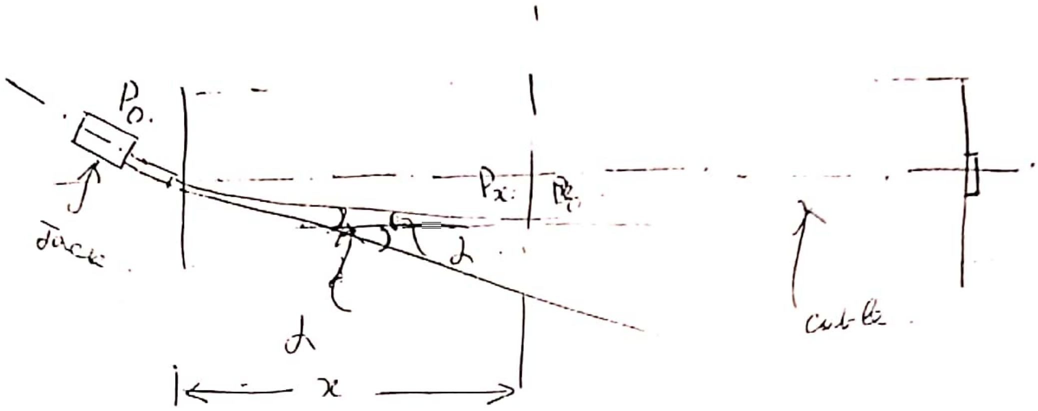
Total Losses allowed for the Design

As recommended by Lin under normal conditions

Type of loss	Percentage loss of Stress	
	Pretensioning	Post-tensioning
Elastic shortening	3	1
Creep of Concrete	6	5
Shrinkage of Concrete	7	6
~~Relaxation~~ creep of Steel	2	3
	18	15

NOTE: It is assumed that temporary over stressing is to compensate for the other losses.

Formulae for loss of stress due to FRICTION



The magnitude of the prestressing force at a distance of 'x' from the tensioning end is given by,

$$P_x = P_0 e^{-(\mu \alpha + kx)}$$

where.

μ : coeff. of friction between cable and duct.

α : Cumulative angle in radians through which the tangent to the cable profile has turned between any two points under consideration.

k : coeff. of friction for wave effect.

$e = 2.7183$.

Coeff. of friction μ.

0.55 for steel on smooth concrete.

0.35 for steel on steel.

0.25 for steel on lead.

Wave effect 'k'

0.15 for 100 m for normal condition.

1.5 for 100 m for thin walled ducts and for heavy vibration.

May be taken as = 0 if large ducts are used and wave effect is eliminated.

Anchorage Slip.

Anchorage slip Δ (mm) $= \dfrac{PL}{AE_s}$

Length of the cable (L); Area of the cable (A).

Loss of stress due to anchorage $= \dfrac{P}{A} = \dfrac{\Delta E_s}{L}$

(Small in long members.)

Ex. A PSC beam of size 100 x 300 mm with a span of 10 m is stressed by 3 cables as follows. Area of each cable is 200 mm² and the initial prestress in the cable is 1200 N/mm²

Cable 1 : Parabolic with eccentricity of 50 mm above the centroid at supports and 50 mm below the centroid at midspan

Cable 2 : @ zero eccentricity at the supports and 50 mm below at mid span.

Cable 3 : Straight with eccentricity of 50 mm below throughout.

Estimate the percentage loss in each cable due to friction. Assume $\mu = 0.35$, $k = 0.0015/m$.

SOLUTION

Equation of the parabola is $y = \frac{4e}{L^2} \cdot x(L-x)$

Hence Slope $= \frac{dy}{dx} = \frac{4e}{L^2}(L-2x)$

Slope at the ends $(x=0) = \frac{4e}{L}$

Cable 1 (Total $e = 50+50 = 100$ mm)

Slope at the end $= \frac{4 \times 100}{10 \times 1000} = \frac{1}{25}$

$\alpha = 0.04$

Total change of angle $2\alpha = 2 \times 0.04 = 0.08$ rad. (over 10 m)

Cable 2. ($e = 50$ mm)

Slope at the ends $= \frac{4 \times 50}{10 \times 1000} = \frac{1}{50} = 0.02 = \alpha$

$2\alpha = 2 \times 0.02 = 0.04$ rad.

Let P_x be the prestressing force at the farther end.

Then $P_x = P_0 e^{-(\mu\alpha + kx)}$.

For small values of $(\mu\alpha + kx)$, we can write

$$P_x = P_0[1-(\mu\alpha + kx)]$$

Hence Loss of stress $= (P_0 - P_x) = (\mu\alpha + kx)P_0$.

Cable 1 Loss $= (0.35 \times 0.08 + 0.0015 \times 10) P_0$

$= 0.043 P_0 = 51.6\ N/mm^2$. (4.3%)

Cable 2 Loss $= (0.35 \times 0.04 + 0.0015 \times 10) P_0 = 0.029 P_0$.

$= 34.8\ N/mm^2 = 2.9\%$

Cable 3 ($\alpha = 0$) Loss $= (0.35 \times 0 + 0.0015 \times 10) P_0 = 0.015 P_0$

$= 18.0 = 1.5\%$

EX. A concrete beam is pretensioned by a cable with an initial stress of $1000\ N/mm^2$. Slip was 5 mm at the jacking end. Length of the beam is 30 m. Estimate the loss of stress due to anchorage slip. $E_s = 210 \times 10^3\ N/mm^2$

SOLUTION

$$\text{Loss due to anchorage slip} = \frac{E_s \Delta}{L} = \frac{210\times10^3 \times 5}{30\times1000} = 35\ N/mm^2$$

$$\%\text{age loss} = \frac{35}{1000}\times100 = 3.5\%$$

EXAMPLE ON TOTAL LOSSES.

EX: A prestressed concrete beam ~~200×300 mm is pretensioned with wires.~~ has the following data. Calculate the total percentage loss of stress if the beam is a) Pretensioned b) Post tensioned.

DATA

Size of the beam = 200 × 300 mm

Span = 10 m.

Total Area of the tendons = $160\ mm^2$.

Eccentricity = 50 mm (constant.)

Initial prestress = $1000\ N/mm^2$.

$E_s = 210\ kN/mm^2$; $E_c = 30\ kN/mm^2$.

Relaxation of steel = 5% of initial stress.

Shrinkage of concrete = 300×10^{-6} and 200×10^{-6} for pretensioning and posttensioning

Ultimate creep strain = 40×10^{-6} and 20×10^{-6} mm/mm per N/mm^2 for pretensioning and posttensioning

Slip at anchorage = 1 mm.

Friction coeff. for wave effect = 0.0015 per/m

SOLUTION: Prestressing force = 160×10^3 N (P)

$A = 200 \times 300 = 6 \times 10^4\ mm^2$.

$\alpha_e = \frac{210}{30} = 7$

$I = \frac{200 \times 300^3}{12} = 45 \times 10^7\ mm^4$.

Stress in concrete at the level of steel = $\left[\frac{P}{A} + \frac{Pe}{Z}\right]$

$$f_c = \left[\frac{160 \times 10^3}{6 \times 10^4} + \frac{160 \times 10^3 \times 50}{45 \times 10^7} \times 50\right] = 7.0\ N/mm^2$$

The various losses are.

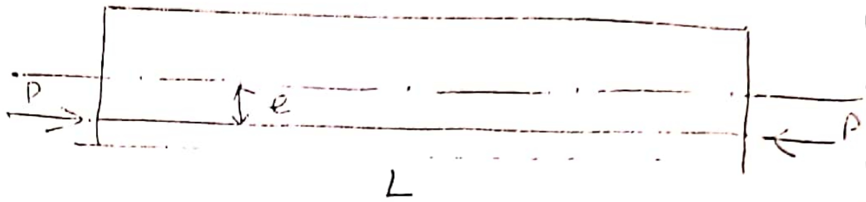
Loss.	Pretensioning	Posttension.
1. Elastic Deformation	$= f_c \times \alpha_e =$	All the wires are stretched at a time and hence no loss.
2. Relaxation of steel	$= \frac{5}{100} \times 1000 = 50 N/mm^2$	$\frac{5}{100} \times 1000 = 50 N/mm^2$
3. creep of concrete	$= \epsilon_c \alpha_e E_s$ $= 40 \times 10^{-6} \times 7 \times 210 \times 10^3$ $= 58.80 N/mm^2$	$20 \times 10^{-6} \times 7 \times 210 \times 10^3$ $= 29.4 N/mm^2$.
4. Shrinkage of concrete	$= 300 \times 10^{-6} \times 210 \times 10^3$ $= 63.0 N/mm^2$	$200 \times 10^{-6} \times 210 \times 10^3$ $= 42.0 N/mm^2$
5. Slip at anchorage	= ——.	$\frac{\Delta E_s}{L} = \frac{1 \times 210 \times 10^3}{10 \times 10^3}$ $= 21.0 N/mm^2$
6. Friction loss.	——	$K x P_o$ $= 0.0015 \times 10 \times 1000$ $= 15.0 N/mm^2$
Total losses	$213.80 N/mm^2$ = 21.38%	15.74% 157.4 N/mm^2

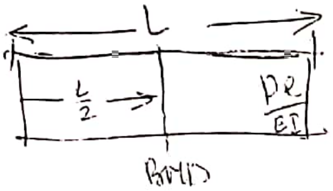
Scanned with CamScanner

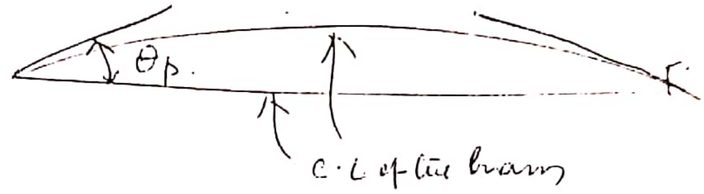
(Due to change of slope and is negligible)

STRESSES IN TENDONS.

Due to the action of the prestressing force and the transverse & loads acting on the member, the curvature of the tendons changes and this results in slight variation of stresses in the tendons.

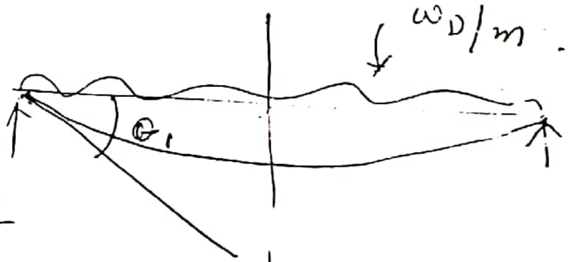






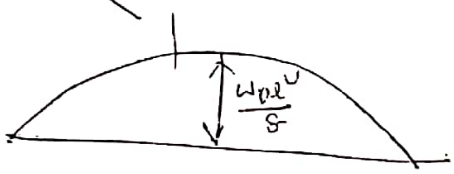
$$\theta_p = \frac{\text{Area of the BMD}}{EI} = \frac{PeL}{2EI}$$ (Due to P) up to the centre.

Due to external loading,

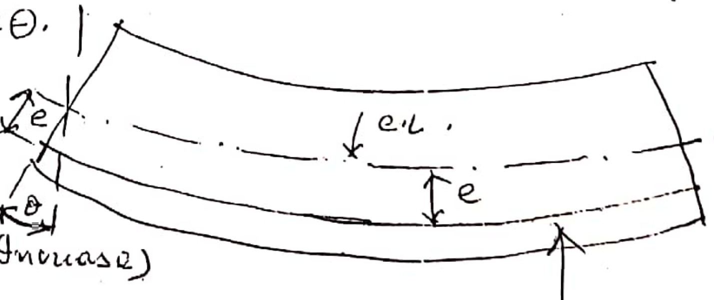


$$\theta_1 = \frac{\frac{1}{2}\times\frac{2}{3}\times L\times \frac{w_D l^2}{8}}{EI} = \frac{w_D l^3}{24EI}$$

Hence net rotation $= \theta = \theta_1 - \theta_p$



Hence the Total elongation in the length of the cable $= 2e\theta$.



Strain $= \cancel{2e\theta}\ \frac{2e\theta}{L}$

Stress $= \frac{E_s \cdot 2e\theta}{L}$. (Increase)

Comments : Generally it is found that in the elastic range, this stress is very small compared to the initial prestressing force. Almost negligible.

VARIATION OF STEEL STRESS IN BONDED AND UNBONDED MEMBERS.

In the case of pretensioned members or posttensioned where grouting of the ducts is done properly, there is good bond between concrete and steel tendons. Sometimes if there is no proper bond between concrete and steel, then the tendons elongate independently under external loads.

In the case of Bonded Beams

Stress in steel = Modular ratio × stress in concrete at the level of steel.

$$= \alpha_e f = \alpha_e \frac{M}{I} y.$$

~~UnBon~~ Unbonded Beams

Total ~~elongation~~ strain of concrete fibre at the level of steel

$$= \frac{M}{E_c I} \cdot y.$$

Hence total elongation $\delta L = \int_0^L \frac{M}{E_c I} y \, dx$

Hence average strain $= \frac{\delta L}{L} = \frac{y}{E_c I L} \int_0^L M \, dx$

Hence stress in steel $= E_s \cdot \frac{y}{E_c I L} \int_0^L M \, dx$

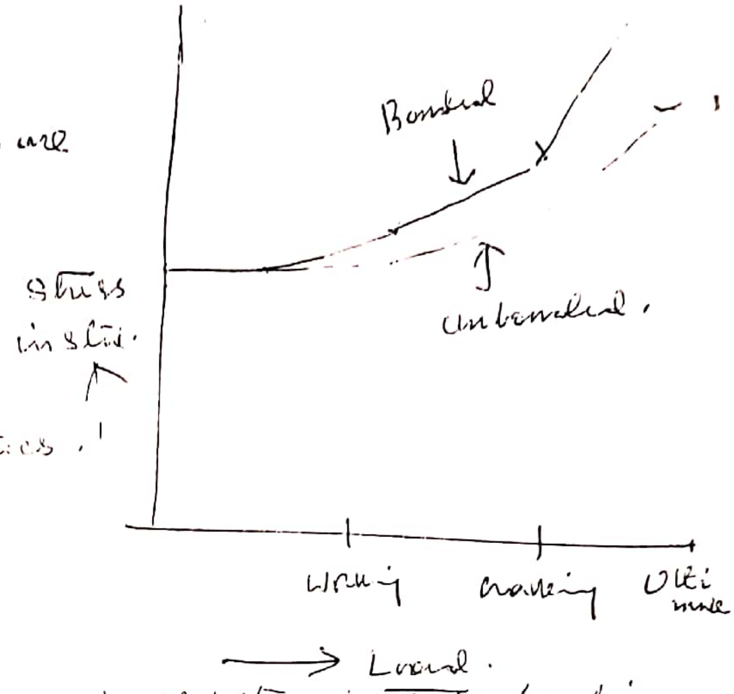
$$= \frac{\alpha_e y}{IL} (A), \quad A : \text{Area of BMD.}$$

Under a U.D.L of W_D/unit, $A = \frac{2}{3} L \cdot \frac{W_d L^2}{8} = \frac{W_d L^3}{12}$.

Hence stress in steel $= \frac{\alpha_e y}{IL} \cdot \frac{W_d L^3}{12} = \frac{W_d \alpha_e y L^2}{I \; 12}$

~~EX~~. Comment

In general bonded beams are preferable because of their higher flexural strength and predictable deformation characteristics.



EX. Calculate the increase in steel stress due to the loading in a) Bonded b) Unbonded cases.

DATA Beam size: 200×300 mm, Eff. Span = 10 m

L.L = 2.56 kN/m, e = 100 mm; $\alpha_e = 6.0$.

SOLUTION. Self wt. = $0.2 \times 0.3 \times 24 = 1.44$ kN/m.

Total load = $2.56 + 1.44 = 4.0$ kN/m = 4.0 N/mm

$$I = \frac{200 \times 300^3}{12} = 45 \times 10^7 \text{ mm}^4$$

$$M = \frac{4 \times 10^2}{8} = 50 \text{ kN m}$$

$y = d/2 = 100$ mm

a) Bonded

$$\text{Stress in Concrete} = \frac{M}{I} \cdot y = \frac{50 \times 10^6 \times 100}{45 \times 10^7} = 11.1 \text{ N/mm}^2$$

$$\text{Stress in steel} = \alpha_e \times 11.1 = 6 \times 11.1 = 66.6 \text{ N/mm}^2.$$

b) Unbonded

$$\text{Stress in steel} = \frac{W_d \alpha_e y L^2}{12 I} = \frac{4 \times 6 \times 100 \times (10 \times 1000)^2}{12 \times 45 \times 10^7}$$

$= 44.4$ N/mm². This is lesser than the previous case.

CRACKING MOMENT

When loads are increased, tensile stresses may develop at the bottom and when this exceeds the modulus of rupture of Concrete, Cracks develop in Concrete. The crack width depends upon the degree of bond between steel and Concrete. Hence tension has to be checked.

EX DATA : Beam Size: 120 x 300 mm

Eff. Prestressing force = 180 kN, $e = 50$ mm (Const.)

Imposed load = 3.14 kN/m , $L = 6$ m.

Modulus of rupture = $5 N/mm^2$.

Evaluate (cracking) the load factor against cracking.

SOLUTION . $P = 180$ kN, $e = 50$ mm, $A = 36 \times 10^3 mm^2$.

$I = 27 \times 10^7 mm^4$, $Z = 18 \times 10^5 mm^3$

Self wt. = $0.12 \times 0.3 \times 24 = 0.86$ kN/m.

W (Total) = $3.14 + 0.86 = 4.0$ kN/m.

Stress due to prestress = $\frac{P}{A} = \frac{180 \times 10^3}{36 \times 10^3} = 5 N/mm^2$.

Due to eccentricity = $\frac{Pe}{Z} = \frac{180 \times 10^3 \times 50}{18 \times 10^5} = 5 N/mm^2$.

Due to loads, M (working) = $\frac{4 \times 6^2}{8} = 18$ kNm.

Stress = $\frac{M}{Z} = \frac{18 \times 10^6}{18 \times 10^5} = 10 N/mm^2$.

Hence net stress at the bottom fibre at working load = $5 + 5 - 10 = 0$.

Extra ~~stress~~ Moment required for cracking stress of $5 N/mm^2$

$= fZ = 5 \times 18 \times 10^5 = 9 \times 10^6 = 9.0$ kNm.

(Cont'd)

Hence total cracking moment = $18 + 9 = 27$ kNm.

Load factor against cracking = $\frac{27}{18} = 1.50$

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