UNIT – 5

END BLOCK (ANCHORGE ZONE)

ANALYSIS AND DESIGN

ANALYSIS OF INDETERMINATE BEAMS

END BLOCK [IN PSC BEAMS]

Purpose of End Block.

In a post tensioned PSC element, in the anchorage zone, the state of stress distribution is complex and three dimensional in nature. After stressing and anchoring the prestressing wires at the ends large compressive forces concentrated over a small area of distribution are applied at the ends. These highly discontinuous forces change progressively to linear distribution over the length of end block which is nearly equal to the depth of the beam. Before becoming linear in the end block portion, the end forces produce transverse and shear stresses.

According to St. Venant's principle, the stress distribution at a point far away from the point where the force is applied can be computed based on simple theory of bending. This distance in PSC beams is the end block portion and its length is nearly equal to the depth of the section, from where linear distribution exists. This end portion is termed as END BLOCK or ANCHORAGE ZONE.

The transverse stresses developed in the anchorage zone are tensile in nature and as a result cracks develop in concrete. Hence adequate reinforcement should be provided in the anchorage zone to resist the tension which causes the bursting of the end block.

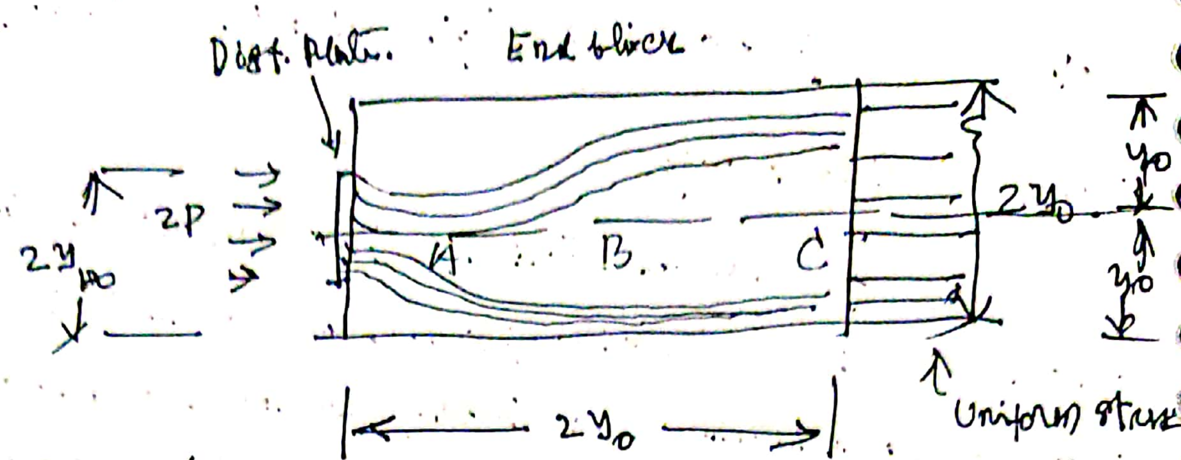


Fig. a

ZONE A.

The lines are convex towards to centre line. Compression occurs

ZONE B

The lines are concave towards centre line tension occurs.

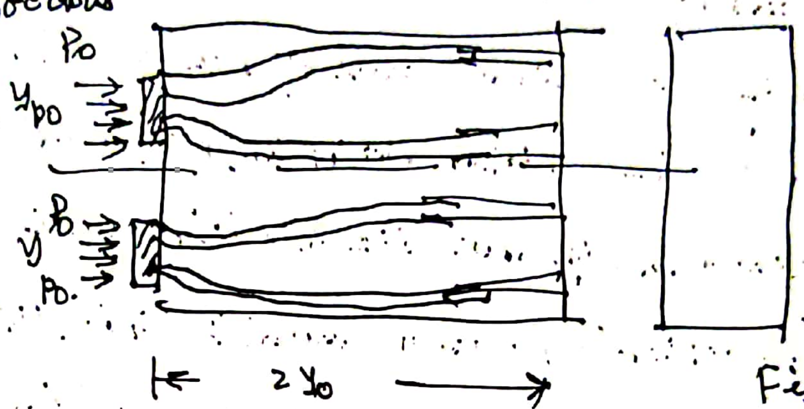
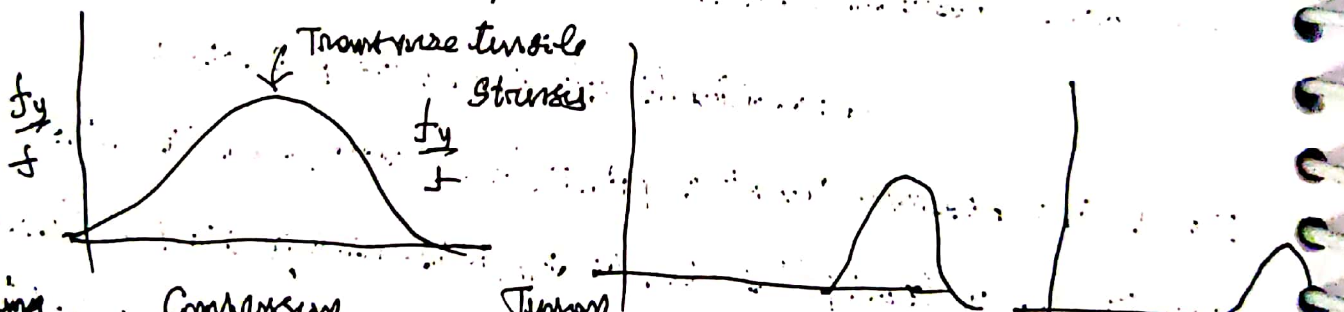


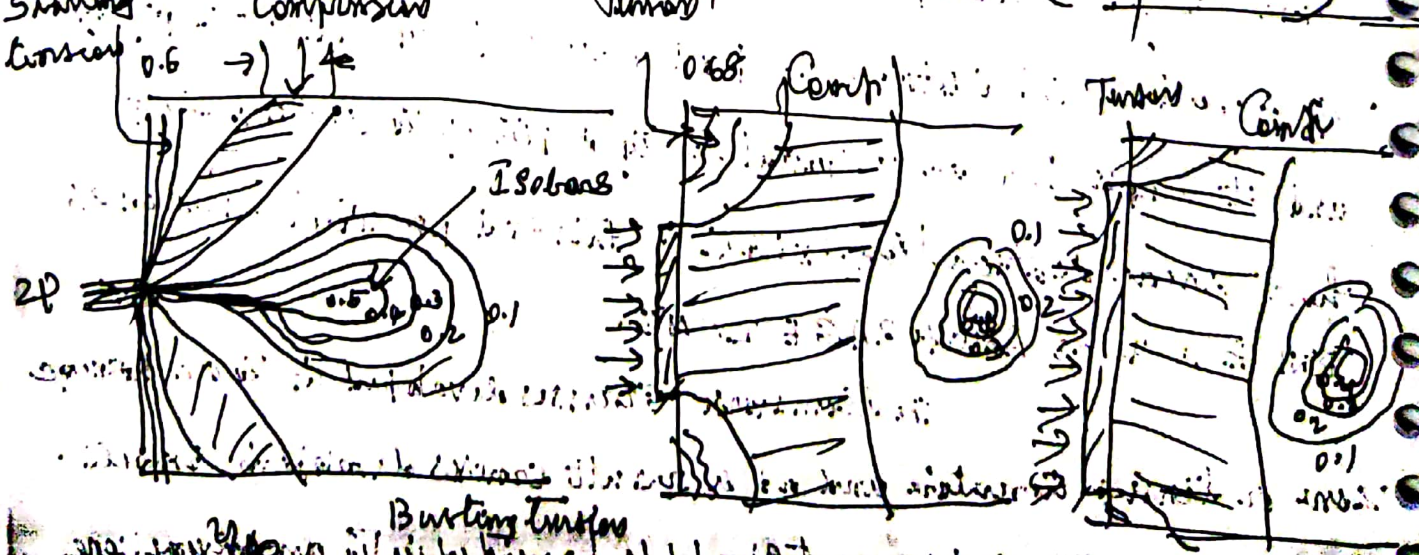
Fig. b

ZONE C

The lines are linear and parallel to the centre line.

No transverse stresses are produced.





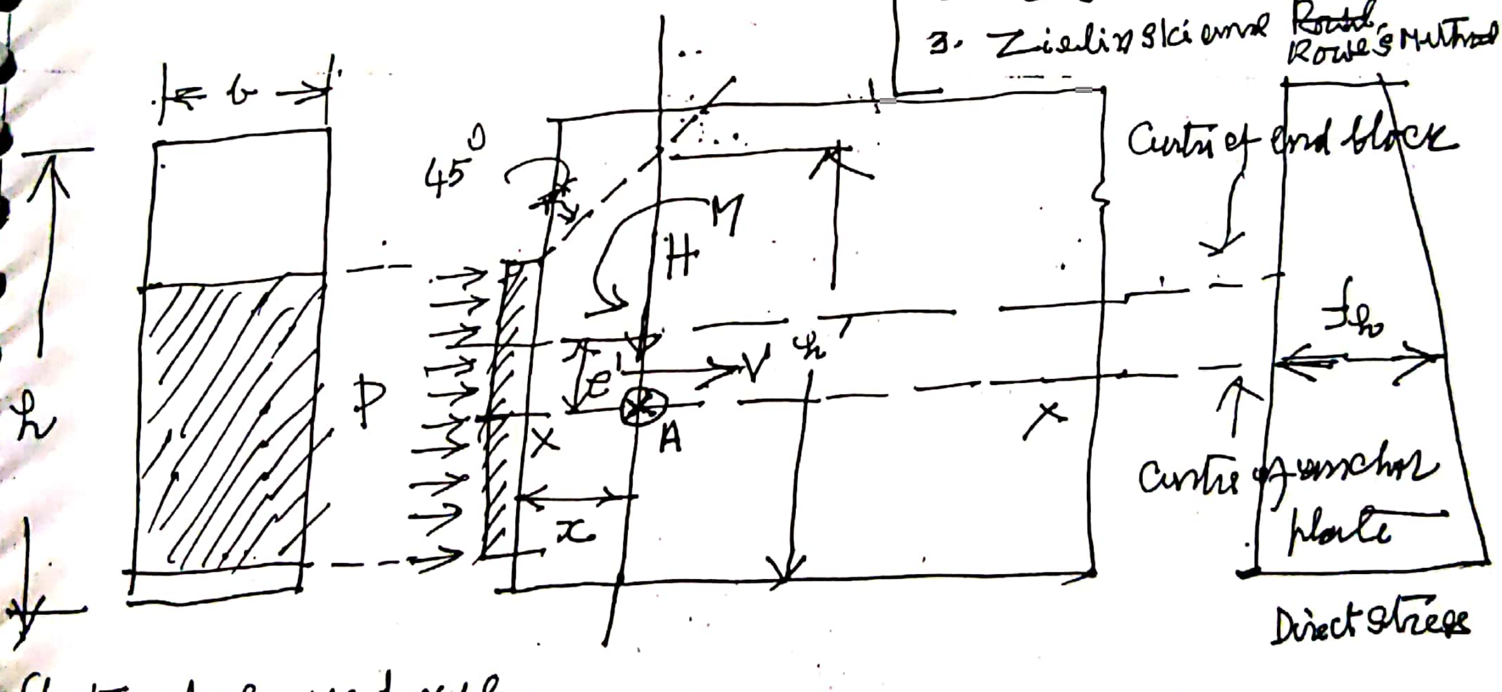
a) $\frac{y_{po}}{y_o} = 0$ b) $\frac{y_{po}}{y_o} = 0.25$ c) $\frac{y_{po}}{y_o} = 0.5$

MAGNEL'S METHOD

In this method the end block is considered as a deep beam subjected to concentrated loads from the anchorages from one side. In addition, there are normal and tangential distributed loads from the linear direct stress and shear stress distribution from the other side. The forces acting on the end block and the stresses acting at any point on the horizontal axis parallel to the beam are shown in the fig.

POPULAR METHODS

1. Magnel Method
2. Guyon's Method
3. Zielinski and ~~Rowd~~ Rowe's Method



In the above, we have,

M: Bending Moment, H: Direct Vertical force;

V: Shear force (Horizontal),

f_b: Vertical stress, f_b: Direct stress, τ: Shear stress.

(These stresses are shown at the point 'A' in the fig)

Magnel has given the following equations for the stress distribution.

$$f_b = K_1\left(\frac{M}{bh^2}\right) + K_2\left(\frac{M}{bh}\right)$$

$$\tau = K_3\left(\frac{V}{bh}\right), \quad f_h = \frac{P}{bh}\left(1+12\frac{e'^2}{h'^2}\right)$$

The constants K_1, K_2 and K_3 are shown in the table.

Table: Constants in Magnel Method

Distance from far end.	K_1	K_2	K_3
0.0	20.00	-2.00	0.00
0.10	7.720	0.00	1.458
0.20	2.560	1.280	2.048
0.30	-1.960	1.960	2.058
0.40	-4.32	2.160	1.728
0.50	-5.00	2.00	~~12.50~~ 1.25
0.60	-4.48	1.60	0.768
0.70	-3.240	1.080	0.378
0.80	-1.760	0.560	0.128
0.90	-0.520	0.160	0.018
1.00	0	0	0

The direct stress 'f_h' is computed by considering the dispersion of the concentrated load at 45°. The depth of the section h' is taken from the intercepted point with the vertical along the section through 'A'.

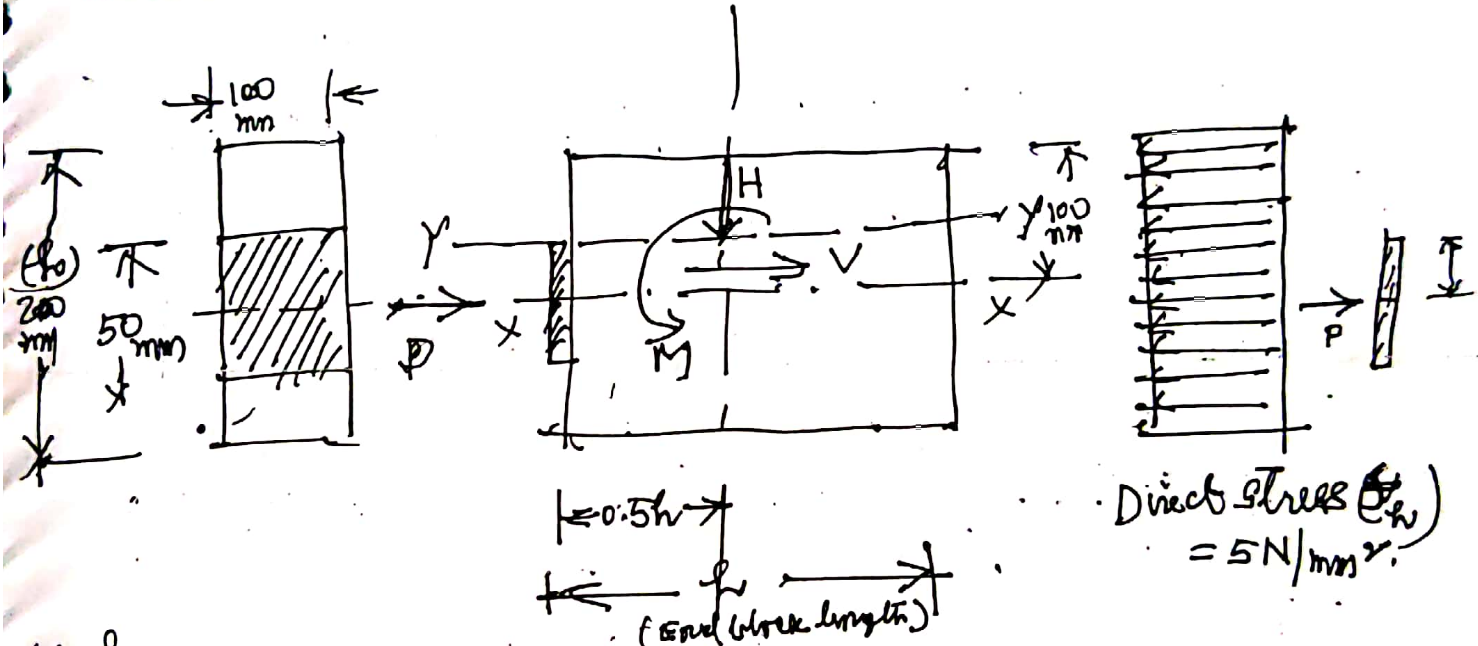
The principal stresses are, $$f_{Max\ or\ Min} = \frac{(f_b+f_h)}{2} \pm \frac{1}{2}\sqrt{(f_h-f_b)^2+4\tau^2}$$

$$\tan 2\theta = \frac{2\tau}{(f_b - f_h)}$$

The bursting tension is calculated from the principal tensile stress. Adequate reinforcement is designed against the bursting tension.

EXAMPLE.: The end block of a P.S.C beam is 100×200 mm. The prestressing force is 100 kN and the distribution plate is 100 mm wide 50 mm deep kept concentric at the ends. Calculate the position and magnitude of max. tensile stress on the horizontal section through the centre and the edge of the distribution plate. Calculate the bursting tensile force on these planes.

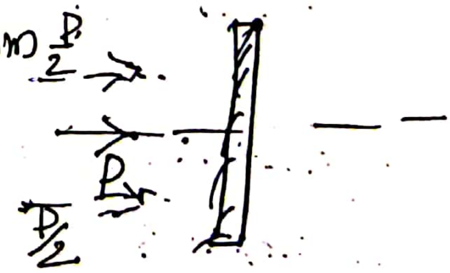
SOLUTION.



We have,

$$P = 100\text{ kN},\quad b = 100\text{ mm},\quad h = 200\text{ mm}$$

$$\text{Direct stress } f_h = \frac{100\times10^3}{200\times100} = 5\text{ N/mm}^2$$



The vertical stress f_v and the shear stress are critical at 0.5 h normally.

a) For Section XX

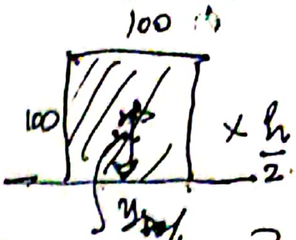
$$x = 0.5h,\quad \frac{x}{h} = 0.5.$$

From table of Magnel, $K_1 = -5.00$, $K_2 = 2.00$, $K_3 = 1.25$.

2. $y_{p_0} = 50\,mm$, $Y_{p_0} = \frac{50}{2}$.

Hence, Force in the xx × Dist. from centre

$M = \left(5\times 100\times \frac{100}{2}\times\frac{100}{2}\right) - \frac{P}{2}\times\frac{Y_{p_0}}{4}$



$= \left(5\times 1000\times 1000\times \frac{100}{2}\right) - \left(\frac{100\times 10^3}{2}\right)\left(\frac{50}{4}\right) = 1875\times 10^3$ Nmm.

In this case $V=0$, $H=0$.

Hence, $f_v = K_1\left(\frac{M}{bh^2}\right) + K_2\left(\frac{H}{bh^2}\right) = -5.00\times\frac{1875\times10^3}{100\times200^2}$

Hence, $f_v = -2.35\ N/mm^2$, $f_h = +5.0\ N/mm^2$.

$\tau = K_3 \cdot \frac{V}{bh} = 0 \cdot (V=0)$

Hence the principal tension (f_{min}) is given by,

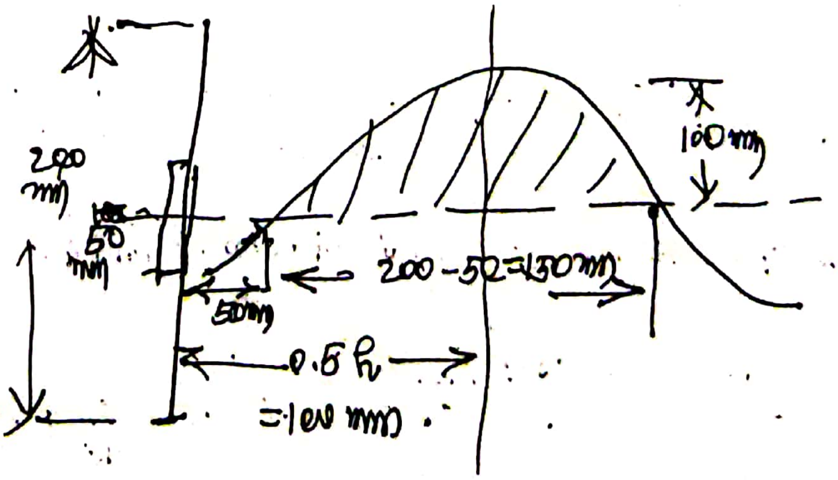
$= \left[\frac{(5-2.35)}{2} - \frac{1}{2}\sqrt{(5+2.35)^2 + 0}\right] = -2.35\ N/mm^2$.

Assuming parabolic distribution of bursting tension,

$= \frac{2}{3}\times 150\times 100\times 2.35$

$= 23{,}500$ N

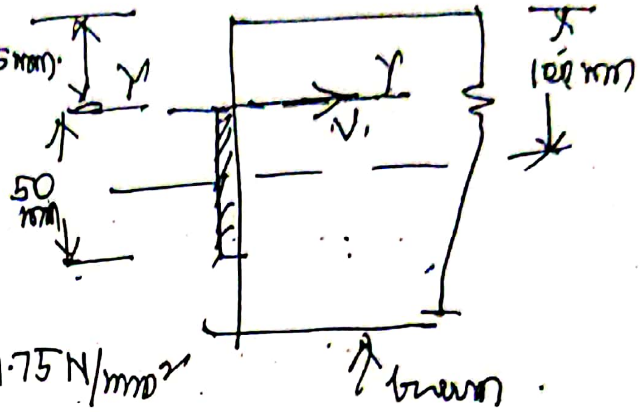
on XX-axis.



b) Section through edge of the dist. Plate (YY').

$M = 100 \times 5 \times 75 \times \frac{75}{2} = 14 \times 10^5$ N mm
(Width)

$V = (100 \times 75 \times 5) = 37,500$ N
(Shear force) $H = 0$.



$f_b = k_1\left(\frac{M}{bh^2}\right) - k_2\left(\frac{H}{bh}\right)$
$=(0)$

$= k_1 \cdot \frac{M}{bh^2} = -\frac{5 \times 14 \times 10^5}{100 \times 200^2} = -1.75$ N/mm²

$f_h = 5.00$ N/mm².

$\tau = k_3 \cdot \frac{V}{bh} = 1.25 \times \frac{(-37,500)}{100 \times 200} = -2.35$ N/mm²

Hence, the principal tensile stress

$$= \frac{(5 - 1.75)}{2} - \frac{1}{2}\sqrt{(5+1.75)^2 + 4(-2.35)^2} = -2.475 \text{ N/mm}^2$$
(Tension).

Angle of inclination of the principal plane w.r.t the vertical plane

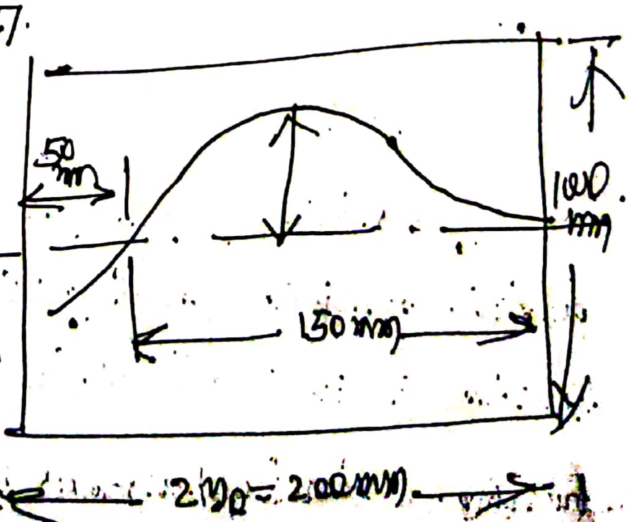
is $\tan 2\theta = \left(\frac{2\tau}{f_b - f_h}\right) = \frac{2 \times (-2.35)}{(-1.75 - 5.0)} = 0.7$.

Hence, $2\theta = 35^\circ$, $\theta = 17.5^\circ$.

Tensile stress component in the vertical direction $= 2.475 \times \sec 17.5^\circ = 2.6$ N/mm²

Hence, bursting tensile force

$= \frac{2}{3} \times 150 \times 100 \times 2.6 = 26,000$ N.



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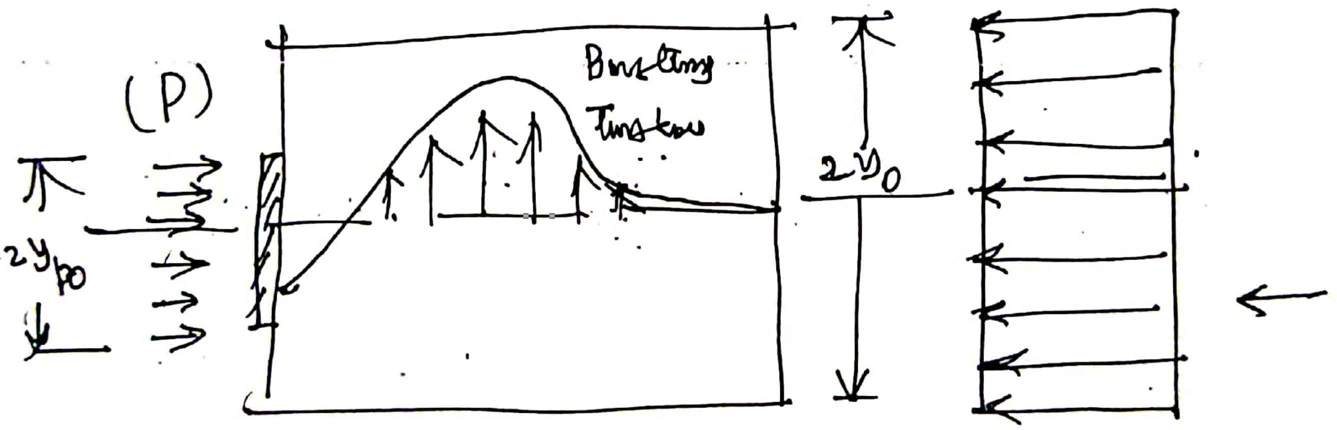
GUYON'S METHOD.

Guyon has developed tables for Computation of bursting tension in end blocks which are based on his previous work. The distribution of forces at the ends are treated as (a) evenly distributed and not evenly distributed.

FORCES EVENLY DISTRIBUTED:

The resultant of the stress distribution at a distance equal to the depth of the beam Coincides with the line of action of force.



Stress distribution.

The position of Zero stress, maximum transverse stress and its magnitude for the forces evenly distributed are Computed by using the Coefficients given in the table. In this Category,

The bursting tension is expressed as $F_{bst} = 0.3P\left[1-\left(\frac{y_{po}}{y_0}\right)^{0.58}\right]$

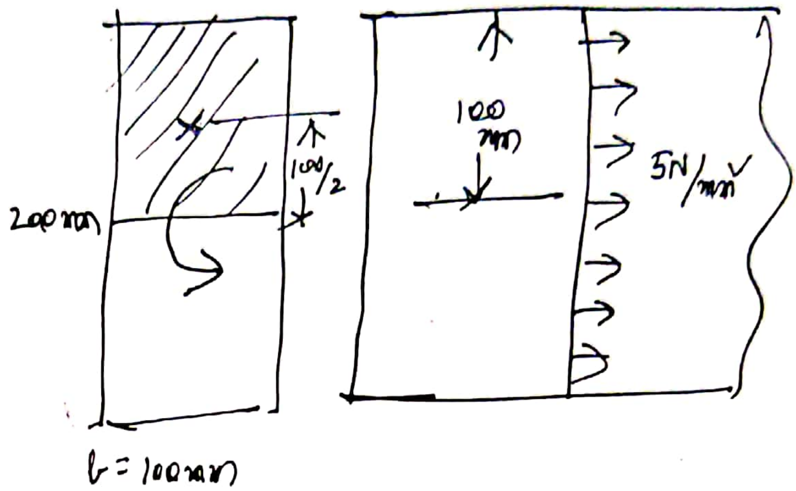
Where P: Anchorage force, $\frac{y_{po}}{y_0}$: distribution ratio.

a) Anchor Plate Concentric. (Explanation for the Example)

For computing the moment 'M'

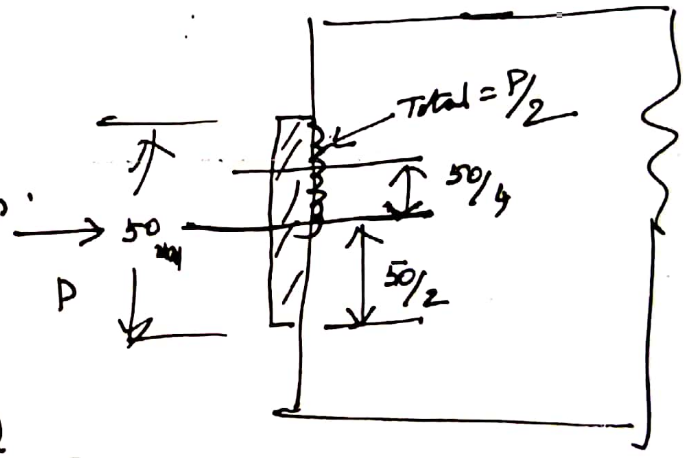
Moment about centre,

$$M = \left(5 \times 100 \times 100 \times \frac{100}{2}\right) - \left(\frac{P}{2} \times \frac{50}{4}\right)$$



$$M = 5 \times 100 \times 100 \times \frac{100}{2} - \frac{100 \times 10^3}{2} \times \frac{50}{4}$$

Hence $M = 1875 \times 10^3$ Nmm.



Anchor Plate end eccentric

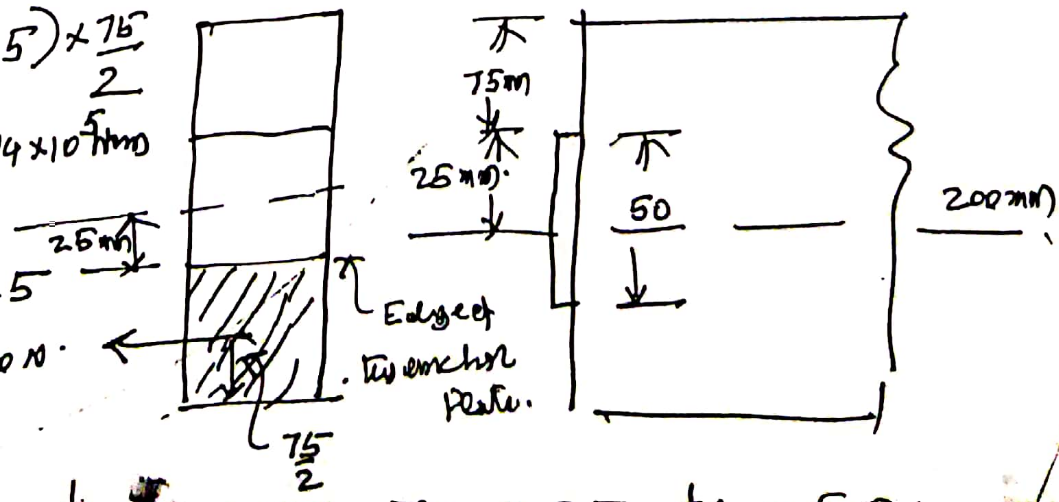
$$M = (100 \times 75 \times 5) \times \frac{75}{2}$$

$$- 0.514 \times 10^5 \text{ Nmm}$$

$V = -100 \times 75 \times 5$

$= -37,500$ N.

$H = 0$.



Calculate, $f_y = -1.75$, $J = -2.35$, $f_h = 5.0$.

Principal tensile stress $= -2.475$, $\tan 2\theta = \frac{2J}{f_v - f_h} = 0.7$, $\theta = 17.5^\circ$.

Table :- Vertical Stress along axis at the ends of prestressed beams.

Sl. No.	Distribution Ratio	Position of Zero stress.	Position of max. stress.	Ratio of max stress (tension) to average stress
1	0.00	0.00	0.17	0.50
2	0.10	0.09	0.24	0.43
3	0.20	0.14	0.30	0.36
4	0.30	0.16	0.36	0.33
5	0.40	0.18	0.39	0.27
6	0.50	0.20	0.43	0.23
7	0.60	0.22	0.44	0.18
8	0.70	0.23	0.45	0.13
9	0.80	0.24	0.46	0.09

EXAMPLE.. Using Guyon's method Compute the position and magnitude of max. tensile stress and bursting tension for the end block ~~and~~ with concentrated anchor force of 100kN. Use the ~~same~~ following data.

~~SOLUTION~~: we have, $P = 100\,kN$.

$2y_{po} = 50\,mm$, $2y_o = 200\,mm$

$\frac{y_{po}}{y_o} = 0.25$

SOLUTION:- From the table, for $\frac{y_{po}}{y_0} = 0.25$.

Position of zero stress from the end face = $0.15 \times 2y_0 = 30\,mm$.

Position of max. stress = $0.33(2y_0) = 66\,mm$.

Max. tensile stress = $0.345 \times$ avg stress $= 0.345 \times \frac{P}{A}$

$$= 0.345 \times \frac{100 \times 10^3}{100 \times 200} = 1.725\ N/mm^2.$$

The formula for bursting tension $= 0.3P\left[1 - \left(\frac{y_{po}}{y_0}\right)^{0.58}\right]$

$$= 0.3 \times 100 \times 10^3 \left[1 - (0.25)^{0.58}\right] = 16,575\ N.$$

Taking the yield stress in mild steel as $260\ N/mm^2$.

Area of steel required $= \frac{16575}{(0.87 \times 260)} = 73\ mm^2$

(Partial Safety factor)

If we use the approximate formula for bursting tension, from the parabolic distribution, we obtain,

The bursting tension $= \frac{2}{3} \times (200-66) \cdot 100 \times 1.725 = 15,550$ nearly.

ZIELINSKI AND ROWE'S METHOD.

As per this method the following equations are recommended for considering tension in the end block of a PSC beam.

$$f_{(t)}\ (\text{Max}) = f_c\left[0.98 - 0.825\,\frac{Y_{po}}{Y_o}\right] \quad (1)$$

$$F_{bst} = P_k\left[0.48 - 0.4\left(\frac{Y_{po}}{Y_o}\right)\right] \quad (2)$$

Where

f_{t} : Transverse tensile stress

F_{bst} : Bursting tensile force

f_c : Average compressive stress and P_k : Applied compressive force (Tendon jacking force on the end block).

I.S. CODE FORMULA. (As per IS 1343-2012).

$$P_{bst} = P_k\left[0.32 - 0.3\,\frac{Y_{po}}{Y_o}\right]$$

(The formula has been adopted by the Code based on Zielinski and Rowe's formula).

EXAMPLE.

The end block of a post tensioned concrete beam 300 x 300 mm section is subjected to a concentric anchorage force of 832.8 kN by Freyssinet anchorage (circular) of area 11720 mm². Design and detail the anchorage reinforcement. Use Zielinski and Rowe's method. Compare the result with that of I.S. Code method.

SOLUTION :

$$f_c = \frac{832.8 \times 10^3}{300 \times 300} = 9.3\ N/mm^2$$

Anchorage diameter $= \sqrt{\frac{11,720 \times 4}{\pi}} = 123\ mm$.

Since $\frac{2Y_{po}}{2Y_o} = \frac{123}{300} = 0.41$

Hence tensile stress $(f_t\ max) = f_c \left[0.98 - 0.825 \frac{Y_{po}}{Y_o}\right]$.

Hence, $f_t\ (Max) = 9.3 \left[0.98 - 0.825 \times 0.41\right] = 6\ N/mm^2$.

$$F_{bst} = P_k \left[0.48 - 0.41 \frac{Y_{po}}{Y_o}\right] = 832.5 \times 10^3 \left[0.48 - 0.41 \times 0.4\right] = 264\ KN.$$

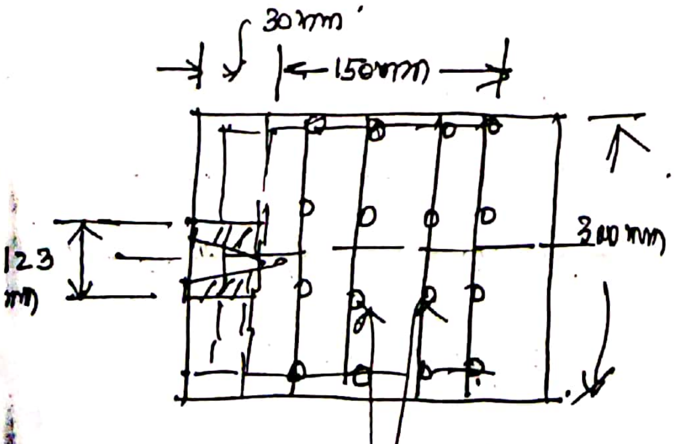
Using 10 mm dia. M.S rods, Area of section $= 79\ mm^2$, $\sigma_y = 260\ N/mm^2$.

Hence, No. of rods required $= \frac{264 \times 10^3}{0.87 \times 260 \times 79} = 15$ Nos, Provide 16 Nos.

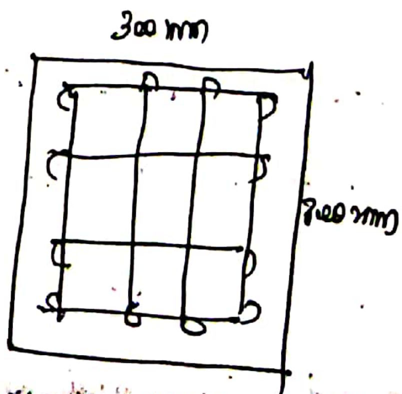
Provide 4 meshes (at 40 mm c/c in the longitudinal direction) with 4 Nos of 10 mm rods kept in the horizontal transverse direction supported by vertical members. The meshes are arranged between $0.2y_o$ to y_o (30 mm to 150 mm)

By I.S. CODE METHOD

$$P_{bst} = 832.8 \left[0.32 - 0.3 \times 0.41\right] = 164.06\ kN.$$ (It is more than the previous value)



4 Nos 10 mm bothways at 40 mm c/c. LONGITUDINAL



MESH IN THE CROSS SECTION

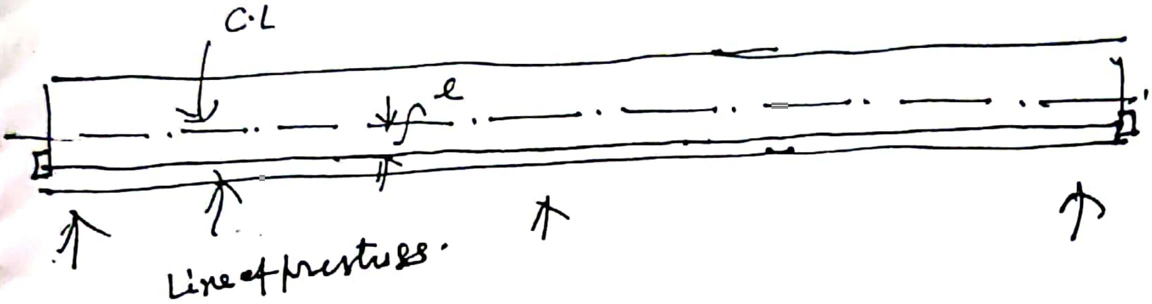
PRESSURE LINE & CONCORDANT CABLE (1)

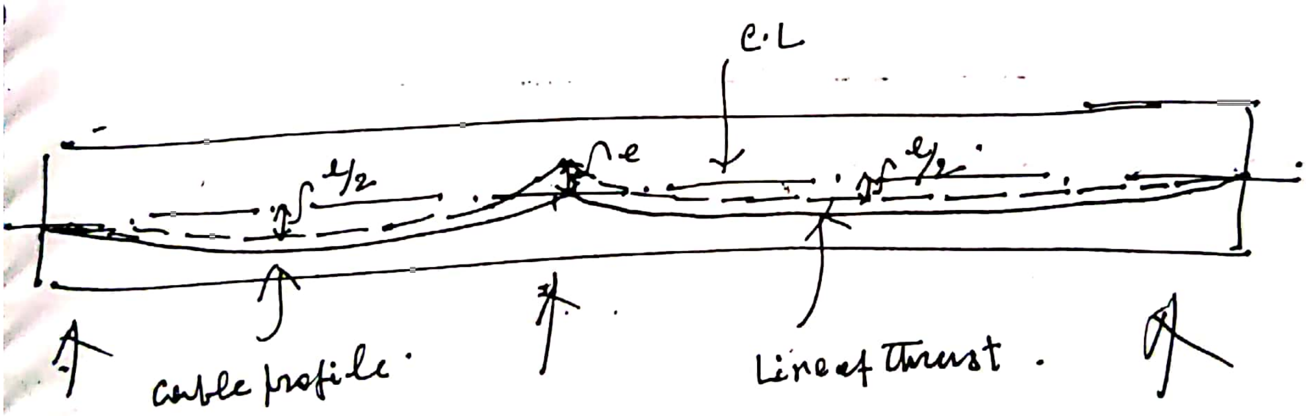
(INDERMINATE BEAMS IN PSC)

Definitions

1) Line of prestress : The locus of the centroid of the prestressing force along the structure, is the line of prestress.

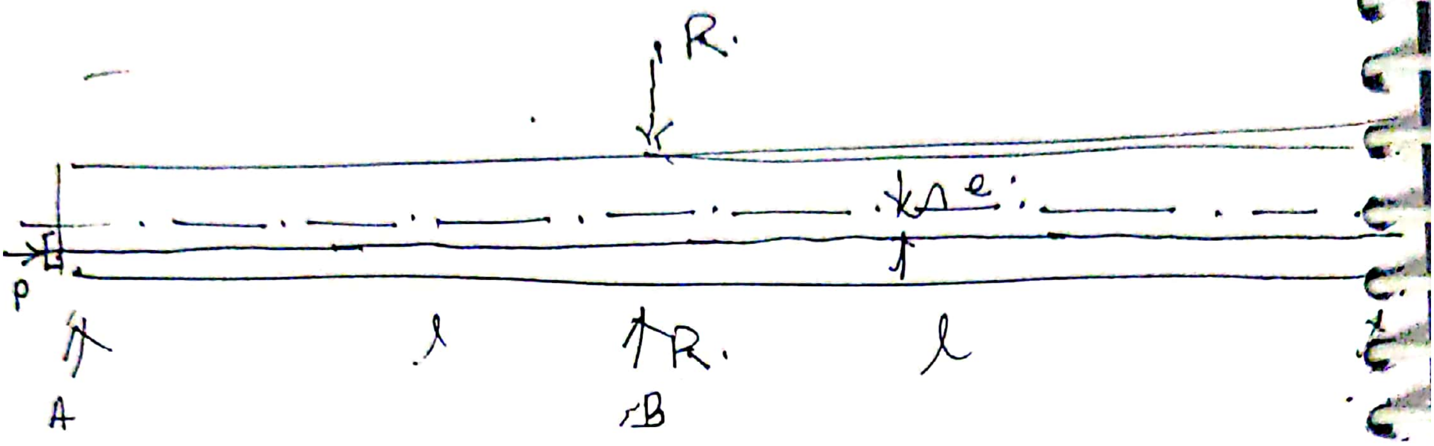
Ex.

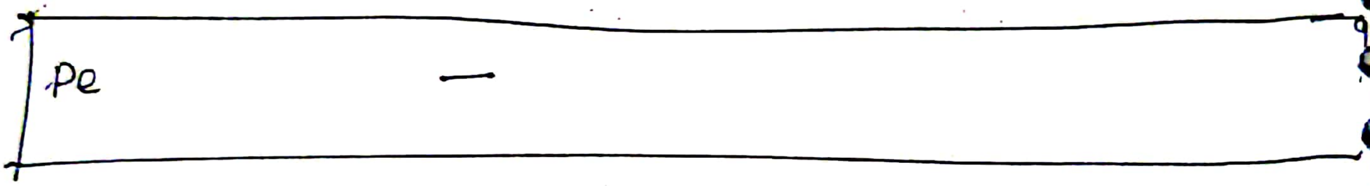




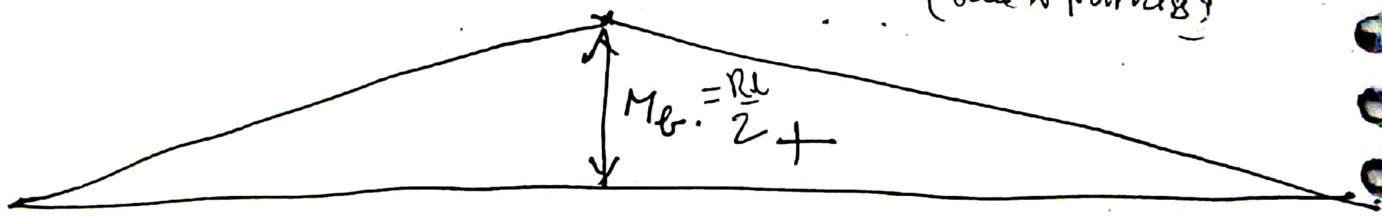
Pressure or thrust line : The pressure line is the locus of the resultant compression at different sections of the given structural member. The pressure line is shifted from the centroidal axis and this shift is obtained as the ratio of the resultant moment and the prestressing force at that section.

Ex: In the case of a 2-span Continuous beam, we have two B.M.Ds





Primary BMD.
(due to prestress)



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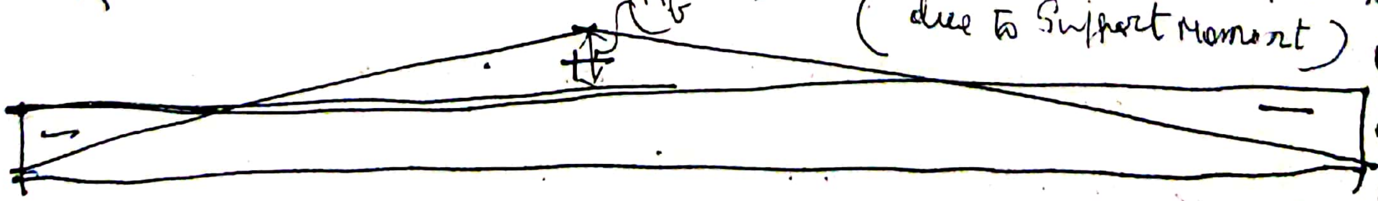
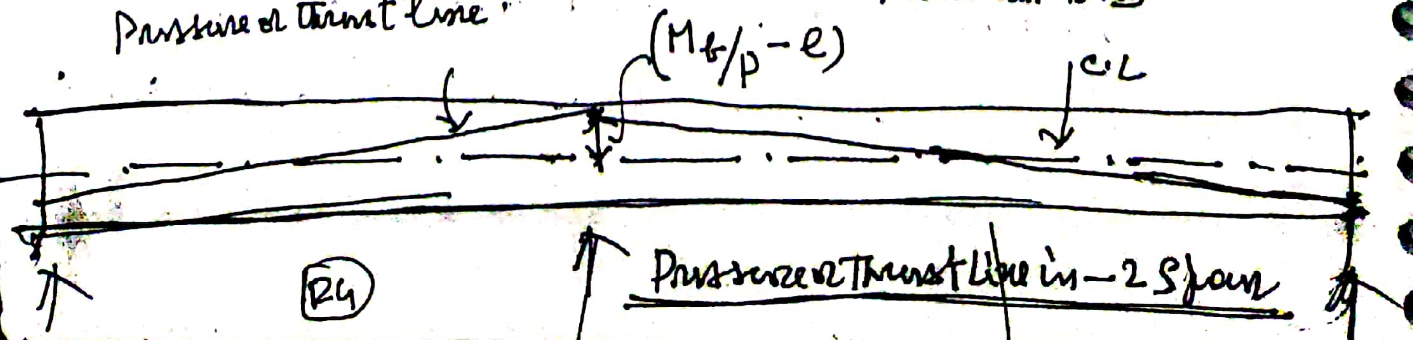
Secondary moment diagram
(due to Support moment)

Net = $(M_b - Pe)$

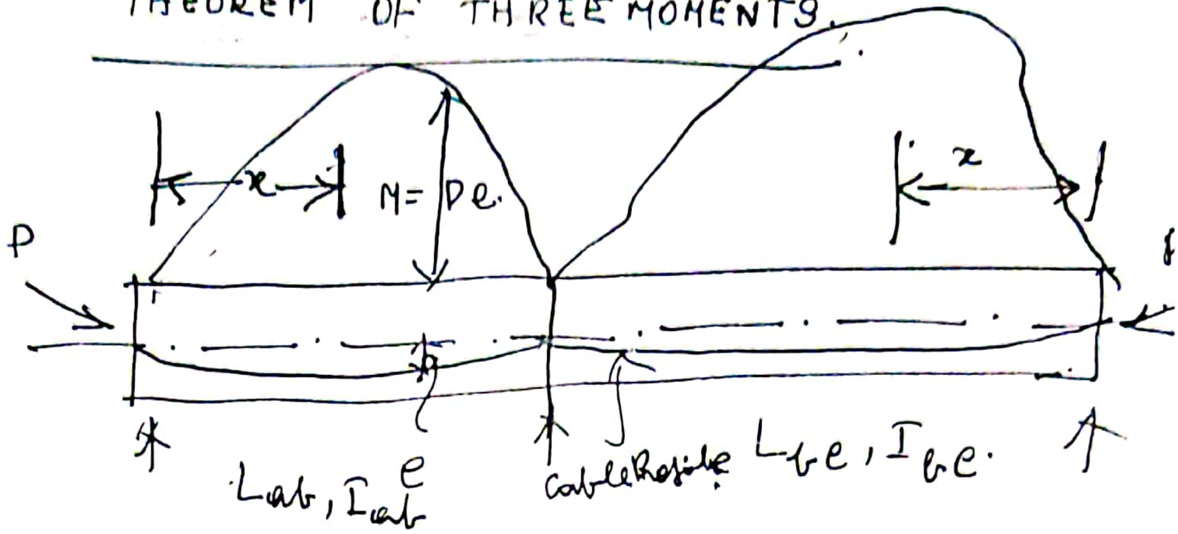


Resultant BMD.



Pressure or Thrust Line in 2 Span

THEOREM OF THREE MOMENTS



The normal 3 moment equation is (For Prismatic beam).

$$M_a l_1 + 2M_b (l_1 + l_2) + M_c l_3 = -\frac{6a_1 \bar{x}_1}{l_1} - \frac{6a_2 \bar{x}_2}{l_2} \quad (a)$$

For a general 3-span Continuous beam, it is

$$\frac{l_{ab}}{I_{ab}} M_{ab} + \frac{l_{ab}}{I_{ab}} . 2M_{ba} + \frac{l_{bc}}{I_{bc}} . 2M_{bc} + \frac{l_{cb}}{I_{cb}} M_{cb}$$

$$= -\frac{6}{l_{ab} I_{ab}} \int_0^{l_{ab}} Mx\,dx - \frac{6}{L_{bc} I_{bc}} \int_0^{l_{bc}} Mx\,dx . \quad (b)$$

$M_{ab}, M_{ba}, M_{bc}, M_{cb}$ are the Secondary moments.

M : Free B.M (Primary moment) at 'x' from the end Support.

$$\int Mx\,dx = \int Pex\,dx = P\int ex\,dx \quad (c)$$

Substituting eq. (c) in eq. (b)

$$\text{The RHS} = -\frac{6P}{l_{ab} I_{ab}} \left(\int_0^{l_{ab}} ex\,dx\right) - \frac{6P}{l_{bc} I_{bc}} \int_0^{l_{bc}} ex\,dx .$$

Putting $K = \frac{I_{ab}}{l_{ab}} \Big/ \frac{I_{bc}}{l_{bc}}$ and $\bar{K} = -\frac{6P}{l^2}\left(\int ex\,dx\right)$

THEOREM OF THREE MOMENTS.

$$\cancel{M} = -\frac{6}{l_{ab} I_{ab}} \int_0^{l_{ab}} M x\,dx - \frac{6}{l_{bc} I_{bc}} \int_0^{l_{bc}} M x\,dx$$

We have $\int M x\,dx = \int P e x\,dx = P \int e x\,dx$

We have $K = \dfrac{I_{ab}/l_{ab}}{I_{bc}/l_{bc}}$.

Multiplying LHS with $\dfrac{I_{ab}}{l_{ab}}$, we get

$$M_{ab} + 2M_{ba} + 2\cdot\frac{l_{bc}}{I_{bc}} \times \frac{I_{ab}}{l_{ab}} M_{bc} + \frac{l_{bc}}{I_{bc}} \times \frac{I_{ab}}{l_{ab}} M_{cb}.$$

RHS,

$$= \cancel{-2P \int e x\,dx \times \frac{I_{ab}}{l_{ab}}}$$

We have $\int M x\,dx = \int P e\,dx$
$= P \int e x\,dx.$

Hence
$$M_{ab} + 2M_{ba} + 2\cdot\left(\frac{I_{ab}}{l_{ab}} \Big/ \frac{I_{bc}}{l_{bc}}\right) M_{bc} + \left(\frac{I_{ab}}{l_{ab}} \Big/ \frac{I_{bc}}{l_{bc}}\right) M_{cb}$$

$$= -\cancel{2P \int e x\,dx \times \frac{I_{ab}}{l_{ab}}} = -\frac{6}{l_{ab}^2} P \int (e x\,dx) - \frac{6P}{l_{bc}^2} \int e x\,dx$$

or $\cancel{M_{ab} + 2M_{ba} + 2KM_{c} + KM_{cb}}$

Taking $l_{ab} = l_{bc} = l$,

$$\boxed{RHS = -\frac{6P}{l^2} \int e x\,dx}$$

Putting $\bar{K} = -\dfrac{6P}{L^2} \int e x\,dx$

$$\bar{K}_{ba} = -\frac{6P}{L^2_{ba}} \int e x\,dx$$

$$\bar{K}_{bc} = -\frac{6P}{L^2_{bc}} \int e x\,dx,$$

(126)

The final equation is written as

$$M_{ab} + 2M_{ba} + 2k\,M_{bc} + k\,M_{cb} = \bar{K}_{ba} + k\bar{K}_{bc}.$$

$$k = \left(\frac{I_{ab}}{l_{ab}}\right) \Big/ \left(\frac{I_{bc}}{l_{bc}}\right), \quad \bar{K} = -\frac{6P}{l^2}\int e\,x\,dx.$$

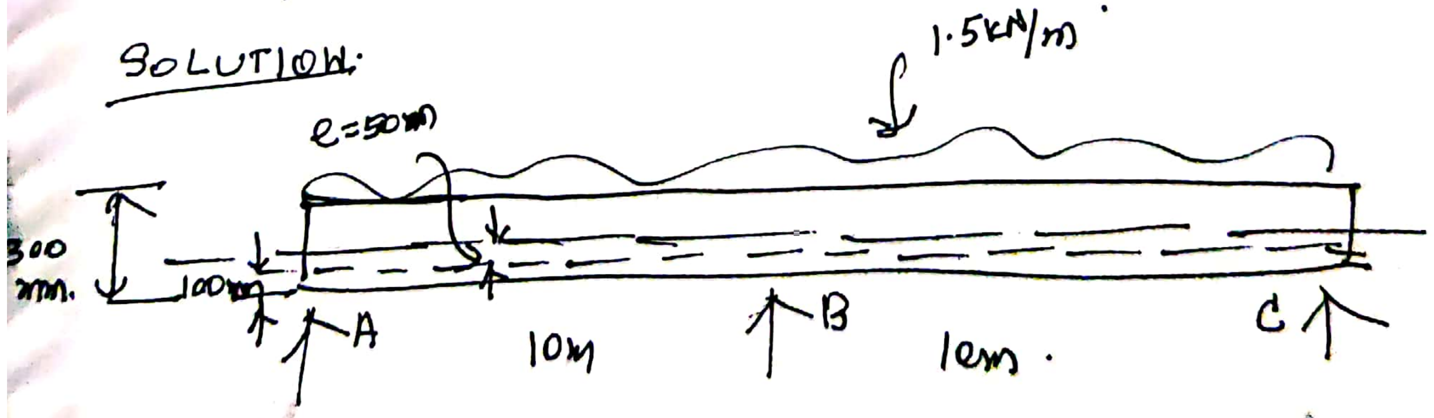
Ex: A continuous beam ABC has AB = BC = 10 m. The section is 100 × 300 mm uniform throught. The cable is parallel to the axis of the beam at 100 mm from the soffit. P = 360 kN.

Determine a) secondary and the resultant moments at the central support.

b) If there is an imposed load of 1.5 kN/m calculate the resultant stresses at top and bottom of the section at the centre 'B'. Density of concrete is 24 kN/m³

c) Locate the position of the pressure line.

SOLUTION:



We have,

$$M_{ab} + 2M_{ba} + 2kM_{bc} + kM_{cb} = \bar{K}_{ba} + k\bar{K}_{bc}.$$

A and c are Simply Supported. Hence $M_{ab} = M_{cb} = 0$.

Here $2M_{ba} + 2kM_{bc} = RHS \left(\bar{K} = -\frac{6P}{l^2}\int ex\,dx \right)$

$$k = \frac{I_{ab}}{l_{ab}} \Big/ \frac{I_{bc}}{l_{bc}} = 1 \quad \text{(Prismatic throught)}$$

Hence, we can take $M_{ba} = M_{bc} = M_b$

Hence, we get $2M_b + 2M_b = \bar{K}_{ba} + \bar{K}_{bc}$.

Since $ab = bc$, $\bar{K}_{ba} = \bar{K}_{bc}$

Hence $2M_b + 2M_b = 2\bar{K}_{ab \text{ or } bc}$.

$$\bar{K}_{ab} = \bar{K}_{bc} = -\frac{6P}{l^2}\int ex\,dx, \quad e = 150-100 = 50\text{mm}.$$

Taking $-ve$ for 'e' below the axis (Hogging)

$$\bar{K}_{ab} = \bar{K}_{bc} = +\frac{6Pe}{l^2} \times \frac{l^2}{2} = +\frac{6Pe}{2} = \frac{6 \times 360 \times 0.05}{2}$$

(e is const.) (m)

$= 54$ kNm.

Substituting, $4M_b = 2 \times 54 = 108$, $M_b = \frac{108}{4} = 27$ kNm $-$ +

(Hogging) (Sagging)

Hence, resultant moment at centre $= -Pe + M_b$

$= -360 \times 0.05 + 27 = +9$ kNm.

Self wt. of the beam $= 0.1 \times 0.3 \times 24 = 0.72$ kN/m.

Imposed load = $1.5 kN/m$

Total load = $2.22 kN/m$ (UDL)

Support moment at 'B' due to UDL = $-\frac{W_d l^2}{8} = -\frac{2.22 \times 10^2}{8}$

$= -27.75 kNm$.

Hence, final resultant moment at B = $+9 - 27.75 = -18.75 kNm$ (Hogging)

Hence, the stresses are

At top $\frac{360 \times 10^3}{100 \times 300} - \frac{18.75 \times 10^6}{\frac{1}{6} \times 100 \times 300^2} = -0.5 N/mm^2$ (Tension)

At bottom $= \frac{360 \times 10^3}{100 \times 300} + \frac{18.75 \times 10^6}{\frac{1}{6} \times 100 \times 300^2} = +24.5 N/mm^2$ (Comp.)

Position of Pressure line:

At A, $e = -50 mm$; At B, $e = \frac{M}{P} = \frac{-18.75 \times 10^3}{360} = -52 mm$. (below)

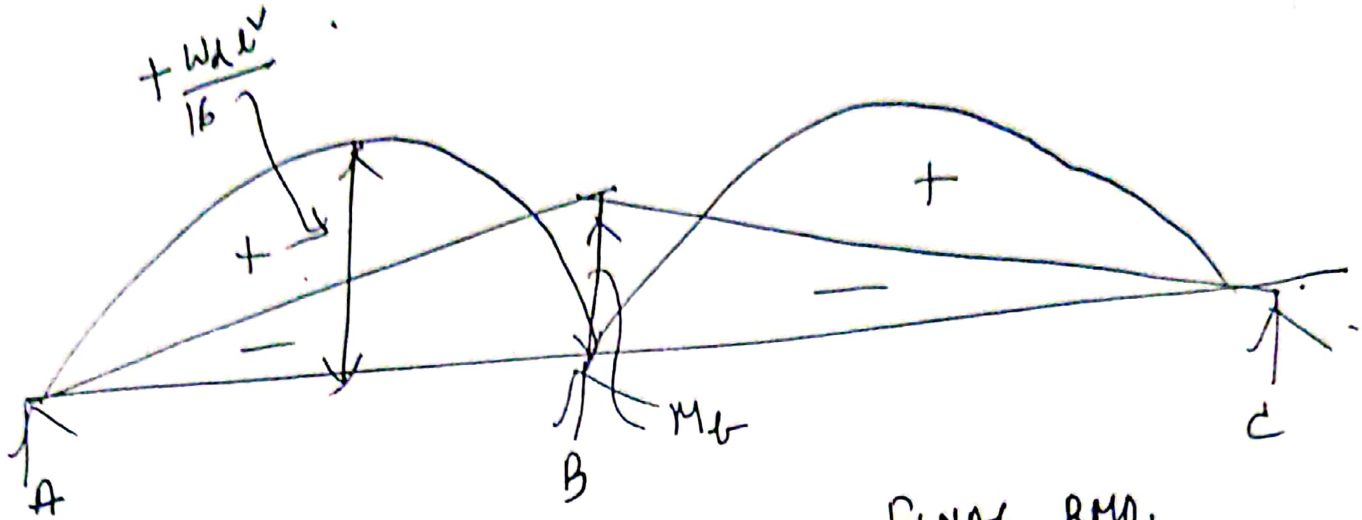
At centre of A and B or B and C, the resultant moment

$= -\frac{Pe}{4} + \frac{Wl^2}{16} = -\frac{360 \times 0.05}{4} + \frac{2.2 \times 10^2}{16} = +9.25 kNm$

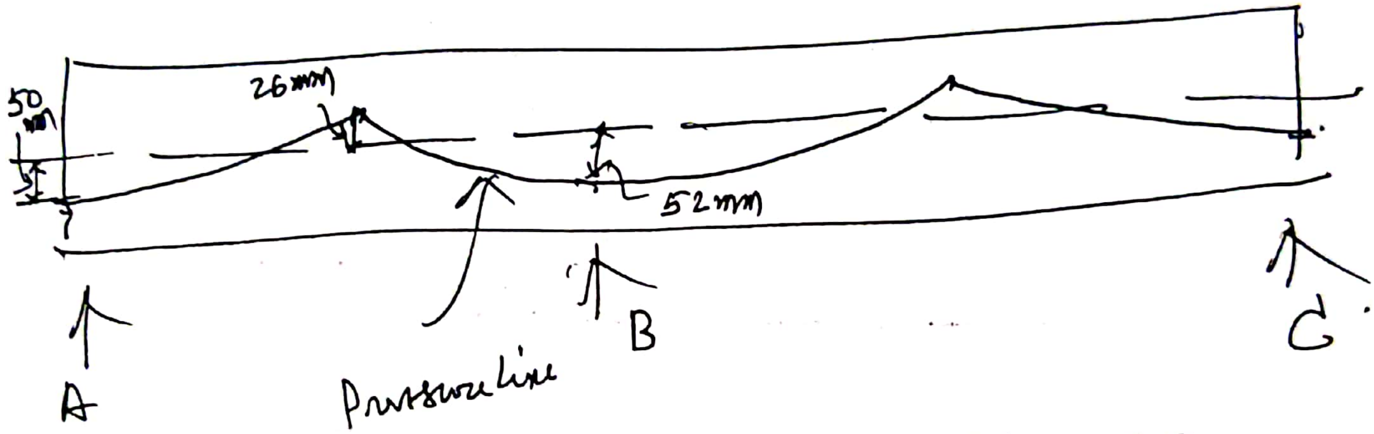
Hence at this point, the shift in the pressure line

$= +\frac{9.25}{360} \times 10^3 = 26 mm$ (above the axis).

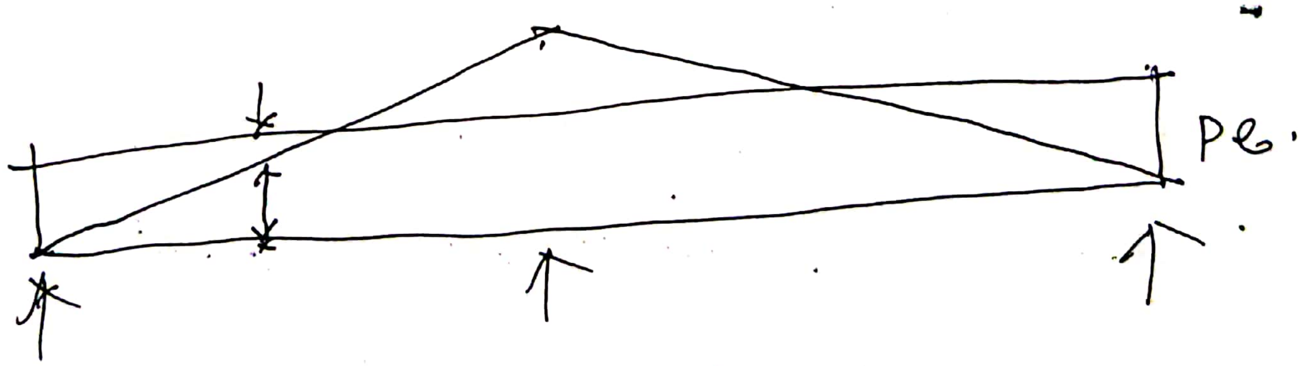
Hence, finally



FINAL BMD.



PRESSURE LINE



~~A.A.S~~

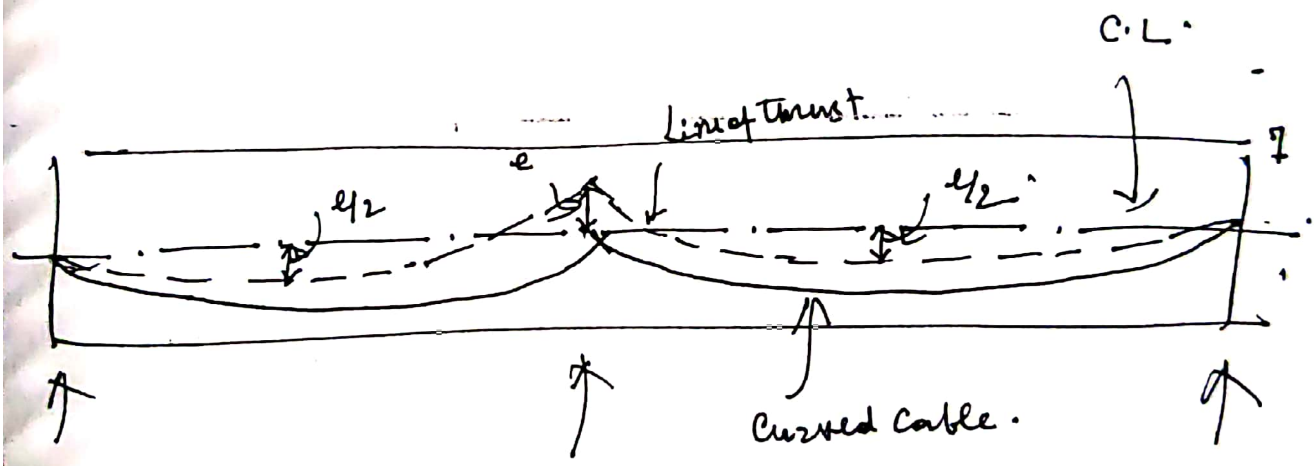
~~(KMM) 7.8~~ (130)

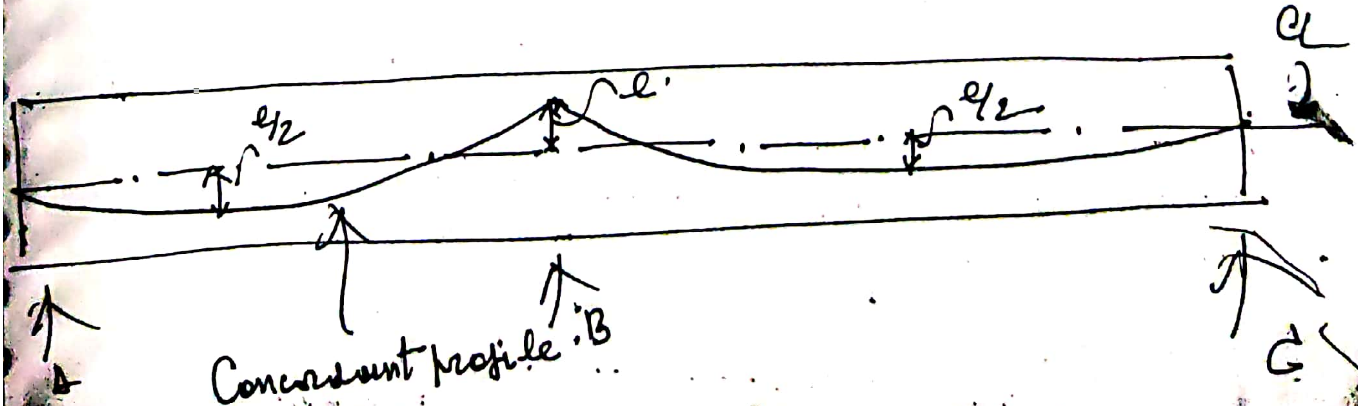
CONCORDANT CABLE PROFILE .

A tendon profile in which the eccentricity is proportional at all sections to the B.M caused by any loading on a rigidly supported statically indeterminate structure is termed as Concordant profile.

According to Guyon, tendons in statically indeterminate structures placed to coincide with the pressure or thrust line do not induce secondary moments.

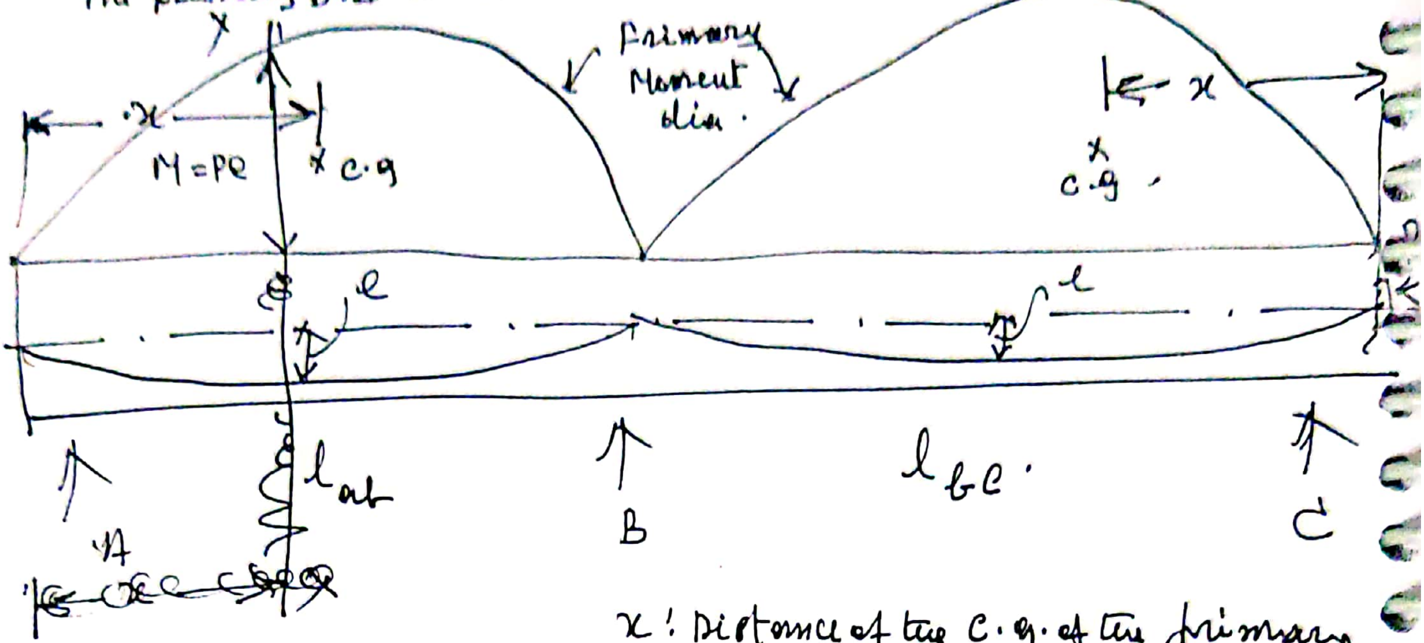
Ex. Consider a 2-span continuous beam with a curved cable.





Consider the general Case of a 2-Span beam with a Curved Cable.

The primary BMD is also curved as shown.



x : Distance of the c.g. of the primary B.M.D's from the ends for each span.

From the above, at any Section, it can be written that

$$\int Mx\,dx = \int Pex\,dx = P\int ex\,dx$$

= P [Moment of the area between the cable profile and the centroidal axis taken about the Support]

Considering Concordant cables, they do not induce Secondary moments. For a 2-Span Continuous beam, we have the general three moments equation ~~as~~ to find secondary moments as,

$$\frac{l_{ab}}{I_{ab}} M_{ab} + \frac{l_{ab}}{I_{ab}} 2M_{ba} + \frac{l_{bc}}{I_{bc}} 2M_{bc} + \frac{l_{bc}}{I_{bc}} M_{cb}$$

$$= -\frac{6}{l_{ab} I_{ab}} \int_0^{l_{ab}} Mx\,dx - \frac{6}{l_{bc} I_{bc}} \int_0^{l_{bc}} Mx\,dx$$

Since the Secondary Moments = 0.

RHS = 0.

Hence, the RHS also can be written as (By Mohr's theorems)

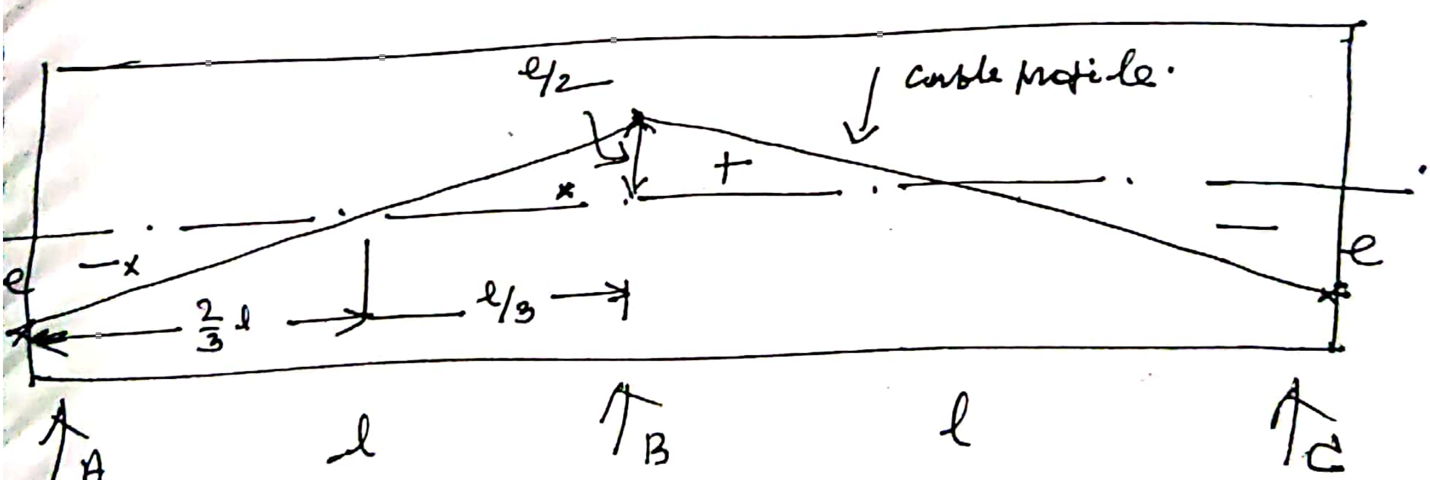
$$\boxed{\frac{A_1 x_1}{(EI)_1} + \frac{A_2 x_2}{(EI)_2} = 0.}$$

If 'EI' is same throughout, Then $\boxed{A_1 x_1 + A_2 x_2 = 0.}$

Ex: A PSC beam ABC (AB = BC) has a uniform cross section throughout. The cable is straight with an effective prestressing force of 'P'. The eccentricity of the cable at the ends is 'e' towards the soffit and it is $\left(\frac{e}{2}\right)$ towards top at the central support. Show that the cable is concordant.

SOLUTION:

EI is const., l is same.



The condition for concordant cable is, $A_1 x_1 + A_2 x_2 = 0$.

$$A_1 x_1 = P \int_0^{ab} e\,x\,dx, \text{ Hence, we get}$$

$$\frac{1}{12} \times \frac{8 l}{9} \qquad \left(\frac{8}{9} l\right)$$

$$A_1 x_1 = P\left[+\frac{1}{2} \times \frac{2l}{3} \times e \times \frac{1}{3} \times \frac{2l}{3} - \frac{1}{2} \times \frac{l}{3} \times \frac{e}{2} \times \left(\frac{2}{3} l + \frac{2}{3} \times \frac{l}{3}\right)\right]$$

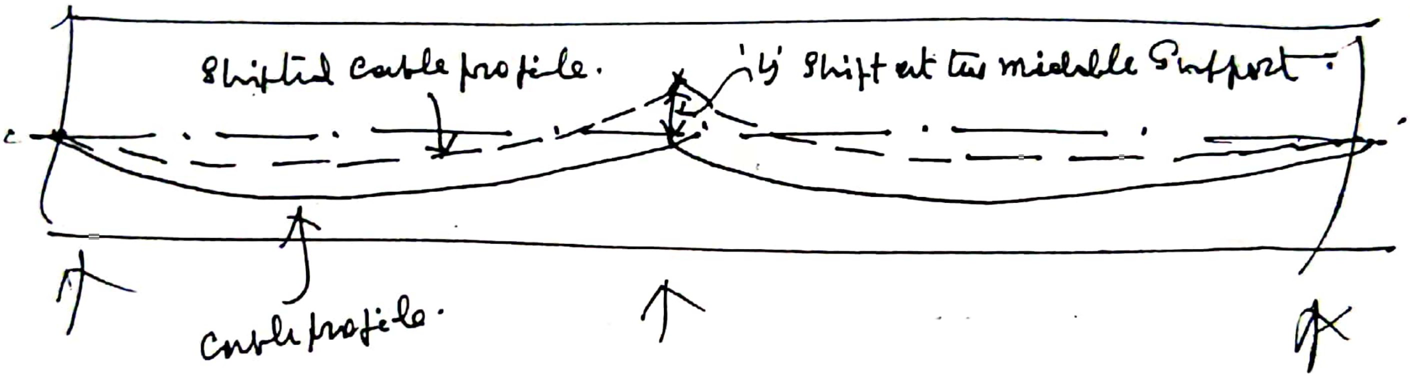
$$= Pe\left[\frac{2l^2}{27} - \frac{2l^2}{27}\right] = 0; \text{ ll}^{ly}\ A_2 x_2 = 0.$$

Hence the profile is concordant.

Guyon's theorem about shifting the tendon profile.

In a continuous PSC beam, if the tendon profile is displaced vertically at any of the intermediate supports, by any amount, but without altering its intrinsic shape between the supports, the resultant line of thrust is unchanged.

Ex.



OTHER METHODS OF ANALYSIS FOR INDETERMINATE STRUCTURES

1. Method of Consistent deformations
2. Flexibility Influence Coefficients
etc.
3. Method of Equivalent Loads.

EXAMPLE:

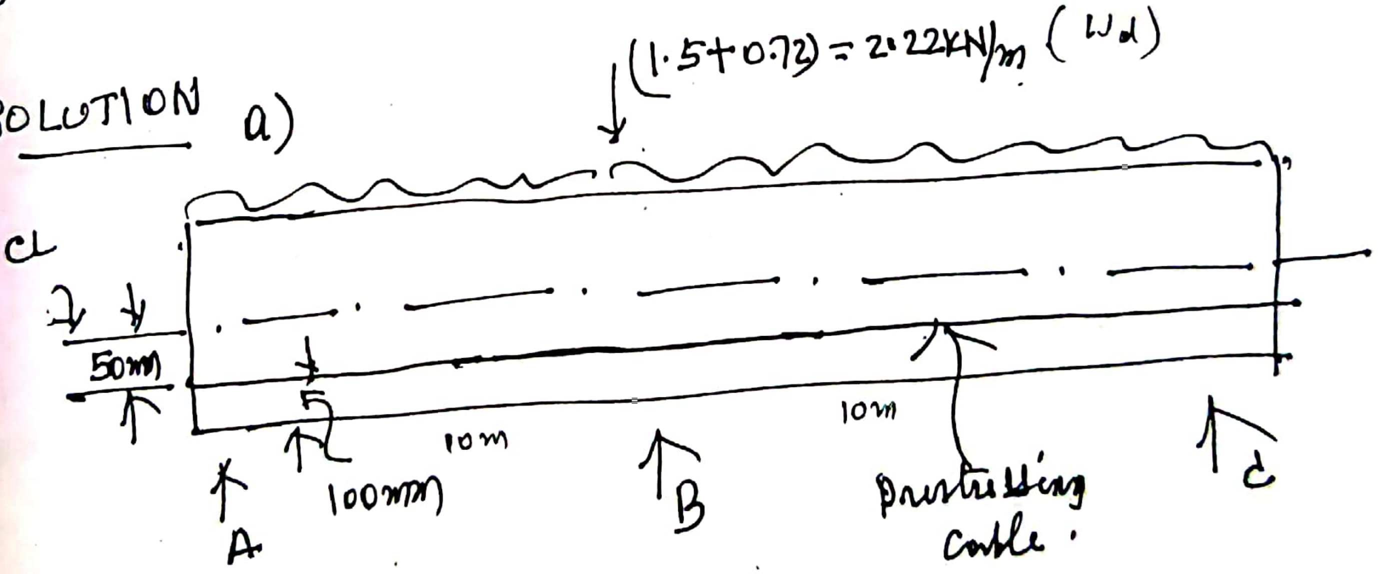
A Continuous PSC beam ABC with AB = BC = 10m has a uniform rectangular Cross Section of 100 × 300mm. The cable carries an effective prestressing force of 360kN. The cable is parallel to the axis of the beam, with an eccentricity of 100mm from the soffit.

a) Determine the secondary and the resultant moments at the central support 'B'.

b) If the beam supports an imposed load of 1.5kN/m, calculate the resultant stresses at the top and bottom of the beam at 'B'. Assume density of concrete as 24 kN/m^3

c) Locate the resultant line of thrust through the beam 'AB'.

SOLUTION a)



Self wt. = $0.1 \times 0.3 \times 24 = 0.72$ kN/m

The theorem of three moments is

$$M_{AB} + 2M_{ba} + 2M_{bc} + \bar{K}M_{cb} = K_{ba} + \bar{K}_{bc}.$$

A and B are Simple Supports, Hence, $M_{AB} = M_{CB} = 0$;

$k = \frac{I_{ab}}{l_{ab}} / \frac{I_{bc}}{l_{bc}} = 1$

$k \cancel{/k'} = 1$, $M_{ba} = M_{bc} = M_b'$, Hence $2M_{ba} + 2M_{bc} = 4M_b'$

$\overline{k}_{ba} = \overline{k}_{bc} = \frac{6P}{-l^2} \int e x\, dx = \frac{6 \times 360}{10^2} \int$ | $e = 50mm = 5cm = 0.05m$

$= -\frac{6P}{l^2}$ [Moment of the area between the Cable profile and the Centroidal axis taken about the Support.]

$= \frac{6 \times 360}{10^2} \left[0.05 \times 10 \times \frac{10}{2} \right] = 54\,kNm$.

Hence, we get $4M_b = \overline{k}_{ba} + \cancel{k}\,k_{bc}' = k_{ba} + k_{bc} = 2 \times 54$.

Hence $4M_b = 2 \times 54 = 108$, $M_b = \frac{108}{4} = 27\,kNm$.

Resultant moment at 'B' $= -360 \times 0.05 + 27 = +9\,kNm$;
(Hogging) (Sagging)

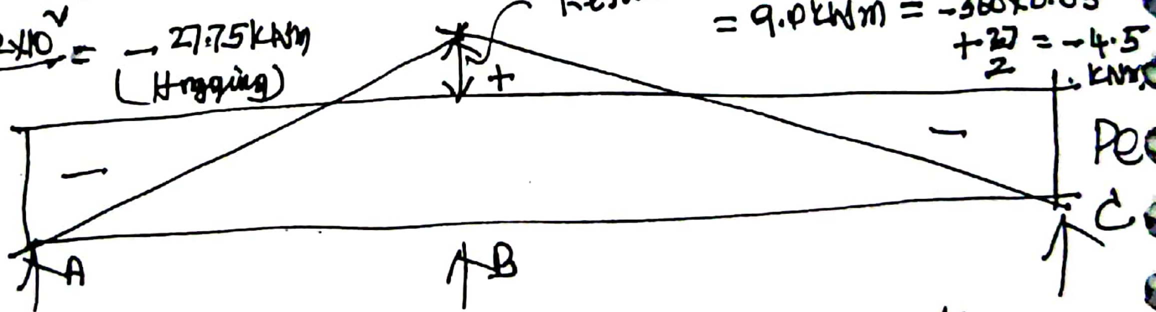
BM at 'B' due to the loads.

$= \frac{w_d l^2}{8} = -\frac{2.22 \times 10^2}{8} = -27.75\,kNm$ (Hogging)

(continuous beam formula)

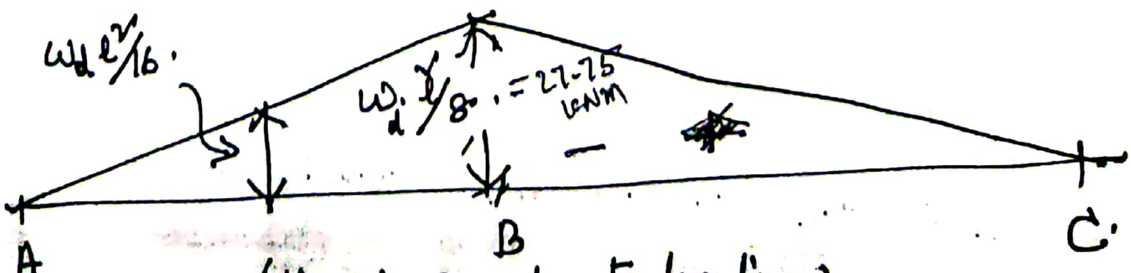
Resultant Moment at centre

Resultant Moment $= 9.0\,kNm = -360 \times 0.05 + \frac{27}{2} = -4.5\,kNm$



(BMD due to only Prestressing force and indeterminacy).

Hence net moment at B $= -27.75 + 9 = -18.75\,kNm$ (Hogging).



(Hogging BMD due to bending)

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Scanned with CamScanner

~~Hinge~~

b) Resultant stresses

$Z =$

$$\text{at top} = \frac{P}{A} - \frac{\text{Resultant Moment}}{Z} = \frac{360\times10^3}{100\times300} - \frac{(18.75\times10^6)\times6}{(100\times300^2)}$$

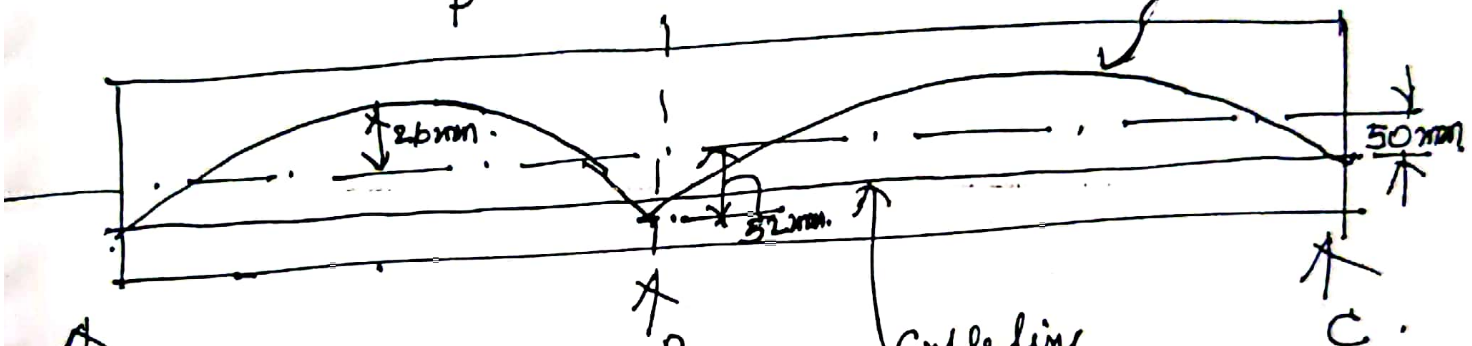
$= -0.5\ N/mm^2$ (at tension)

at bottom $= +24.5\ N/mm^2$ (Compression).

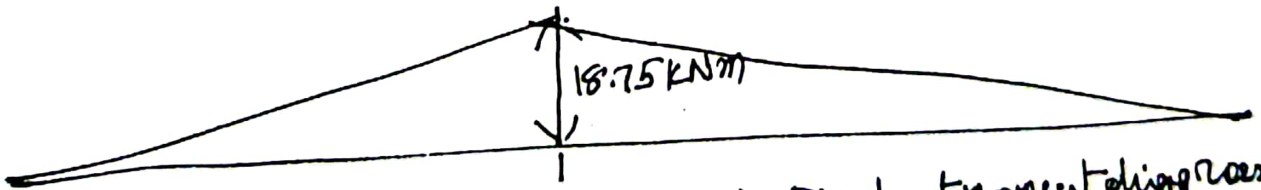
C) To determine the pressure line

At 'A' = 50 mm below the axis = -50 mm.

$$\text{At } B = \frac{\text{Resultant Moment at } B}{P} = \frac{-18.75\times10^3}{360} = -52\ mm.$$



Pressure line diagram



(Final net moment diagram)

Resultant Moment at centre of span

$$= -4.5 + \frac{wl^2}{16} = -4.5 + \frac{2.2\times10^2}{16} = -4.5 + 13.75$$

$$\frac{220}{16} = \frac{110}{8}$$

$= +9.25$ kNm.

~~13.75~~

Shift of the pressure line from the centroidal axis at mid span point $= +\dfrac{9.25\times10^3}{360} = +26$ mm (above).

EXAMPLE : A PSC beam having a rectangular Section 120x300mm is Continuous over AB = BC = 8m. The cable has zero eccentricity at the end supports and 50mm towards the top fibres of the beam over the central support. The eff. prestressing force is 500kN.

a) Calculate the Secondary moment at 'B'

b) If the beam supports concentrated loads of 20kN each at mid points of spans AB and BC, evaluate the resultant stresses at the central support section at 'B'.

c) Locate the position of the pressure line.

SOLUTION

a) $P = 500\,kN$; $Z_t = Z_b = I/\frac{d}{2} = 18 \times 10^5\ mm^3$.

$Q = 20\,kN$, $e = 50\,mm$, $A = 36 \times 10^3\ mm^2$, $L = 8\,m$.

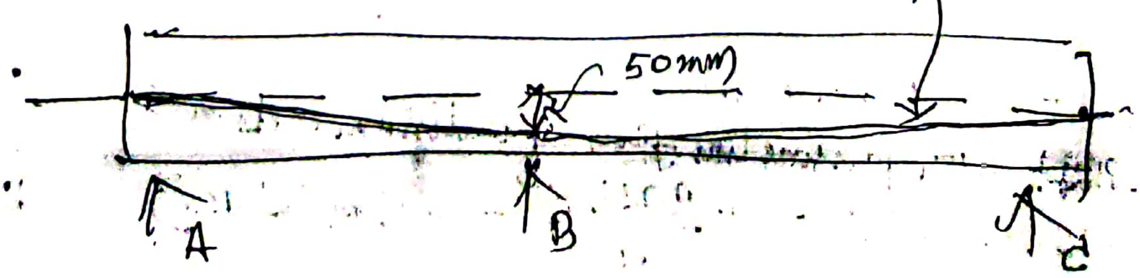
Self wt. 'g' = $0.12 \times 0.3 \times 24 = 0.86\ kN/m$.

In the present case, $M_a = M_c = 0$, $l_1 = l_2 = l = 8\,m$.

I is same, Hence $k = 1$, To calculate the Secondary moments

$$\bar{K}_{ab} = \bar{K}_{bc} = -\frac{6P}{l^2}\int e x\,dx = -\frac{6P}{l^2}\left[\frac{1}{2} l \times e \times \frac{2l}{3}\right] = -2Pe$$

Triangular Cable Profile.



(38)

Hence,

By theorem of three moments,

$$4 M_B = -2Pe - 2Pe = -4Pe.$$

Hence $M_B = +Pe$. (Secondary Moment is +ve.)

Primary moment at 'B' $= -Pe$

Hence, net moment at 'B', $-Pe + Pe = 0.0$

Moment at B, due to Self wt. $= -\frac{gl^2}{8}$.

" due to imposed loads $= -\frac{3Ql^2}{16}$ (Hogging)

Hence, total due to the loads $= \left[-\frac{gl^2}{8} - \frac{3Ql^2}{16}\right]$.

$$= -\frac{0.86 \times 8^2}{8} - \frac{3 \times 20 \times 8}{16} = -36.88 \text{ kNm}.$$

Hence, the final resultant moment at B $= -36.88$ kNm.

Hence, the stresses are,

at top $= +\frac{500 \times 10^3}{36 \times 10^3} - \frac{36.88 \times 10^6}{18 \times 10^5} = -6.6 \text{ N/mm}^2$ (Tension)

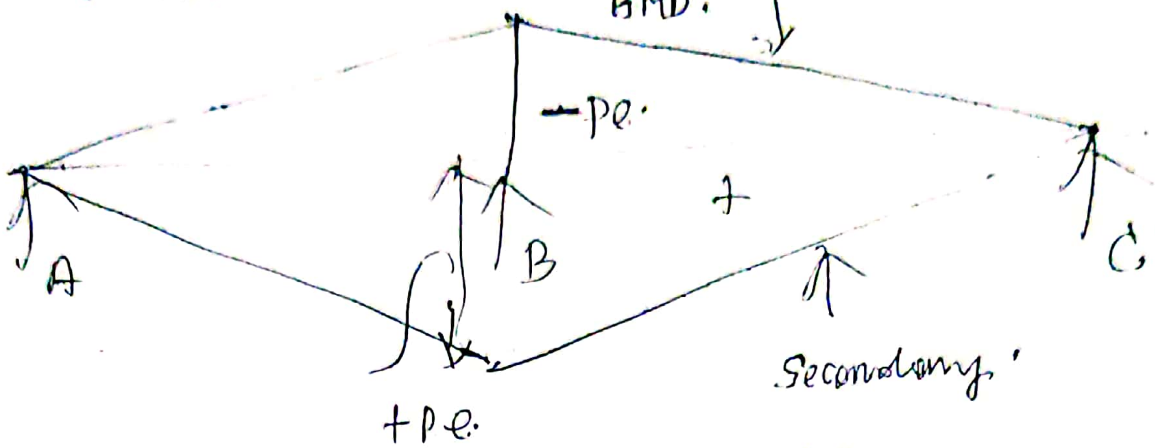
at bottom $= (\quad) + (\quad) = +34.4 \text{ N/mm}^2$ (Compression)

c)

Position of pressure line at centre 'B'

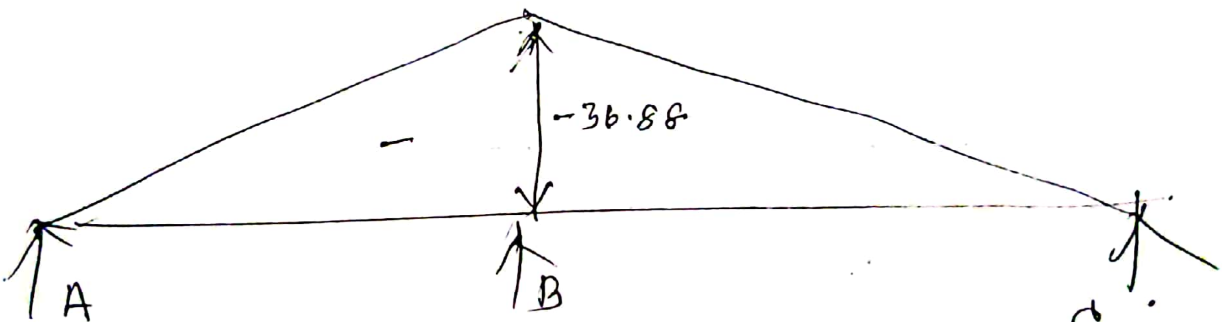
$$= \frac{M_t}{P} = \frac{-36.88 \times 10^3}{500} = -73.5 \text{ mm below the axis}.$$

Fig. a: Primary and Secondary BMD.



Net moment at B = 0.

Fig. b: Resultant BMD.



Resultant BMD due to loads and primary and secondary Moments

At the centre of span, the moment $= +\frac{M_b}{2} - \frac{Pe}{2} = \frac{Pe}{2} - \frac{Pe}{2} = 0$.

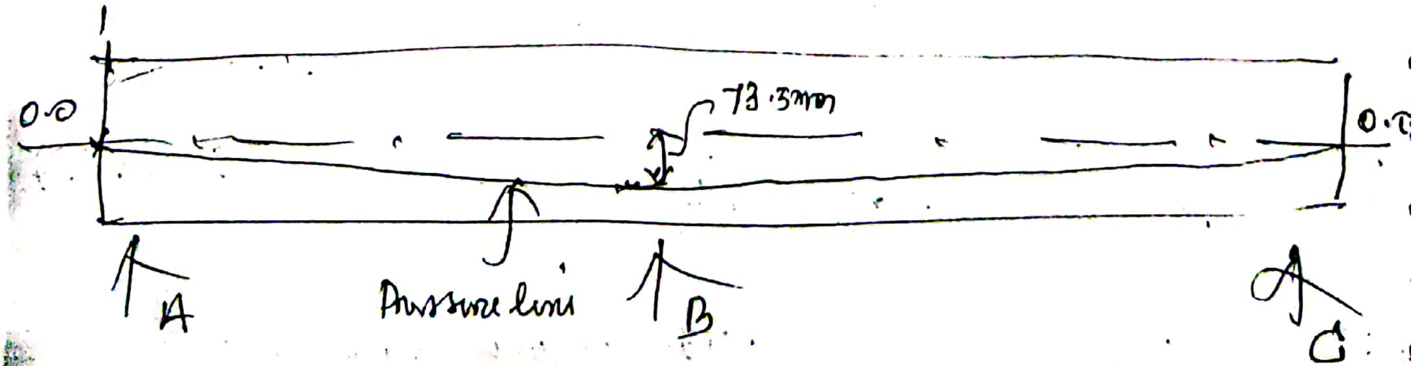


Fig. c: Pressure line Diagram.

CONCORDENT CABLE PROFILE.

"A tendon profile in which the eccentricity is proportional at all sections to the B.M. caused by any loading on a rigidly supported indeterminate structure is a concordant profile."

According to Guyon tendons kept coinciding with the pressure line do not produce secondary moments.

INTRODUCTION AND ADVANTEGES OF CONTINUOUS BEAMS

For PSC continuous members the following are the advantages.

1) The B.M^S are more evenly distributed between the centre and supports
2) There is reduction in the size of the members and hence it is lighter
3.) There is redistribution of moments in the ultimate stages and hence the load carrying capacity is more compared to determinate structures.
4. There is increased stability because of continuity of the members
5. Continuous curved cables can be positioned suitably and effectively.
6. ~~R~~ A reduction in the number of anchorages and stressing operation
7. Deflections are smaller as compared to simply supported members.