UNIT – IV

DELECTIONS IN PSC BEAMS

DEFLECTIONS IN P.S.C BEAMS.

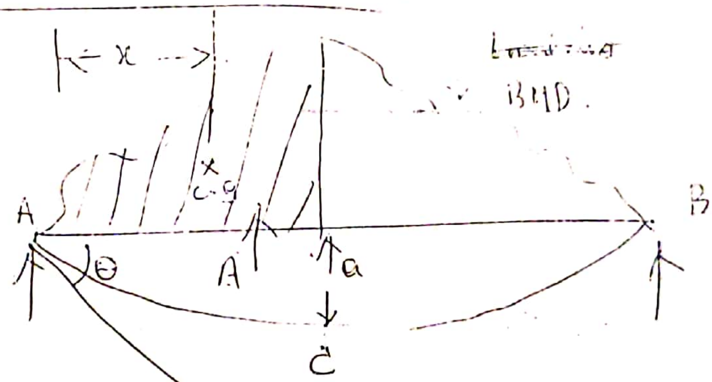
UNIT – III

Deflections are to be controlled to avoid undesirable effect.

Factors influencing Deflections

1) Imposed load and ...
2) Prestressing force
3) Cable profile
4) I of the section and E
5) Shrinkage, creep and relaxation
6) Span
7) Fixity conditions.

Computation of Short term Deflections.



According to Mohr's theorem

$$\text{Slope} = \frac{\text{Area of BMD}}{EI} \quad (a)$$

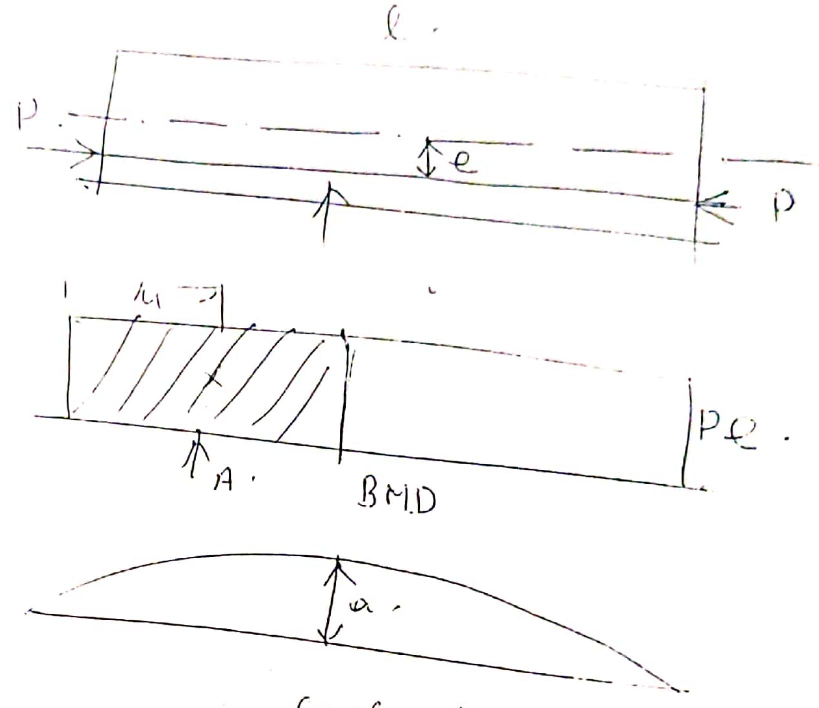
$$\theta = A/EI$$

$$\text{Deflection 'a'} = \frac{\text{Moment of 'A'}}{EI} = \frac{Ax}{EI} \quad (b)$$

C.

Deflections with various tendon profiles.

1) Straight tendons with eccentricity.



Conjugate beam

Deflection due $A = \frac{Pel}{2}$, $x = l/4$

Hence Deflection $a = \left(\frac{Pel}{2}\right)\left(\frac{l}{4}\right) \Big/ EI = \frac{Pel^2}{8EI}$

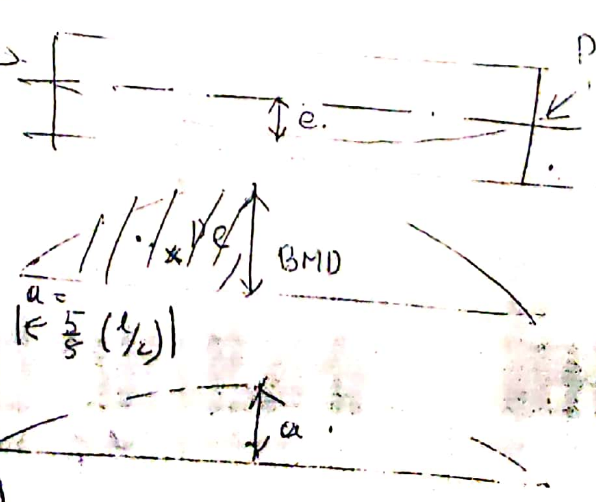
2) Parabolic Tendons. (central)

$A = \frac{1}{2}\left(\frac{2}{3} Pel\right)$

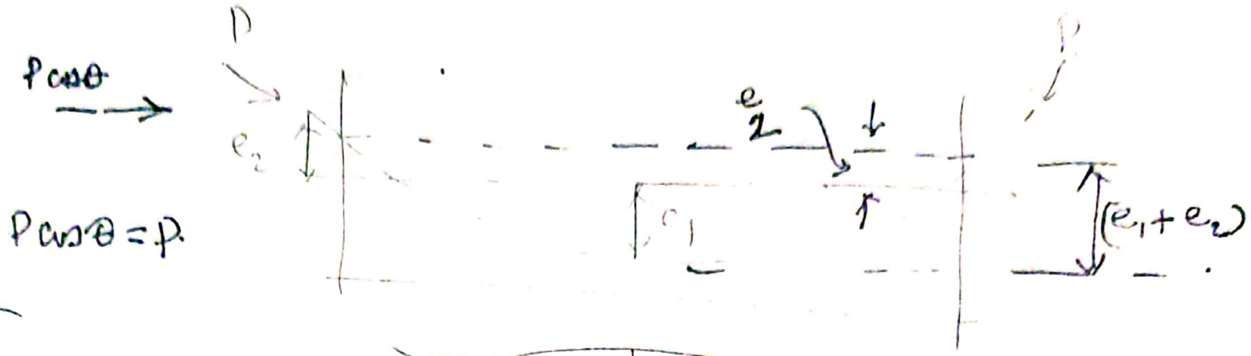
$x = \frac{5}{8}\left(l/2\right)$

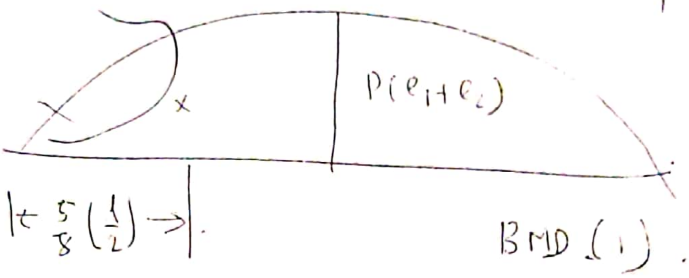
Hence $a = \dfrac{\frac{1}{2} \times \frac{2}{3} Pel \times \frac{5}{8} \cdot \frac{l}{2}}{EI}$

$a = \frac{5}{48} \frac{Pel^2}{EI}$

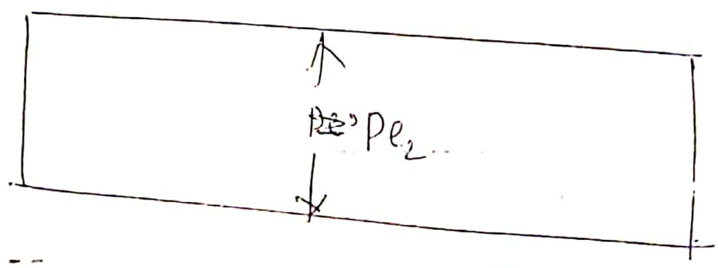


17) Parabolic ([illegible])





BMD (1).

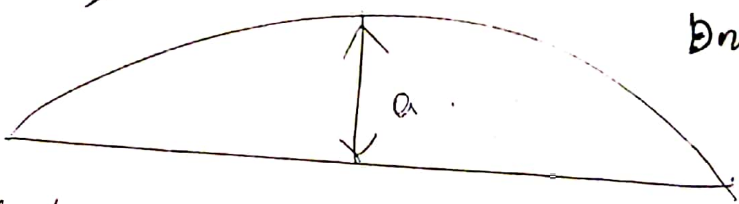


$(e = e_1+e_2)$

BMD (2)

Upward : −ve

Downward = +ve.



Upward deflection due to (e_1+e_2) at centre $= -\frac{5}{48}\frac{Pel^2}{EI}$ (upward)

Downward " due to $P.e_2$ throughout $= \frac{Pe_2l^2}{8EI}$ (Downward)

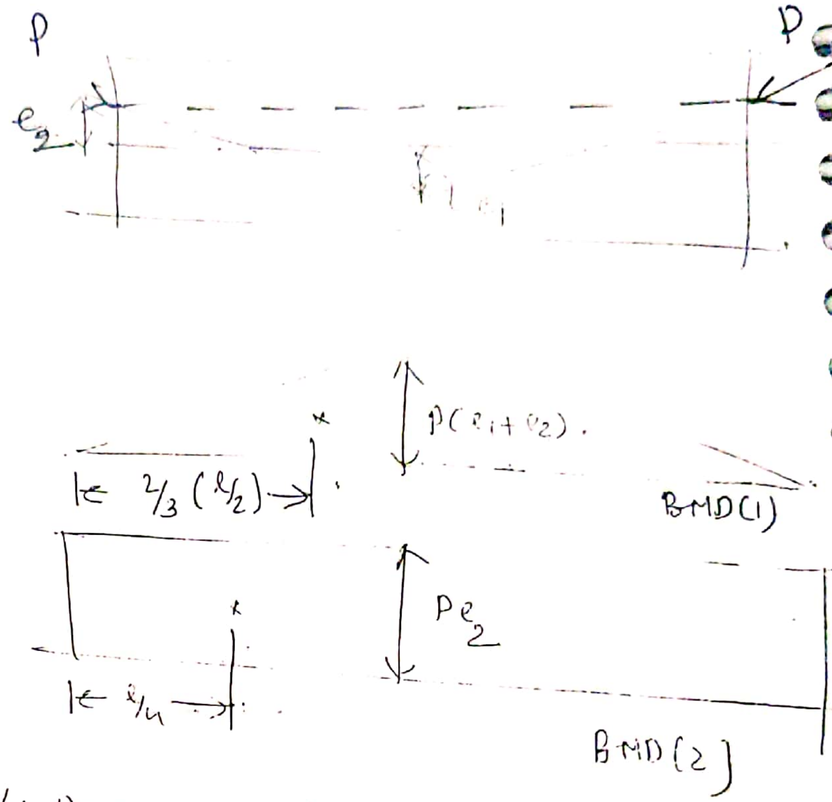
$(e = e_1+e_2)$

Hence net 'a' $= -\frac{5}{48}\frac{Pel^2}{EI} + \frac{Pe_2l^2}{8EI} = \frac{Pl^2}{48EI}(-5e+6e_2) = \frac{Pl^2}{48EI}(-5e+6e_2)$

$-\frac{5}{48}\left[\frac{Pe_1l^2}{EI} + \frac{Pe_2l^2}{EI}\right] + \frac{Pe_2l^2}{8EI} = -\frac{5}{48}\frac{Pe_1l^2}{EI} + \left(\frac{6Pe_2l^2}{48EI} - \frac{5Pe_2l^2}{48EI}\right)$

(96) $= \frac{Pl^2}{48EI}(-5e_1 - 5e_2 + 6e_2) = \frac{Pl^2}{48EI}\left[-5e_1 + e_2\right)$

4) Sloping Tendons (Eccentric)



Upward $= \dfrac{\left(\frac{1}{2} P(e_1+e_2)\cdot\frac{l}{2}\right)\frac{2}{3}\,\frac{l}{2}}{EI} = \dfrac{P(e_1+e_2)\,l^2}{12\,EI}$ (Upward) $-ve$

Downward $= \left(Pe_2\,\frac{l}{2}\right)\frac{l}{4} = +\dfrac{Pe_2 l^2}{8}$ (Downward) $+ve$.

Net) $a = \dfrac{Pl^2}{24EI}\left[-2e_1 + e_2\right]$

EXAMPLE: A PSC beam has the following data (IMP).

DATA:

Section: 300 x 500 mm

Prestressing steel: 2 post tensioned cables of each 600 mm²

Initial prestress - 1600 N/mm²

e = 100 mm (const.) (downward), l = 10 m

E_c = 38 kN/mm², E_s = 210 kN/mm².

a) Neglecting all losses calculate the deflection at centre under self wt.

b) Allowing 20% losses, calculate final deflection at centre under an imposed load of 18 kN/m.

SOLUTION.

Self wt. = $0.3 \times 0.5 \times 24 = 3.6$ kN/m.

= 0.0036 kN/mm := 3.6 kN/m.

$I = \frac{1}{12} \times 300 \times 500^3 = 3125 \times 10^6$ mm⁴.

$P = 2 \times 600 \times 1600 = 1920 \times 10^3$ N.

a) Deflection (Downward due to Self Wt.) $= \frac{5}{384} \frac{\omega l^4}{EI} = 3.9$ mm

upward deflection due to P' $= \frac{Pel^2}{8EI} = 20.2$ mm.

Net upward deflection = 20.2 - 3.9 = 16.3 mm

(b)

Due to LL, downward deflection $= \frac{18}{3.6} \times 3.9$

$= 19.50\ \text{mm}$

After 20% losses, upward deflection $= 0.8 \times 20.2 = 16.16$

Final downward deflection $= (3.9 + 19.50 - 16.16)$

$= 7.3\ \text{mm}$ Ans

<u>Curvature</u>

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

or $$\frac{M}{EI} = \frac{1}{R} = \phi$$

PREDICTION OF LONG TERM DEFLECTIONS

Net curvature at any given section at any given time

is given by, $\phi_t = \phi_{mt} + \phi_{pt}$

ϕ_{mt} (Due to transverse loads) ϕ_{pt} (Due to prestress)

Due to creep ϕ_{mt} changes.

Hence $\phi_{mt} = (1+\phi)\,\phi_i$

ϕ: creep coeff.

ϕ_i: Initial curvature, immediately after applying the transverse loads.

Due to prestressing force,

Loss of prestress after a time is

$L_p = (P_i - P_t)$ due to relaxation of steel

P_i: Initial, P_t: After a time 't'

Hence, after time 't' the curvature due to prestress can be expressed as,

$$\phi_{pt} = -\frac{P_i e}{EI}\left[1 - \frac{L_p}{P_i} + \left(1 - \frac{L_p}{2P_i}\right)\phi\right]$$

e: eccentricity, L_p: loss in steel, ϕ: Creep coefficient.

If a_{it} and a_{ip} are the initial deflections due to transverse load and prestress, then the total long time deflection after time 't' is given by,

$$\boxed{a_t = a_{it}\,(1+\phi) - a_{ip}\left[\left(1 - \frac{L_p}{P_i}\right) + \left(1 - \frac{L_p}{2P_i}\right)\phi\right]}$$

$a_{it}(1+\phi)$: (Downward), $a_{ip}[\ldots]$: (Upward)

Lin's approximate formula.

$$a_{t\,(\text{Total})} = \left(a_{it} - a_{ip}\frac{P_t}{P_i}\right)(1+\phi)$$

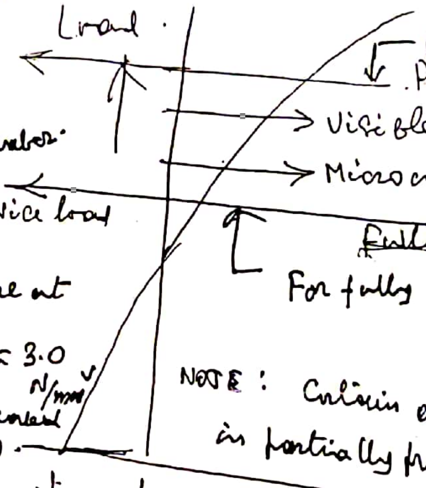
P_i : Initial prestress

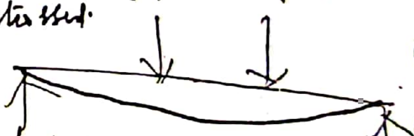
P_t : After attrition "

DEFLECTIONS IN CRACKED MEMBERS.

(SHORT TERM)

In the case of partially prestressed members, Cracks of limited width are acceptable even under working loads as per (CEB-FIP) recommendations.





NOTE : Certain amount of cracking is allowed in partially prestressed members even under working condition.

(The curve is linear up to visible cracking)

a) In Unilinear Method

The deflection of cracked members is calculated as,

$$a = \frac{\beta l^2 M}{E_c I_r}$$

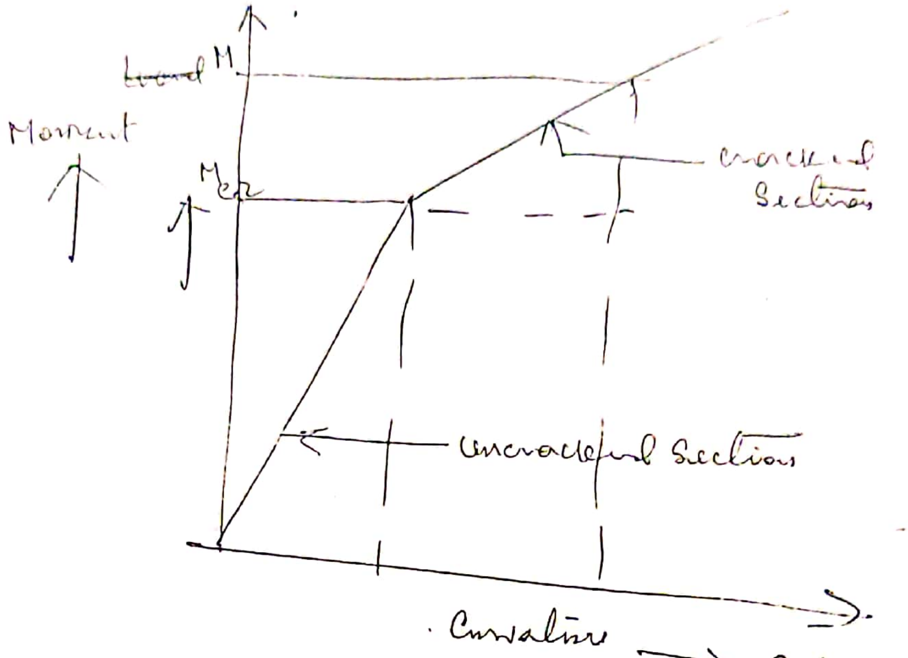
I_r ← M.I. of equivalent cracked section

β : a const. $\frac{5}{48}$ for UDL and $\frac{1}{12}$ for central conc. load.

This method gives gross overestimates in the working range and underestimates in the higher load ranges.

). Bilinear Method (As per B.S. Code)

The load deflection of Moment - Curvature Curve is approximated as follows.



Based on the above, in this case.

$$a = \beta l^2 \left[\frac{M_{cr}}{E_c I_c} + \frac{(M - M_{cr})}{0.85 E_c I_r} \right]$$

Long time. Deflections of cracked Members.

It is complicated in this case.

Hence empirically, by applying a multiplication factor such as 2 to the short time deflection, the long time deflection can be calculated.

Ex: A PSC beam has the following data.

size: 100 mm x 300 mm.

Prestressing cable is parabolic.

Initial prestressing force = 240 kN.

e (at centre) = 50 mm

e (at the ends) = 0

$l = 10$ m, L.L = 2 kN/m

E_c = 35 kN/mm, $\phi = 2.0$, Loss of prestress = 20%

a) Estimate the short term deflection.

b) Estimate the long term deflection after 6 months

Assume that D.L and L.L are simultaneously applied after the release of the prestress.

SOLUTION

a) Short term

$P_i = 240$ kN, $I = 225 \times 10^6$ mm^4, D.L = 0.72 kN/m

L.L = 2 kN/m.

Deflection due to prestress = $\frac{5}{48} \frac{P_i e\, l^2}{EI}$ = 14.7 mm (upward)

" due to (D.L + L.L) = $\frac{5(D.L + L.L) l^4}{384\, EI}$ = 41.5 mm (Downward)

Net deflection = 41.5 − 14.7 = 26.8 mm (Downward)

b) Long term

Initial due to transverse loads = 41.5 mm → (a_{it})

" " prestressing force = 14.7 mm → (a_{ip})

$$a_t = a_{it}(1+\phi) - a_{ip}\left[\left(1 - \frac{L_p}{P_i}\right) + \left(1 - \frac{L_p}{2P_i}\right)\phi\right]$$

Hence final, $a_t = 41.5(1+2) - 14.7\left[1 - \frac{0.2P_i}{P_i} + \left(1 - \frac{0.2P_i}{2P_i}\right)2\right]$

$P_i = 240\,kN$.

$= 86.5$ mm (Downward)

Due to Lin's approach, We get

$$a_t = \left[a_{it} - a_{ip} \cdot \frac{P_t}{P_i}\right](1+\phi)$$

$$= (41.5 - 0.8 \times 14.7)(1+2) = 89.1 \text{ mm}$$

$\cancel{-14.7\left[1 - 0.2 + (1 - 0.2)2.\right]}$

$= \cancel{-14.7[2.4]} =$

As per IS 1343.

1. Final deflection including the effects of creep, shrinkage and temperature not to exceed $l/250$

2. In the case of application of finishes and partition $\not> l/350$ or 20mm whichever is less

3. Max. upward deflection (camber) at transfer $\not> l/300$

Scanned with CamScanner

Ex: A PSC beam has the following data.

Section : 100x200 mm, Span = 2.76 m

Prestressing Cable : 1 NO. with 5 Nos. of wires of 5 mm dia. each

Initial prestress : 1200 N/mm², e = 37 mm w.r.t to centre (below)

$\alpha_e = 6.2$, $E_c = 34$ kN/mm², $f_{cr} = 4$ N/mm² (Modulus of rupture)

Imposed load = 8.4 kN/m.

Calculate the max. deflection of the beam at following stages

a) Prestress + self wt. b) prestress + self wt. + imposed loads (of 8.4 kN/m) (working)

c) cracking load. d) 1.46 x working load e) 1.8 x working load.

SOLUTION

We have the properties of the section as

$A = 2\times10^4$ mm², e = 37 mm, $I_c = \frac{100\times200^3}{12} = 666\times10^5$ mm⁴

$Z = 666\times10^3$ mm³, $E_c = 34$ kN/mm², L = 2760 mm, $\alpha_e = 6.2$

$P = 5\times19.6\times1200 = 12\times10^4$ N = 120 kN.

$f_{cr} = 4$ N/mm².

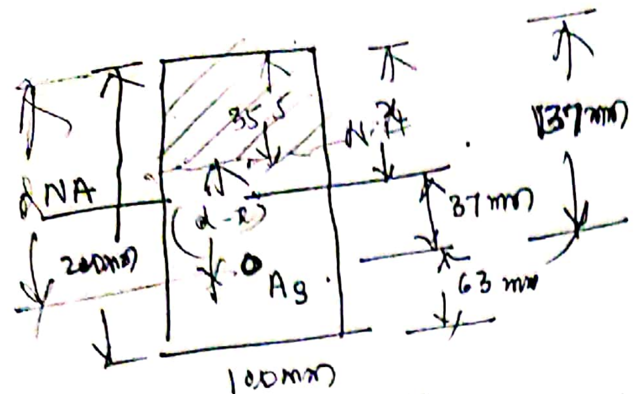
Self wt. = $0.1\times0.2\times24 = 0.48$ kN/m = 0.48 N/mm.

Deflection due to prestress = $\frac{PeL^2}{8E_cI_c} = \frac{12\times10^4\times37\times2760^2}{8\times34\times10^3\times666\times10^5}$ = 1.88 mm (upward)

" Due to self wt. = $\frac{5wL^4}{384E_cI_c}$ = 0.16 mm (Downward)

Working load on beam = $0.48 + \cancel{8.84}\ 8.4 = 8.88$ kN/m.

Deflection due to working load = $\frac{5\times8.88\times2760^4}{384\times34\times10^3\times666\times10^5}$ = 2.96 mm. (Downward)



Working moment $= \dfrac{8.88 \times 2.76^2}{8} = 8.447$ kNm.

Stress at bottom fibre due to (prestress + working load)

$$\frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z} = \frac{120 \times 10^3}{2 \times 10^4} + \frac{120 \times 10^3 \times 37}{666 \times 10^3} - \frac{8.447 \times 10^6}{666 \times 10^3}$$

$= -0.014\ N/mm^2$ (very small)

(To be continued.)

To cause a tensile stress of $4\,N/mm^2$, Extra moment Required

$M = fZ = 4 \times Z = 4 \times 666 \times 10^3 = 2664 \times 10^3$ N mm $= 2.664$ kNm

Hence total cracking moment $= 8.447 + 2.664 = 11.111$ kNm.

Hence if W_{cr} is the cracking load, then $\dfrac{W_{cr} \times 2.76^2}{8} = 11.111$

Hence, $W_{cr} = 11.7$ kN/m $= 11.7$ N/mm.

Deflection due to cracking load $= \dfrac{5}{384} \times \dfrac{11.70 \times 2760^4}{34 \times 10^3 \times 666 \times 10^3}$

$= 3.80$ mm.

Hence, <u>for cracked Transformed section</u>, we have

If 'x' is the depth of the N.A from top, then $\dfrac{bx^2}{2} = \alpha_e A_s (d-x)$

Hence, $100 \times \dfrac{x^2}{2} = 6.2 \times 100 \times (137 - x)$ ($A_s = 100\ mm^2$)

Nearly

Solving, we get $x = 35.5$ mm.

If I_r is the M.I of the cracked section, then,

$$I_r = b \cdot \frac{x^3}{3} + \alpha_e A_g r^2$$ (concrete below the N.A is not considered).

$r = d - x = 137 - 35.5 = 101.5$ mm.

$$= \frac{100 \times 35.5^3}{3} + 6.2 \times 100 \times 101.5^2$$

$= \cancel{29 \times 10^5}\ 79 \times 10^5\ mm^4.$

Deflection due to 1.46 times the working load.

Load at this stage $= 1.46 \times 8.88 = 13$ kN/m

Corresponding moment $= \frac{13 \times 2.76^2}{8} = 12.4$ kNm.

Deflection due to the above

$$= \frac{\beta l^2 M}{E_c I_r (\text{Unilinear})}, \quad \beta = \frac{5}{48} \text{ for UDL.}, \text{ Substituting,}$$

$$\text{the deflection} = \frac{5l^2}{48} \cdot \left[\frac{M_{cr}}{E_c I_g} + \frac{M - M_{cr}}{0.85 E_c I_r} \right]$$ (Bilinear)

(Beyond the cracking stage, the deflections increase and hence E_c decreases. Hence $0.85 E_c$ is taken above beyond the cracking stage)

Substituting in the above,

$$\text{Deflection} = \frac{5}{48} 2760^2 \left[\frac{11.111 \times 10^3}{34 \times 666 \times 10^5} + \frac{12.4 \times 10^3 - 11.111 \times 10^3}{0.85 \times 34 \times 79 \times 10^5} \right]$$

$= 8.3$ mm.

Deflections due to 1.8 times the working load

Load $= 1.8 \times 8.88 = 16 \text{ kN/m}$

Moment $= \dfrac{16 \times 2.76^2}{8} = 15.1 \text{ kN/m}$

Corresponding Deflection is

$$= \frac{5}{48} \times 2760^2 \left[\frac{11.111 \times 10^3}{34 \times 666 \times 10^5} + \frac{15.1 \times 10^3 - 11.111 \times 10^3}{0.85 \times 34 \times 79 \times 10^5} \right] = 17 \text{ mm}$$

Hence at various stages, the final deflections are

a) Prestress + self wt. $= -1.86 + 0.16 = -1.72$ mm (upwards)

b) Prestress + working load $= -1.88 + 2.96 = 1.08$ mm (Downwards)

c) Cracking load $= -1.88 + \cancel{8.3}\ 3.90 = \cancel{6.42 \text{ mm}}\ 2.02$ mm

~~d) 1.8 times the working load = −1.88 +~~

d) 1.46 times the working load $= -1.88 + 8.3 = +6.42$ mm

e) 1.80 times the working load $= (-1.88 + 17.0) = 15.12$ mm.