Design for Shear

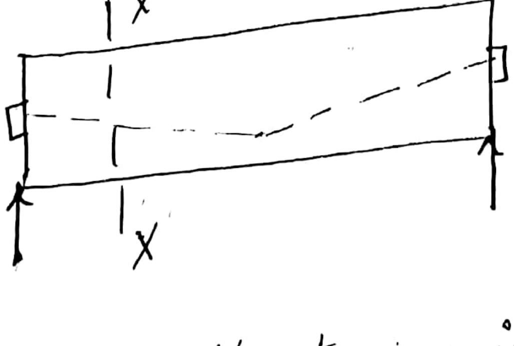
In case of R.C beams the design for Shear was made on the assumption that the failure is liable to be caused by diagonal tension i.e the failure is taken to be due to principal tensile stresses.

But in case of PSC members, due to the introduction of comp. stress by prestressing, the principal tensile stress are reduced. Further if cables are inclined or curved the vertical component of the tensions in the cables also will resist shear.

Consider the beam prestressed by a St tendon

Consider any arbitrary section x-x. The shear force at the section has to be resisted entirely by concrete alone.

Now consider the beam prestressed by an inclined tendon



Consider the section XX.
Let the SF at Sec XX due to dead load and liveload be $V_d + V_l$

Let P be the tension in the tendons.
Let θ be the inclination of the tendon with the horizontal

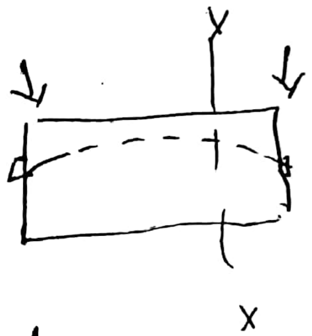
Net actual S.F to be resisted by the Concrete

$= V_c = V_d + V_L - P\sin\theta$

principal tensile stress in a beam section at any point can be determined as follows:

i) Det. the net S.F V_c to be resisted by concrete at the section.

$V_c = V_d + V_L - P\sin\theta$



in some cases

$V_c = V_d + V_L + P\sin\theta$

ii) find the shear stress 'q' at any level using

$$q = \frac{V \cdot A\bar{y}}{I \cdot b}$$

where q = shear stress intensity at le[vel]

$A\bar{y}$ = static moment of c/s area of beam above/below that level about the centroidal axis

I = M.I of the beam about CA

b = width of beam at the leve[l]

iii) Now find the ~~principal~~ normal stress distribution due to prestress, eccentricity and the external B.M (p).

iv) Now find the principal tensile stress (p_2) from

$$p_2 = \frac{p}{A} - \sqrt{\frac{p^2}{4} + q^2}$$

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1) A prestressed Concrete beam of rectangular section is 150mm wide, 375 deep is S.S over a span of 8m. The beam is concentrically prestressed by a cable carrying an eff. prestressing force of 337.50 kN. The beam supports an all inclusive load of 8kN/m. Compare the principal tensile stress induced in the beam with and without the prestress, at the support section.

Sol:

$A = 150 \times 375 = 56250 mm^2$

i) S.F @ support $= S = \frac{WL}{2} = \frac{8 \times 8}{2} = 32 kN$.
(S or V)

ii) Shear stress $q = \frac{3}{2} \cdot \frac{S}{b \cdot d} = \frac{3}{2} \times \frac{32 \times 10^3}{56250} = 0.85 N/mm^2$

iii) find normal stress $= P/A = p$

a) when prestress is not applied → no normal stress $= p = 0$

(iv) ∴ final principal stress $p_2 = \frac{p}{2} \pm \sqrt{\frac{p^2}{4} + q^2}$

$\Rightarrow p_2 = \sqrt{q^2} = \pm 0.85 N/mm^2$ Ans

b) when prestress is applied

$p = \frac{337.50 \times 10^3}{56250} = 6 N/mm^2$

∴ final principal stress $p_2 = \frac{p}{2} \pm \sqrt{\frac{p^2}{4} + q^2}$

$p_2 = \frac{6}{2} \pm \sqrt{\frac{36}{4} + 0.85^2} = 3 \pm 3.12$

$p_2 = 6.12 N/mm^2$ (comp) or $-0.12 N/mm^2$ (tension) Ans

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v) % reduction in principal stress due to axial prestress (prestress force) $= \frac{0.85 - 0.12}{0.85} \times 100$

$= 88.9\%$

#2) A post tensioned concrete beam of rectangular C.S 250mm X 500mm has a span of 12.5m and carries a S.I load of 8.5 kN/m. The tendons is provided with a parabolic profile with a central dip of 180mm and with no eccentricity at ends. The eff. prestressing force in tendon is 750 kN. Determine

i) The principal stresses at the supports.

ii) The principal stresses at supports without prestress.

Take wt of concrete = 24 kN/m3

Sol $b = 250$; $d = 500$; $l = 12.5$; $w = 8.5$ kN/m

$h = 0.18m$; $P = 750kN$

Slope of cable @ each end $\theta = \frac{4h}{L} = \frac{4 \times 0.18}{12.5} = 0.0576$ rad

$= 3.3°$

Vertical Component of P = $P\sin\theta = 750 \sin 3.3° = 43.20$ kN

Horizontal Component of P = $P\cos\theta = 750\cos 3.3 = 748.756$ kN

Dead load of beam of beam = $0.25 \times 0.5 \times 24 = 3$ kN/m

S.I load = 8.5 kN/m

Total load on the beam = $8.5 + 3 = 11.50$ kN/m

step i $V_c = V_d + V_L - P\sin\theta$

Net S.F @ support = $\frac{11.50 \times 12.5}{2} - 43.20 = 28.675$ kN

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(ii) Max Shear Stress $q = \frac{3}{2} \frac{V_c}{bd}$

$$q = \frac{3}{2} \times \frac{28.675 \times 1000}{250 \times 500} = 0.344 \, N/mm^2$$

(iii) Normal Stress due to prestressing force P (Direct)

$$p = \frac{P}{A} = \frac{P\cos\theta}{A} = \frac{748.756 \times 1000}{250 \times 500} = 5.99 \, N/mm^2$$

(iv)

∴ Principal Stresses $p_{1,2} = \frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 + q^2}$

(a) when P is applied

$$p_2 = \frac{5.99}{2} \pm \sqrt{\frac{5.99^2}{4} + 0.344^2}$$

$= 2.995 \pm 3.015$

$= 6.01 \, N/mm^2$ (comp)

$= -0.02 \, N/mm^2$ (Tensile)

(b) when P is not applied i.e P = 0

i.e $p_{1} = q$ SF at support $= \frac{wL}{2} = \frac{11.50 \times 12.5}{2}$

$p_2 =$ $= 71.875 \, kN$

Max Shear Stress $q = \frac{3}{2} \times \frac{71875 \times 1000}{250 \times 500} = 0.8625 \, N/mm^2$

∴ $p = \pm q = \pm 0.8625 \, N/mm$ Ans

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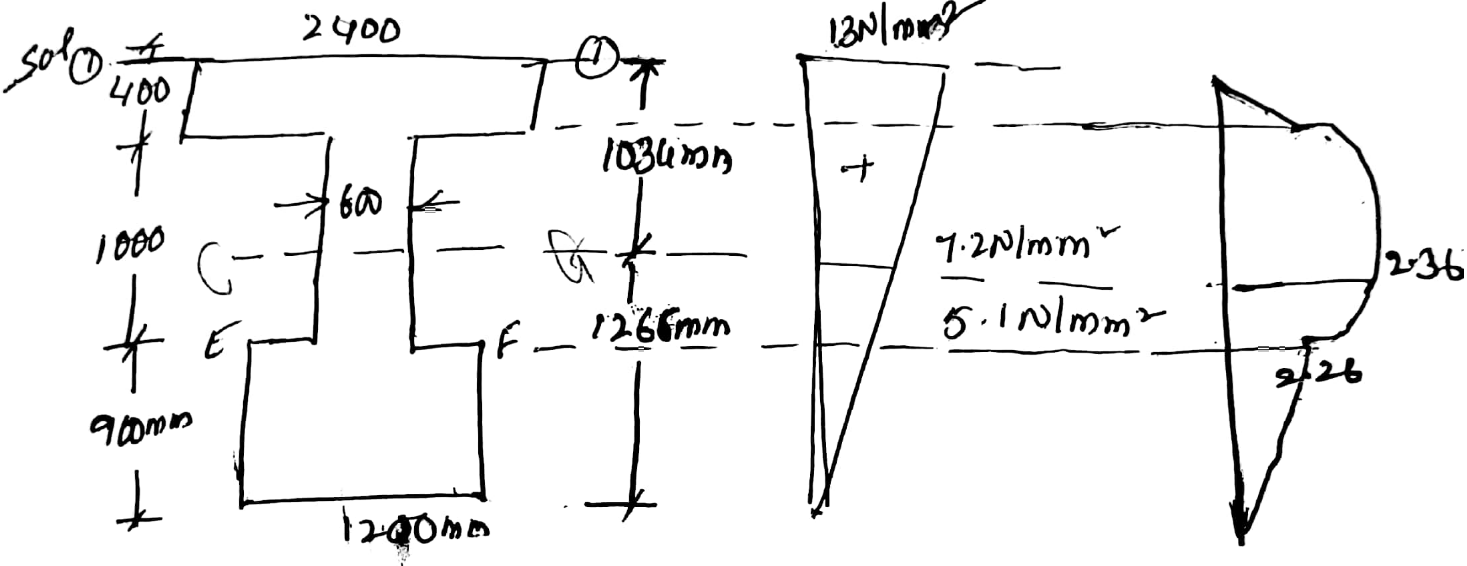
③ I-beam

find stresses in the extreme fibres of I-section

A PSC beam section when subjected to a certain BM has a fibre stress distribution as shown in fig. The total vertical shear in concrete at the section is 2360 kN. find the principal tensile stress at the junction of the web with lower flange and at the level of centroidal axis.

Sol:



Note *

First determine the centroidal axis & MI of beam about C.

Component	Area mm^2	Centroidal dist. y from 1-1	ay mm^3	ay^2 mm^4	Iself
flange	9.6×10^5	200	1.92×10^8	3.84×10^{10}	$\frac{24\times4^3}{12}\times10^{10} = 1.28\times10^{10}$
web	6×10^5	900	5.40×10^8	48.6×10^{10}	$\frac{600\times1000^3}{12} = 5\times10^{10}$
bottom flange	10.8×10^5	1850	19.98×10^8	369.63×10^{10}	$\frac{1200\times900^3}{12} = 7.29\times10^{10}$
Total	26.4×10^5		27.3×10^8	422.07×10^{10}	13.57×10^{10}

Distance of the Centroidal axis from the top edge $\bar{y} = \frac{\Sigma ay}{A} = \frac{27.3 \times 10^8}{26.4 \times 10^5} = 1034 mm$

M.I about ①-①

$I_{1-1} = \Sigma I_{self} + \Sigma ay^2$

$= (13.57 \times 10^{10}) + (422.07 \times 10^{10})$

$I_{1-1} = 435.64 \times 10^4$

But $I_{aa} = I_{CG} + A\bar{y}^2$

$I_{CG} = I_{11} - A\bar{y}^2$

$= (435.64 \times 10^4) - (26.4 \times 10^5 \times 1034^2)$

$= 153.383 \times 10^{10} mm^4$

Shear stress in the web at the Centroidal axis

$a\bar{y} = (1200 \times 900 \times 816) + (600 \times 366 \times 183)$

$= 9.214668 \times 10^8 mm^3$

$$\begin{array}{r} 1266 \\ -450 \\ \hline 816 \end{array}$$

Shear stress $q = \frac{V a\bar{y}}{I \cdot b} = \frac{2360 \times 10^3 \times 9.214668 \times 10^8}{153.383 \times 10^{10} \times 600} = 2.36 N/mm^2$

Shear stress in the web at the level EF

$a\bar{y} = 1200 \times 900 \times 816 = 8.8128 \times 10^8 mm^3$

Shear stress $q = \frac{S a\bar{y}}{I \cdot b} = \frac{2360 \times 10^3 \times 8.8128 \times 10^8}{153.388 \times 10^{10} \times 600} = 2.26 N/mm^2$

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$\therefore$ principal tensile stress in the web at N.A (G)

$$p_2 = \frac{p}{2} - \sqrt{\frac{p^2}{4} + q^2}$$

$$= \frac{7.2}{2} - \sqrt{\frac{7.2^2}{4} + 2.36^2} = -0.705\ N/mm^2 \quad \text{Ans.}$$

$\Rightarrow$ Principal tensile stress in the web at level EF

$$p_2' = \frac{5.1}{2} - \sqrt{\frac{5.1^2}{4} + 2.36^2}$$

$$= -0.857\ N/mm^2 \quad \text{Ans.}$$

ble bela

$\frac{p}{2} - \frac{p^2}{9}$ N

VERTICAL PRESTRESSING

Besides the longitudinal prestressing, sometimes it may be desirable to provide vertical prestressing to reduce or eliminate principal tensile stresses.

Vertical prestressing is done by providing high tension vertical steel wires of small dia at suitable pitch and stressed adequately.

If at a point in the section of the beam

p = longitudinal comp stress

p' = vertical comp stress

q = shear stress

Then principal stresses will be

$$\text{Major Principal stress} = \frac{p+p'}{2} + \sqrt{\frac{(p-p')^2}{4} + q^2}$$

$$\text{Minor principal stress} = \frac{p-p'}{2} - \sqrt{\left(\frac{p-p'}{2}\right)^2 + q^2}$$

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The horizontal stress at the centroid of a PSC. of C.S 125mm x 250mm is 7N/mm² and the max SF on the beam section is 68kN. Find the principal tensile stress. Also find the min. vertical prestress reqd to eliminate this principal tensile stress.

Sol $A = 125 \times 250 = 31250\, mm^2$; $p = 7N/mm^2$

$q = \frac{3}{2} \times \frac{V}{bd} = \frac{3}{2} \times \frac{68 \times 1000}{31250}$; $p' = ?$

$q = 3.264 N/mm^2$

principal tensile stress $p_2 = \frac{p}{2} - \sqrt{\frac{p^2}{4} + q^2}$

$= \frac{7}{2} - \sqrt{\frac{49}{4} + 3.264^2}$

$= -1.285 N/mm^2$

Let p' be min vertical prestress reqd to eliminate the principal tensile stress.

$\therefore \frac{p+p'}{2} - \sqrt{\left(\frac{p-p'}{2}\right)^2 + q^2} = 0$

$\left(\frac{7+p'}{2}\right) - \sqrt{\frac{(7-p')^2}{4} + 3.26^2} = 0$

$\Rightarrow \left(\frac{7-p'}{2}\right) + 3.264^2 = \left(\frac{7+p'}{2}\right)^2$

on solving $p' = 1.522 N/mm^2$

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DESIGN OF SHEAR REINFORCEMENT

The effect of shear is to induce tensile stresses on diagonal planes and prestressing is beneficial since it reduces the magnitude of the principal tensile stress in concrete. Curved cables are advantageous in reducing the effective shear and together with the horizontal compressive prestress, reduce the magnitude of the principal tension.

IS 1343-1980 Recommendations:

~~At any~~

SHEAR & PRINCIPAL STRESSES

The shear stress at a point is expressed as

$$\tau_v = \left[\frac{VS}{Ib}\right]$$

where τ_v = shear stress due to transverse load

V = Shear force.

S = statical moment (first moment of area $A\bar{y}$)

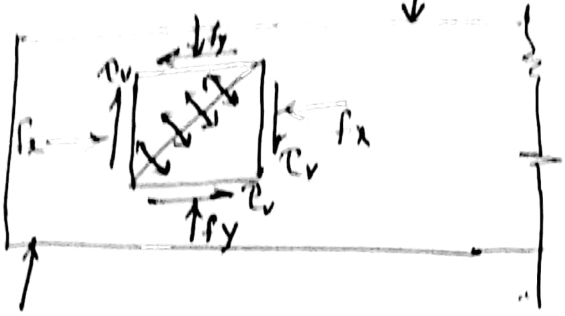
I = second moment of area of section about its centroid (M.I)

b = breadth of section at the given point

The effect of this shear stress is to induce principal tensile stresses on diagonal planes.

In a PSC member, the shear stress is generally accompanied by a direct stress in the axial direction of the member and if vertical prestressing is adopted comp. stresses in the direction ~~perpendicular~~ to the axis of the member will be present in addition to the axial prestress.

The most general case of an element subjected to a 2 Dimensional stress is as shown in fig below



principal tensile stresses in a PS member.

* The max. & min principal stresses developed are given by

$$f_{\max,\min} = \left[\left(\frac{f_x + f_y}{2}\right) \pm \frac{1}{2}\sqrt{(f_x - f_y)^2 + 4\tau_v^2} \right]$$

Where f_x & f_y are the direct stresses and τ_v is the shear stress acting at the point.

In PSC members, the direct stress f_x & f_y being compressive, the magnitude of principal tensile stress is considerably reduced and in some cases even eliminated, so that under working loads both major & minor principal stresses are compressive thereby eliminating the risk of diagonal tension cracks in concrete.

In general, there are 3 ways of improving the shear resistance of structural concrete members by prestressing techniques

1. Horizontal or axial prestressing
2. Prestressing by inclined or sloping cables
3. Vertical or transverse prestressing

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#1 A PSC beam of span 10m of rectangular section 120mm wide and 300mm deep is axially prestressed by a cable carrying an effective force of 180kN. The beam supports a total udl of 5kN/m which includes the self wt of the member. Compare the magnitude of the principal tension developed in the beam with and without the axial prestress

Sol $A = (120 \times 300) = 36 \times 10^3 mm^2$; $I = \frac{120 \times 300^3}{12} = 27 \times 10^7 mm^4$

$w = 5kN/m$

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SF at support, $V = \frac{wL}{2} = \left(\frac{5 \times 10}{2}\right) = 25kN$

Max shear stress at support, $\tau_v = \left(\frac{3V}{2bh}\right) = \left(\frac{3}{2} \times \frac{25 \times 10^3}{120 \times 300}\right)$

$= 1.05 N/mm^2$

principal stress $= \pm \frac{1}{2}\sqrt{4\tau_v^2} = \pm \tau_v = \pm 1.05 N/mm^2$ (comp & Ten)

Axial prestress, $f_x = \frac{P}{A} = \left(\frac{180 \times 10^3}{36 \times 10^3}\right) = 5 N/mm^2$

Max. & Min. principal stress (with axial prestress) $= \frac{f_x}{2} \pm \sqrt{f_x^2 + 4\tau_v^2}$

$= \frac{5}{2} \pm \sqrt{\left(\frac{5}{2}\right)^2 + 4(1.05)^2}$

$= 5.23 N/mm^2$ comp &

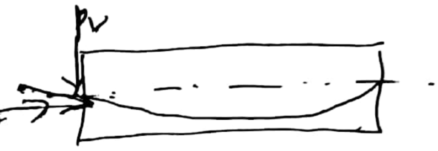
$-0.23 N/mm^2$ Ten.

Hence with axial prestress, the principal tension is reduced by $\left(\frac{1.05 - 0.23}{1.05}\right) \times 100 = 78\%$

#2 A post tensioned beam of Rectangular c/s 200mm wide and 400mm deep is 10m long and carries an applied load of 8kN/m, uniformly distributed on the beam. The effective prestressing force in the cable is 500kN. The cable is parabolic with zero eccentricity at the supports and a max. eccentricity of 140mm at the centre of span.

(a) calculate the principal stresses at the supports

(b) what will be the magnitude of the principal stresses at the supports in the absence of Prestress?



Sol $DL, g = 0.2 \times 0.4 \times 24 = 1.92\, kN/m$

$LL\ q = 8\, kN/m$; $P = 500\, kN$; $e = 140\, mm$; $L = 10\, m$

Slope of Cable at Support, $\theta = \left(\frac{4e}{L}\right) = \frac{4 \times 140}{10000} = 0.056\, rad$

vertical Component of P = $P \sin\theta = (500 \times 0.056) = 28\, kN$

Horizontal Component = $P \cos\theta = (500 \times 0.998) \simeq 500\, kN$

Max. Shear at support due to loads $= \frac{wl}{2} = \frac{(1.92 + 8) \times 10}{2}$

$= 49.60\, kN$

Net Shear at support = Max Shear $- P_v$

$= 49.60 - 28 = 21.6\, kN$

Shear stress $= \tau_v = \left(\frac{3}{2} \cdot \frac{V}{bh}\right) = \frac{3}{2}\left(\frac{21.6 \times 10^3}{200 \times 400}\right) = 0.405\, N/mm^2$

Direct Stress $= \frac{P}{A} = \left(\frac{500 \times 10^3}{200 \times 400}\right) = 6.25\, N/mm^2$

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(a) Max & Min. principal stresses

$$= \left[\frac{6.25}{2} \pm \frac{1}{2}\sqrt{6.25^2 + 4(0.405)^2}\right] = 6.275\ N/mm^2\ (COMP)$$
$$= -0.025\ N/mm^2$$

(b) In the absence of prestress, vertical shear = 49.60 kN

∴ Max & min. principal stress = Max. shear stress

$$= \left(\frac{3}{2} \times \frac{49.6 \times 10^3}{200 \times 400}\right) = 0.93\ N/mm^2$$

#3. A PS I-section has the following properties:

Area = $55 \times 10^3\ mm^2$; $I = 189 \times 10^7\ mm^4$; ~~Ay~~

~~secti~~ Statical Moment about the Centroid = $468 \times 10^4\ mm^3 = A\bar{y}$.

Thickness of web = 50 mm

It is prestressed by horizontally by 24 wires of 5 mm ∅ and vertically by similar wires at 150 mm centres. All the wires carry a tensile stress of 900 N/mm². Calculate the principal stresses at the centroid when a shearing force of 80 kN acts upon the section.

Sol Shear stress $= \frac{VA\bar{y}}{I \cdot b} = \left(\frac{80 \times 10^3}{189 \times 10^7 \times 50}\right) 468 \times 10^4 = 3.95\ N/mm^2$

Hor. prestress at centroid $= \left(\frac{24 \times 19.7 \times 900}{55 \times 10^3}\right) = 7.75\ N/mm^2$

Vertical prestress $= \left(\frac{19.7 \times 900}{150 \times 50}\right) = 2.46\ N/mm^2$

Hence $f_x = 7.75\ N/mm^2$; $f_y = 2.46\ N/mm^2$; $\tau_v = 3.95\ N/mm^2$

Max. & Min. principal stresses $= \left(\frac{f_x + f_y}{2}\right) \pm \frac{1}{2}\sqrt{(f_x - f_y)^2 + 4\tau_v^2}$

$$= \left(\frac{7.75 + 2.46}{2}\right) \pm \frac{1}{2}\sqrt{(7.75 - 2.46)^2 + 4 \times 3.95^2} = 9.8\ N/mm^2$$
$$= 0.4\ N/mm^2$$

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Design of Shear Reinforcement

INDIAN CODE (IS 1343-1980) Recommendation

At any section, the ultimate shear resistance V_c of the Concrete alone should be taken as the lesser of the values of V_{co} and V_{cf} (flexural shear reinforcement). When 'V' along the SF due the ultimate loads is less than V_c, the SF which can be carried by concrete, minimum shear reinforcement should be provided in the form of stirrups such that

$$S_v = \left[\frac{A_{sv} \times 0.87 f_y}{0.4 b}\right]$$

where S_v = spacing of stirrups along the length of the member

A_{sv} = Total c/s area of stirrups legs effective in shear

b = breadth of member = b_w for T, I & L beams

f_y = Characteristic strength of stirrups reinforcement $\ngtr 415 \, N/mm^2$

If the SF $V < 0.5 V_c$, shear reinforcement need not be provided.

When $V > V_c$, shear reinforcement is required confirming to the relation

$$S_v = \left[\frac{A_{sv}\, 0.87 f_y \cdot d_t}{V - V_c}\right]$$

Where d_t = depth from the Extreme compression fibre either to the longitudinal bars or to the centroid of tendons which ever is greater.

Max. Shear Stress (IS 1343 1980)

grade	M30	M35	M40	M45	M50	M55 & above
max shear stress N/mm^2	3.5	3.7	4.0	4.3	4.6	4.8

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Spacing of stirrups should not exceed neither $0.75 d_t$ or 4 times the web thickness for flanged members.

When V exceeds $1.8 V_c$ the max spacing should be reduced to $0.5 d_t$. The lateral spacing of the individual legs of the stirrups provided at a c/s should not exceed $0.75 d_t$.

If nominal shear stress $\left(\frac{V}{bd}\right)$ exceeds these (max per shear stress) values, the section has to be ~~redesigned~~ Redesigned.

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The support section of a ps concrete beam 100mm wide and 250mm deep, is required to support an ultimate SF of 60kN. The compressive prestress at the centroidal axis is $5 N/mm^2$. The characteristic strength of concrete is $40 N/mm^2$. The cover to the tension reinforcement is 50mm. If the characteristic tensile strength of steel in stirrups is $250 N/mm^2$, design the sections using suitable reinforcements at the sections using the IS code 1343 recommendations.

sol $b_w = 100mm$; $f_{cp} = 5 N/mm^2$; $h = 250mm$; $\underline{\underline{d = 200mm}}$

$V = 60kN$; $f_y = 250 N/mm^2$

For the support section uncracked in flexure,

$V_c = V_{cw} =$ ultimate shear resistance of concrete in a section due to web shear cracks

$$= 0.67 b_w h \sqrt{f_t^2 + 0.8 f_{cp} f_t}$$

$$f_t = 0.24 \sqrt{f_{ck}} = 0.24\sqrt{40} = 1.517 N/mm^2$$

$$V_c = 0.67 \times 100 \times 250 \sqrt{1.517^2 + (0.8 \times 5 \times 1.517)}$$

$$= 48457 N$$

$$V_c = 48.4 kN$$

$\therefore$ Balance Shear $= (V - V_c)$

$$= (60 - 48.4)$$

$$= 11.6 kN$$

using 6mm dia. two legged stirrups, the spacing is obtained as

$$S_v = \left[\frac{A_{sv} \cdot 0.87 f_y d}{(V - V_c)} \right] = \left[\frac{2 \times \overset{\pi d^2/4}{28.2} \times 0.87 \times 250 \times 200}{11600} \right]$$

$$= 211.5 mm.$$

Max. per. spacing $= 0.75 d = (0.75 \times 200) = 150 mm$

$\therefore$ Adopt 6mm ϕ 2-legged stirrups @ 150mm c/c

PRESTRESSED CONCRETE MEMBERS IN TORSION

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DESIGN OF REINFORCEMENT FOR TORSION, SHEAR & BENDING

A post tensioned bonded prestressed concrete beam of rectangular cross section, 400mm wide by 550mm deep, is subjected to a service load BM of 166.6kN-m, torsional moment of 46.6 kNm and SF of 66.6kN. The section has an eff. PS force, determined from service load requirements of magnitude 500kN at an eccentricity of 150mm, provided by 5 nos of 12.5mm stress relieved strand of CS area 506 mm^2 with an ultimate tensile strength of 1820 N/mm^2. If the cube strength of concrete is 40 N/mm^2. Design suitable longitudinal and transverse reinforcement using IS 1343-1980 recommendations.

Given data:

Ultimate BM = M = $1.5 \times 166.6 = 250$ kN-m

" TM = T = $1.5 \times 46.6 = 70$ kN-m

" SF V = $1.5 \times 66.6 = 100$ kN.

Area of ~~PSR~~ prestressing strands $A_p = 506$ mm^2

Ultimate tensile strength of strand $= f_p = 1820$ N/mm^2

Cube strength of concrete $f_{ck} = 40$ N/mm^2

Yield strength of transverse reinforcement $f_y = 250$ N/mm^2

PS force P = 500 kN

Eccentricity e = 150 mm

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$b = 400mm;\ D = 550\,mm$

$A_c = 22 \times 10^4\ mm^2$

$I = \frac{400 \times 550^3}{12} = 5.54 \times 10^9\ mm^4$

Eff stress in steel $= \frac{P/A_s}{} \quad \frac{500 \times 10^3}{506} = 988\,N/mm^2$

Eff depth $d = 425\,mm$

I) DESIGN OF LONGITUDINAL REINFORCEMENT (IS1343)

$M_{e1} = M + M_T$

M_{e1} = Equivalent ult. BM
M = applied BM
$M_T = T\sqrt{1 + (2D/b)}$

$M_T = 70\sqrt{1 + \left(\frac{2 \times 550}{400}\right)}$

$= 136\,kNm$

$M_{e1} = 250 + 136 = 386\ kNm$

The longitudinal steel in the section is designed for the ult. BM of 386 kNm.

The ultimate moment capacity of the section (M_U) with steel is first determined and for balance moment mild steel inforcement will be designed.

for the prestressed section, we have

$f_p = 1820\,N/mm^2$; $b = 400\,MM$
$A_p = 506\,mm^2$; $d = 425\,mm$
$f_{ck} = 40\,N/mm^2$

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Ratio $\left[\frac{f_p \cdot A_p}{f_{ck}\, b\, d}\right] = \left(\frac{1820 \times 506}{40 \times 400 \times 425}\right) = 0.135$

Ref. IS1343 (A11)

for $0.135 \Rightarrow \frac{f_{pu}}{0.87 f_p} = 1.0$

f_{pu} = stress in tendons at the failure stage of the beam

$\Rightarrow f_{pu} = 0.87 \times 1820$

$= 1583\ N/mm^2$

& $\frac{x_u}{d} = 0.2787 \Rightarrow x_u = 0.2787 \times 425$

$= 122 mm$

$M_u = f_{pu} A_p (d - 0.42 x_u)$

$= 1583 \times 506 (425 - 0.42 \times 122) = 299\ kN\,m.$

But the total moment to be resisted by the section is 386 kN m

$\therefore$ Balance moment $= (386 - 299) = 87\ kN\,m$

providing ms reinforcement at an Eff depth of 500mm, we get

$M_u = 0.87 f_y A_{st} d \left[1 - \left(\frac{A_{st} f_y}{b d f_{ck}}\right)\right]$

$87 \times 10^6 = 0.87 \times 250 \times A_{st} \times 500 \left[1 - \left(\frac{A_{st} \times 250}{400 \times 500 \times 40}\right)\right]$

on solving $A_{st} = 840\ mm^2$

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Provide 2 bars of 25mm dia. each at an Eff depth of 500mm ($A_{st} = 982\ mm^2$)

ii) DESIGN OF TRANSVERSE REINFORCEMENT

Avg intensity of prestress at the Centroid

$$f_{cp} = \frac{P}{A} = \left[\frac{500\times10^3}{22\times10^4}\right] = 2.27\ N/mm^2$$

Tensile strength of Concrete $f_t = 0.24\sqrt{f_{ck}} = 0.24\sqrt{40}$
$= 1.51\ N/mm^2$

The reduced torsional moment T_{c1} Carried by the Concrete is given by $T_{c_1} = T_c\left[\frac{e}{e+e_c}\right]$

Where $T_c = 0.15 b^2 D\left[1 - \left(\frac{b}{3D}\right)\right]\lambda_p\sqrt{f_{ck}}$

$$\lambda = \sqrt{\left(1+\frac{12 f_{cp}}{f_{ck}}\right)} = \sqrt{1+\left(\frac{12\times2.27}{40}\right)} = 1.296$$

$$T_c = 0.15\times400^2\times550\left[1-\left[\frac{400}{3\times550}\right]\right]1.296\sqrt{40}$$

$= 82\ kN\text{-}m.$

$$e = \frac{T}{V} = \frac{70}{100} = 0.7m = 70mm$$

$e_c = \left[\frac{T_c}{V_c}\right]$ where V_c is smaller of V_{cw} & V_{cf}

$$V_{cw} = 0.67 bD\sqrt{f_t^2 + 0.8 f_{cp}\cdot f_t}$$

$$= 0.67\times400\times550\sqrt{1.51^2 + (0.8\times2.27\times1.5)} = 330kN$$

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$$V_{cf} = \left[1 - 0.55\frac{f_{pe}}{f_p}\right] \xi_c\, b_w \cdot d + \left(\frac{M_0}{M}\right) V \nless 0.1\, b_w d \sqrt{f_{ck}}$$

$$A_p = 506 + A_{us}\left(\frac{f_y}{f_p}\right) = 506 + 942\left(\frac{250}{1820}\right)$$

$$= 635\ mm^2$$

$$\left[\frac{100 A_p}{bd}\right] = \left(\frac{100 \times 635}{400 \times 425}\right) = 0.373$$

from table 8.1 for M40 grade concrete, the design shear strength

$\xi_c = 0.44 N/mm^2$

$$M_0 = 0.8 f_{pt}\left(\frac{I}{y}\right)$$

$$f_{pt} = \left[\left(\frac{P}{A}\right) + \left(\frac{Pey}{I}\right)\right] = \left[\left(\frac{500 \times 10^3}{400 \times 550}\right) + \left(\frac{500 \times 10^3 \times 150 \times 275}{5.54 \times 10^9}\right)\right]$$

$$= 6 N/mm^2$$

$$M_0 = 0.8 \times 6\left(\frac{5.54 \times 10^9}{\underset{d/2}{275}}\right) = 96.6 \times 10^6\ N\text{-}mm$$

$$V_{cf} = \left[\left(1 - 0.55\left(\frac{988}{1820}\right)\right)\right](0.44 \times 400 \times 425) + \left[\left(\frac{96.6 \times 10^6}{250 \times 10^6}\right)(100 \times 10^3)\right]$$

$= 91.1 kN$

Smaller value of V_{cw} + V_{cf} is $91.1 kN = V_c$

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$$\therefore e_c = \left(\frac{T_c}{V_c}\right) = \frac{82}{91.1} = 0.90m = 900mm$$

$$T_{c1} = T_c\left(\frac{e}{e+e_c}\right) = 82\left(\frac{150}{150+900}\right) = 11.7\,kN\,m$$

$$V_{c1} = V_c\left(\frac{e}{e+e_c}\right) = 91.15\left(\frac{150}{150+900}\right) = 13.02\,kN\,m$$

$$(V-V_{c1}) = (100-13.02) = 87\,kN$$

$$(T-T_{c1}) = (70-11.7) = 58.3\,kN\,m$$

The C.S A_{sv} of closed stirrups enclosing the longitudinal bars is taken as the larger of the following two values

$$A_{sv} = \left[\frac{M_t S_v}{1.5\, b_1 d_1 f_y}\right] \;\&\; A_{sv} = (A_v + 2A_T)$$

Where $A_v = \dfrac{(V-V_1)S_v}{0.87 f_y d_1}$ & $A_T = \dfrac{(T-T_{c1})S_v}{0.87 b_1 f_y d_1}$

$\Rightarrow$ $b_1 = 350mm$; $d_1 = 450mm$

using $S_v = 80mm$; $f_y = 250\,N/mm^2$

$$A_{sv} = \left[\frac{136 \times 10^6 \times 80}{1.5 \times 350 \times 450 \times 250}\right] = 184\,mm^4$$

$$A_v = \left[\frac{87 \times 10^3 \times 80}{0.87 \times 250 \times 450}\right] = 71\,mm^2$$

$$A_T = \left(\frac{58.3 \times 10^6 \times 80}{0.87 \times 350 \times 450 \times 250}\right) = 136\,mm^2$$

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$A_{sv} = [A_v + 2A_T]$

$= [71 + 2 \times 136] = 343 mm^2$

The large value of $A_{sv} = 343 mm^2$

Provide 12mm ϕ 2 legged stirrups @ spacing of 50mm

Which provide $A_{sv} = 343 mm^2$

<u>Check for spacing</u> $x_1 = 382mm$; $y_1 = 482mm$

$\left(\frac{x_1 + y_1}{4}\right) = \left(\frac{382 + 482}{4}\right) = 216mm > 50mm$

<u>Safe</u>

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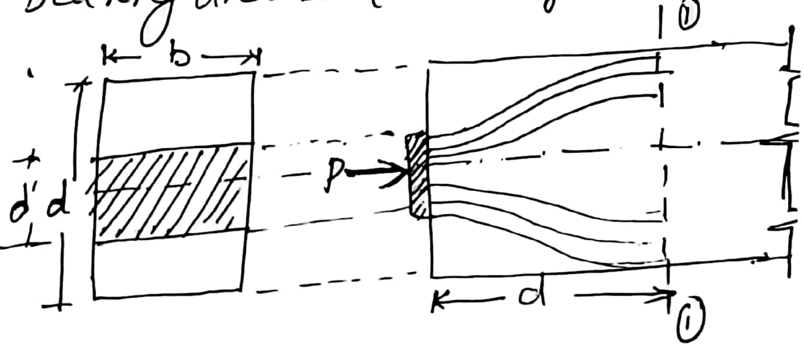
UNIT-VI

END BLOCKS

End block means the portion of the PSC beam surrounding the anchorages of the tendons. Large concentrated forces are transmitted on the bearing surfaces at the ends of the beam by the anchorages. The stresses spread out into the concrete setting up complicated stress patterns. The stress distribution close to the anchorage is different from the stress distribution at sections away from anchorage.

Consider a beam of c-s "b×d". Let the prestressing force P be applied on a bearing area bd' (i.e. anchorage area) on the end face of the beam.



At sections close to the end face, the stress on the section will be greater than $\frac{P}{bd}$. There exists a section 1-1 beyond which the stress is $\frac{P}{bd}$. The part of the beam from the end face to section 1-1 is called the END BLOCK. The length of end block is usually taken equal to 'd'. Stress distribution in the end block is convex towards the centroidal axis near the end face to concavity towards the centroidal axis at a dist. 'd' from the end. Transverse stresses are caused by these curvatures. Transverse comp. stresses are developed for a certain length and beyond this zone transverse tensile stresses are developed.

(PTO)

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END BLOCKS

End block means the portion of the PSC beam surrounds the anchorages of the tendons. Large Concentrated forces are transmitted on the bearing surfaces at the ends of the beam by the anchorages. The stresses spread out into the Concrete setting up complicated stress patterns. The stress distribution close to the anchorage is different from the stress distribution at sections away from anchorage.

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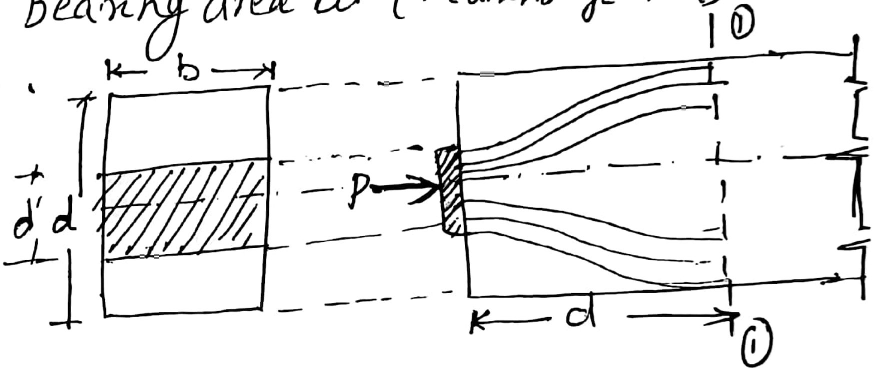

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(P.T.O)

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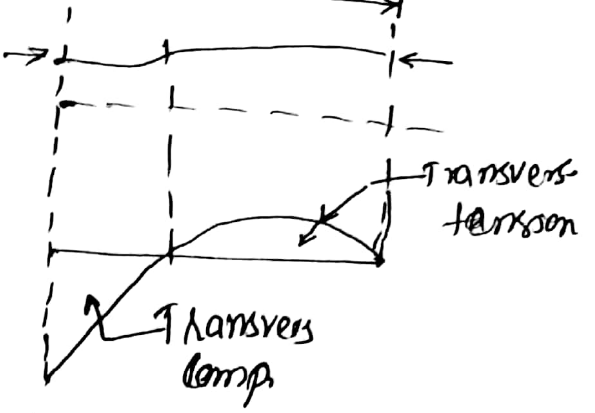
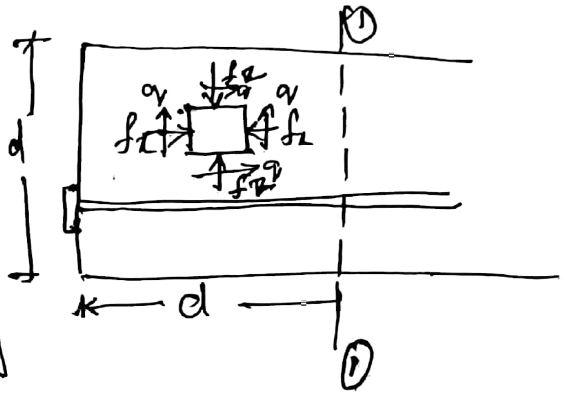
<u>MAGNEL'S METHOD</u>



MAGNEL'S METHOD

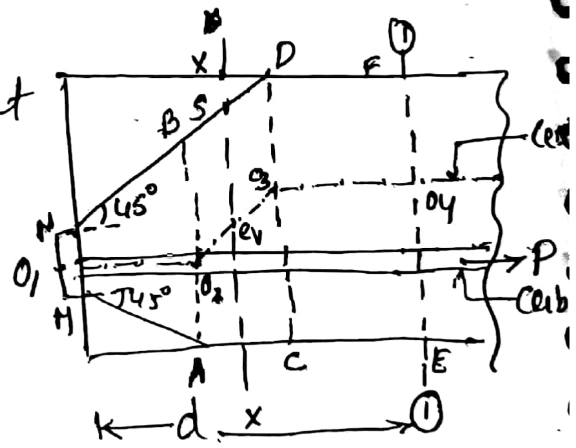
In this method the stresses due to achorage bearing are taken to be dispersed over a length of beam Equal to depth of beam.

Consider any Elemental Unit situated with in this end zone.



This Elemental Unit is subjected to three Stress f_x, ~~f_y~~ & ~~q~~. f_x & ~~f_y~~ are horizontal & vertical stresses and q is a shear stress.

In this method it is assumed that Comp force P from the anchorage will ~~be~~ dispersed at 45°. Let the extreme lines of dispersion meet ~~at D & A~~ the upper and lower faces at D & A



Now locate the centroids of stressed zone for all sections from NM to FE. locus of the centroids of stressed sections is given by $O_1 O_2, O_3 O_4$.

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Thus for any general vertical section XX, the cable eccentricity e_v is therefore known

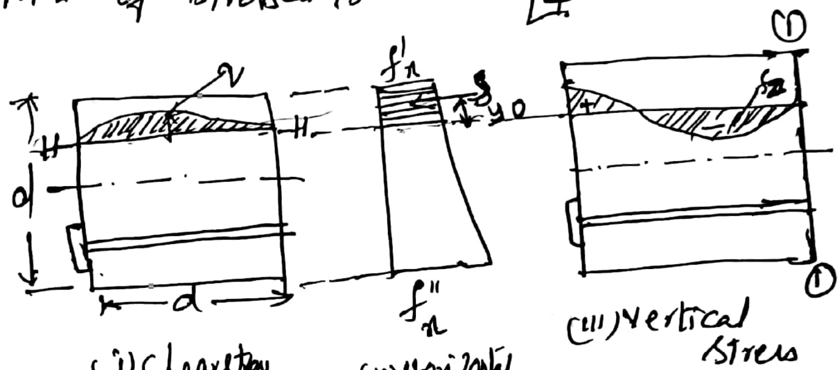
For the section X-X the stress at any point, distant y from centroidal line is given by $\frac{P}{A} - \frac{Pe}{Z}$

$$f_x = \frac{P}{b \cdot d_v} \pm \frac{P \cdot e_v \cdot y}{I_v} \quad Z = \frac{I}{y} \quad \text{(Horizontal Stress)}$$

where d_v = depth of stress zone at XX = XS

I_v = M.I of stressed section

(ii) Shear Stress (q)



According to Magnel the shear stress on any horizontal plane say H-H is given by

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$$q = k_q \cdot \frac{S}{bd} \qquad k\frac{V}{bd} \quad (1.5)$$

where k_q = shear stress factor which can be determined from tables

S = part of prestressing force at the section 1-1 acting above the level of H-H.

'S' can be determined from the horizontal stress distribution for the section ①①.

'S' can be calculated by summing up the stresses f_x above the level H-H.

Note: If cable force 'P' acts above the level H-H, then this cable force 'P' should be deducted)

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(iii) Vertical Stress f_v

According to Magnel, the vertical stress distribution on any horizontal plane H-H is given by

$$\boxed{f_v = K_z \cdot \frac{M}{bd^2}}$$

Where K_z = vertical stress factor

M = moment due to S

$= S_x \cdot y_0$

y_0 = dist. of the line of action of S from H-H

* Principal Stresses :

$$\boxed{\text{Principal stress} = \frac{f_x + f_z}{2} \pm \sqrt{\left(\frac{f_x - f_z}{2}\right)^2 + q^2}}$$

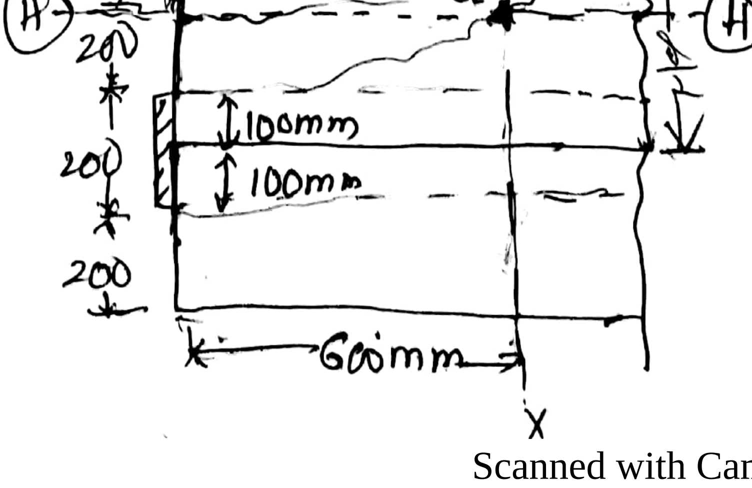
position of principal plane

$$\tan 2\theta = \frac{2q}{f_x - f_z}$$

P1 fig. shows a PSC beam 400mm wide & 800mm deep. Det. the horizontal, vertical and shear stresses at the point "Q" in the end block. Find also the principal stresses at Q. The tendons are placed at an eccentricity of 100mm. The anchor plate is 300mm wide and 200mm deep. prestressing force in tendons is 1000kN.

allow a 45° stress dispersion from top & bottom of anchor plate.

Length of end block = depth of beam = 800mm



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i) Horizontal stress at Q $= \frac{P}{b \cdot d_v} - \frac{P \cdot e \cdot y}{I_v}$

d_v = depth of stressed zone at XX

from fig. it is clear that the depth of stressed zone = 800mm ; y = dist of Q from centroidal axis = 200

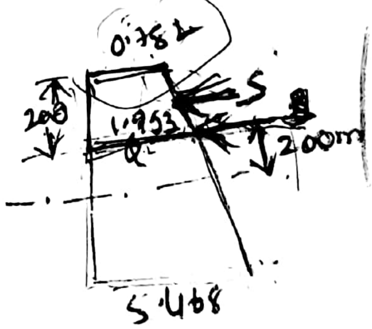
$$\therefore f_x = \frac{1000 \times 10^3}{400 \times 800} - \frac{1000 \times 10^3 \times 100 \times 200}{\frac{400 \times 800^3}{12}}$$

$= 3.125 - 1.172 = 1.953\ N/mm^2$

Extreme horizontal stress at sec(1-1) (at the end of end block)

$$= \frac{P}{bd} \mp \frac{P \cdot e}{I_v} \times \frac{d}{2}$$

$$= \frac{1000 \times 10^3}{400 \times 800} \mp \frac{1000 \times 10^3 \times 100}{\frac{400 \times 800^3}{12}} \times \frac{800}{2}$$



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$= 3.125 \mp 2.343$

$= 0.782\ N/mm^2$ @ top

$= 5.468\ N/mm^2$ @ bottom.

Total horizontal force above the level of H-H

$= S = \left(\frac{0.782 + 1.953}{2}\right)(400 \times 200) = 109400\ N$

for Trapezium $y_0 = \frac{x + 2y}{x + y} \times \frac{d}{3}$

This force acts at $y_0 = \frac{1.953 + 2 \times 0.782}{1.953 + 0.782} \times \frac{200}{3} = 85.733$ mm above H.

The given section is 600mm from the end face of beam.

depth of end block = 800mm

Dist. of the given section from the end face of beam

$\frac{600}{800} d = 0.75d$ from end face.

$800 = d$
$600 = x$
$x = \frac{600 \times d}{800}$

... from end face

$K_z = -2.47$; $K_q = 0.251$

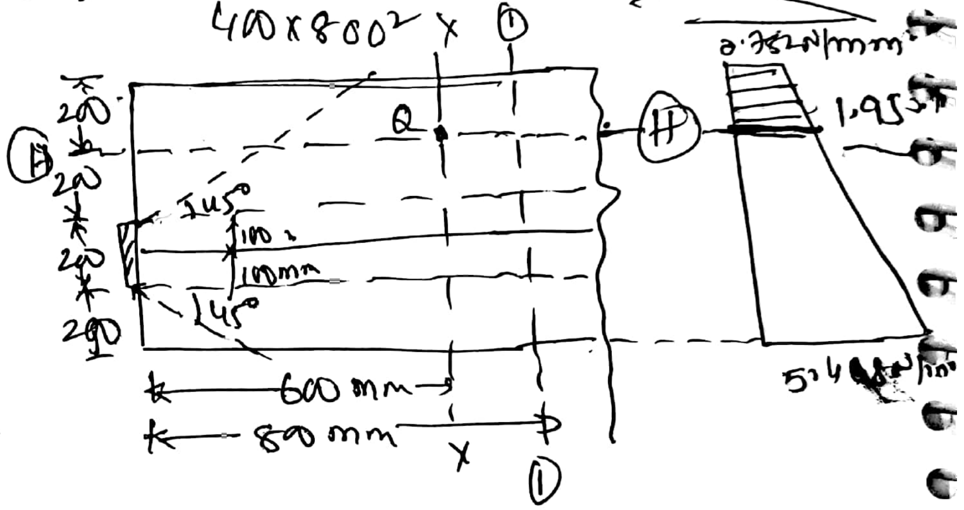
ii) $\therefore$ Shear stress @ Q $q' = K_q \cdot \frac{S}{bd}$

$= 0.251 \times \frac{10,9400}{400 \times 800} = \underline{0.086 N/mm^2}$

(iii) vertical stress $f_z = K_z \cdot \frac{M}{bd^2}$

where $M' = S \cdot y_0 = 10,9400 \times 85.73$ N-m

$\therefore f_z = -2.47 \times \frac{10,9400 \times 85.73}{400 \times 800^2} = \underline{-0.09 N/mm^2}$



Thus at Q we have

$f_x = 1.953 N/mm^2$

$f_z = -0.09 N/mm^2$

$q = 0.086 N/mm^2$

Principal stress $= \frac{f_x + f_z}{2} \pm \sqrt{\left(\frac{f_x - f_z}{2}\right)^2 + q^2}$

$f_{max} / f_{min} = \frac{1.953 - 0.09}{2} \pm \sqrt{\left(\frac{1.953 + 0.09}{2}\right)^2 + 0.086^2}$

$= \underline{1.957 N/mm^2}$ and $\underline{-0.94 N/mm^2}$

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END BLOKS

SYLL: End Blocks Nature of stresses, Stress distribution, Magnel & Guyon methods - Design by Guyon method

ANALYSIS

Design of End blocks

1) Magnel's method → Analysis
2) GUYON'S METHOD → Design

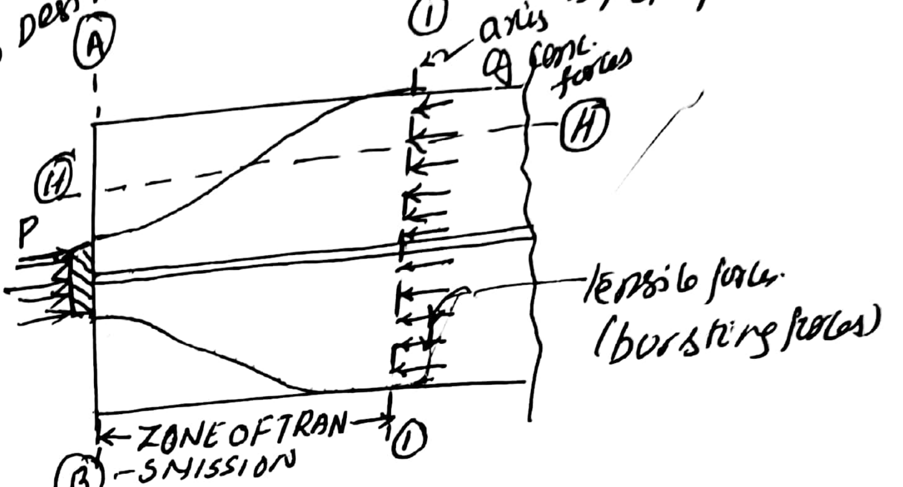

fig:

...ON'S METHOD

* When the Prestressing force (P) is applied through the anchor plate, the force is dispersed in a certain length of beam to the entire c/s.

The length of beam within which the dispersion of the prestressing force takes place is called the **Zone of transmission** *

Within the zone of transmission, the distribution of stresses can be analysed as follows:

In a direction transverse to the axis of this concentrated force, tensile forces are developed. These are called **bursting forces**

Further at the endsection the surface adjacent to the anchor plate is also subjected to a tensile force called the **spalling force.**

The fig above shows the end of a beam to which a prestressing force P is applied through a centrally placed

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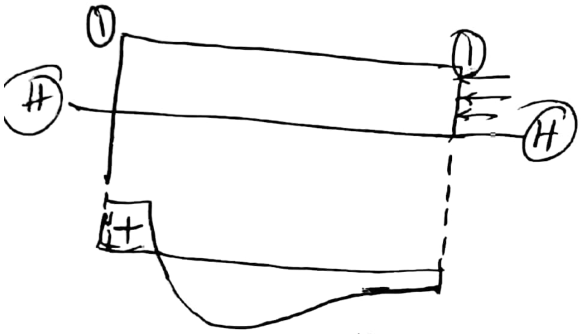
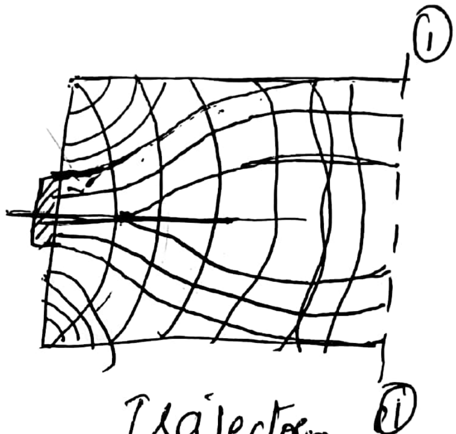
anchor plate. Let AB be the end face of the beam. Let 1-1 be the end of the zone of transmission.

Consider any Horizontal H-H. This horizontal section is subjected to transverse stresses & shear stresses.

The fig. below shows the trajectory of principal stresses due to Horizontal, vertical and shear stress. The depth of anchor plate influences the length of transmission.



Transverse stresses



Trajectories of principal stresses

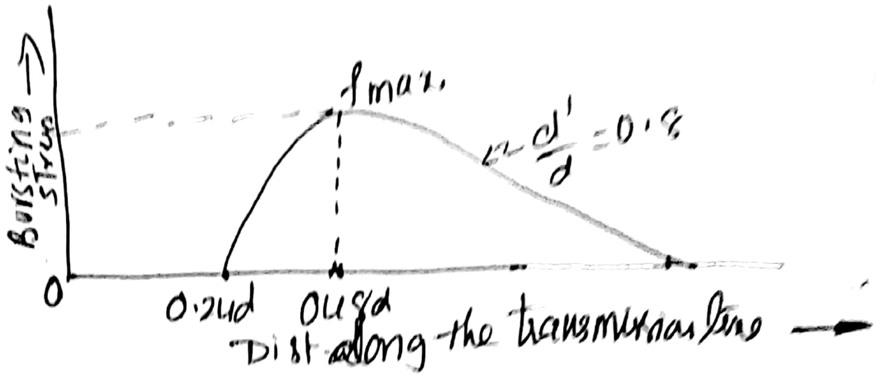
The length of transmission and the bursting forces (i.e transverse tensile force) depend on the ratio of the depth of the anchor plate (d') to the depth of beam (d).

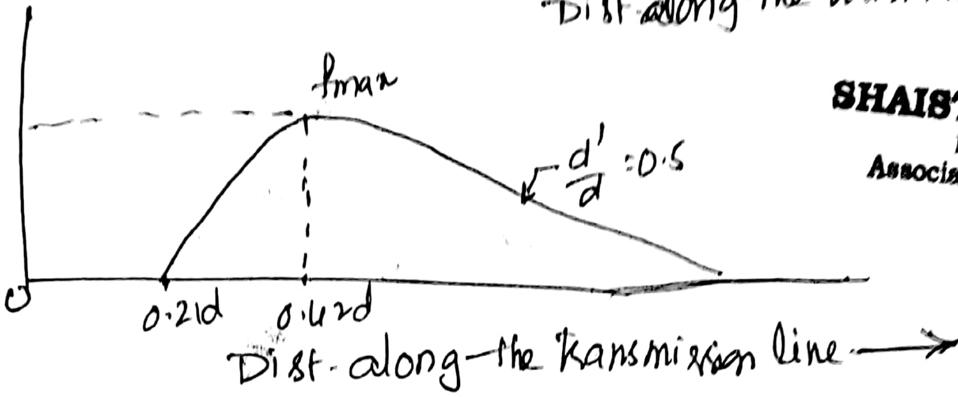
The bursting force is approximately given by (IS 1343)

$$P_b' = 0.3P\left[1 - \frac{d'}{d}\right]$$

e bursting stress distribution, along the length of nsmission for different values of $\frac{d'}{d}$ are give by separate Charts. The bursting stress distribution for

$\frac{d'}{d} = 0.8$ & $\frac{d'}{d} = 0.5$ are shown below





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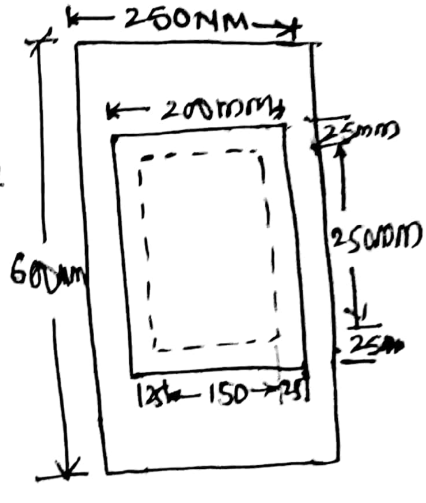
#1) A prestressed Concrete beam 250mm x 600mm is subjected to an axial P.S force of 1500kN. Design the end block.

sol Beam Section ; $b = 250mm$; $d = 600mm$

Anchor plate ; $b' = 0.8b = 0.8 \times 250 = 200mm$

$d' = 0.5d = 0.5 \times 600 = 300 mm.$



Design of Anchor plate:

Assume all cables pass through one duct and are anchored to one anchor plate ~~at with 25mm~~ an an overhang (cover) of 25mm on all sides.

Intensity of bearing pressure on the Anchor plate $= \frac{1500 \times 10^3}{300 \times 200} = 25N/mm^2$

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... from the cantilever strip of the anchor plate,

Max BM for strip $\left(\frac{wl^2}{2}\right) = 25 \times \frac{25^2}{2} = 7812.5$ N-mm

Allowing a bending stress (f_b) of 165 N/mm², equating the MR to BM

BM = M.R

$M = \frac{1}{6} \cdot f_b \times 1 \times t^2$

$7812.5 = \frac{1}{6} \times 165 \times 1 \times t^2$

$\Rightarrow t = 16.8$ mm

f_b = Per. bending stress for anchor plate = 165 N/mm²

∴ Provide 18mm thick anchor plate.

Analysis for bursting force

There will be bursting forces on (i) horizontal planes and (ii) vertical planes.

1) Bursting force on horizontal plane:

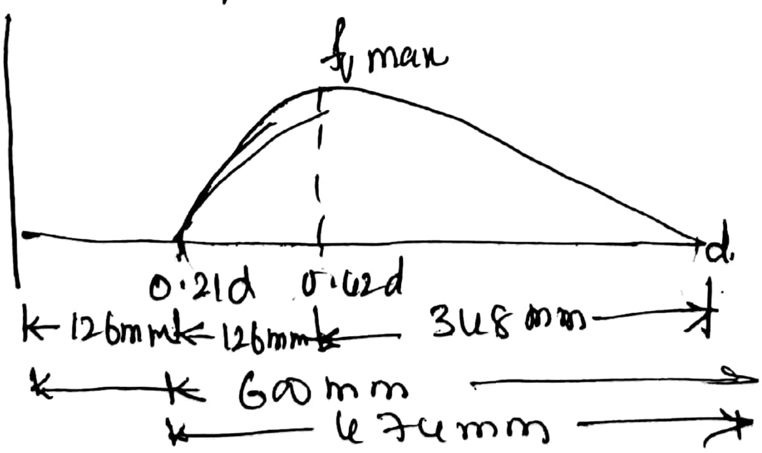
$= 0.3P\left[1 - \frac{d'}{d}\right]$

$= 0.3 \times 1500\left[1 - 0.5\right]$

$= 225$ kN

0.21 × 60
126
126
348
00

bursting force per mm width $= \frac{225 \times 10^3}{1 \times 200} = 1125$ N/mm



Equating the area of the bursting stress diag to the bursting force per mm

$1125 = \frac{2}{3} \times 474 \times f_{t\,max}$

$\Rightarrow f_{t\,max} = 3.56$ N/mm² (Note if $f_{t\,max} < f_t$ then just provide an...

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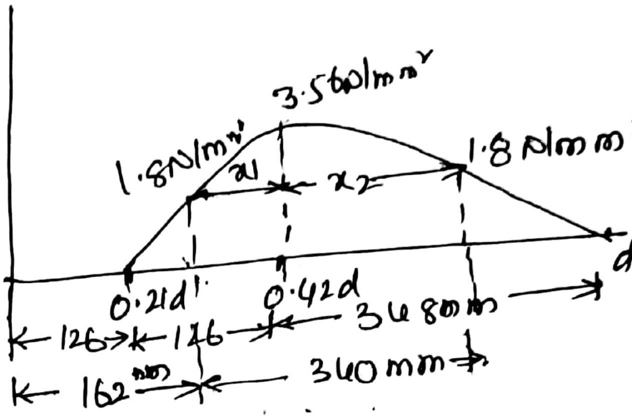
permissible bursting Stress = 1.80 N/mm²

Let this stress at distance x_1 & x_2 from the point of Max. stress



Assuming parabolic law

$$\frac{3.56-1.80}{3.56} = \frac{x_1^2}{126^2} = \frac{x_2^2}{348^2} = 0$$

on solving

$x_1 = 88.6$ say 90 mm

$x_2 = 244.7 \simeq 250$ mm

$\therefore x_1 + x_2 = 340$ mm

Total tension for which Reinforcement is necessary

$= \frac{2}{3} \times 340 \times 1.76 \times 200 = 79790$ N/mm ... $(3.56-1.8) = 1.76$

Steel Reqd $= \frac{79790}{140} = 570$ mm²

140 = f safe stress in steel = 140 N/mm²

Provide 8 bars of 10mm ϕ

Provide 4 stirrups of two legs each

9) Bursting force on Vertical plane

$= 0.3P\left[1 - \frac{b'}{b}\right] = 0.3 \times 1500 [1 - 0.8]$

$= 90$ kN $= 90 \times 10^3$ N.

bursting force per mm height

$= \frac{90 \times 10^3}{300} = 300$ N/mm.

Equating the area of the bursting stress diag. to the bursting force per mm height

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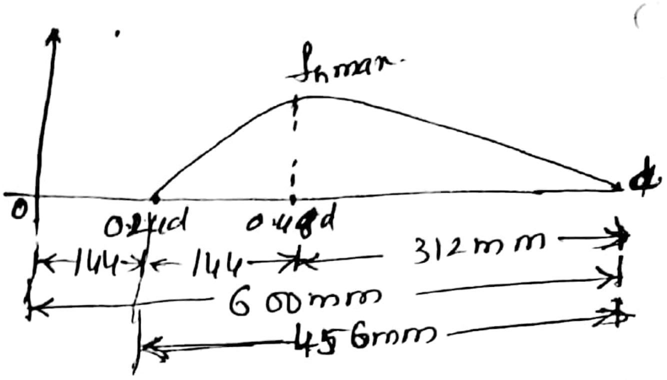
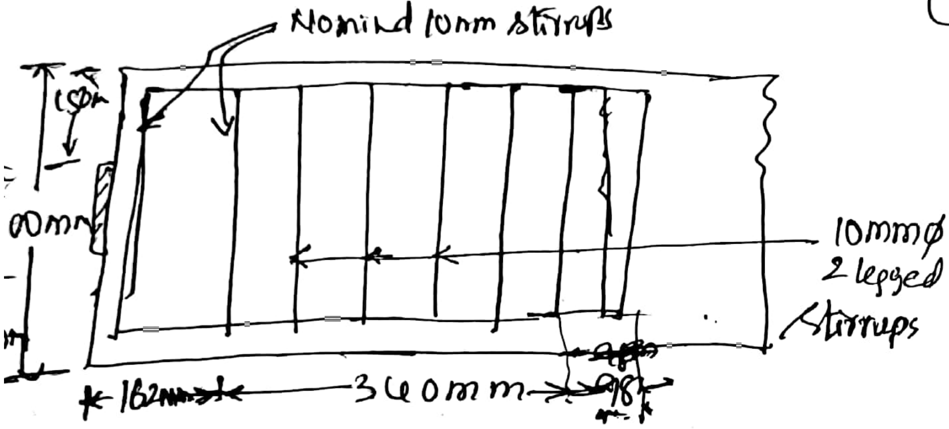
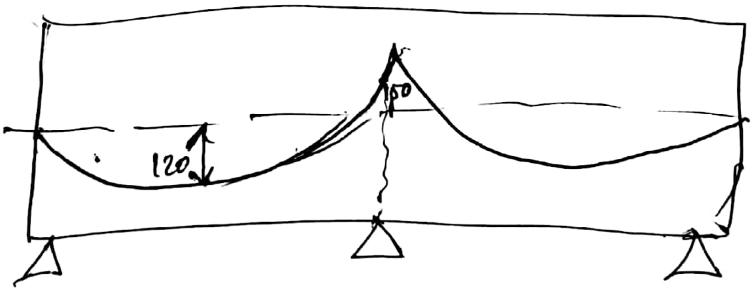


$300 = \frac{2}{3} \times 456 \times f_{h max}$

$\Rightarrow f_{h max} = 0.99 \, N/mm^2$
$< f_{per} = 1.8 \, N/mm^2$

Provide nominal horizontal steel. The horizontal legs of vertical stirrups are sufficient for providing necessary resistance. (Note: Always bursting on vertical plane will be less than that on horizontal plane,





CONTINUOUS BEAMS

SYLLABUS:
→ Advantages of Continuos beams
→ Codal Provisions
→ Analysis of 2 span continuos beams
→ Concordant Cable Profile

Advantages

i) For Each Span there is an even distribution of moments b/w the middle and ends of the span.

) Sectional requirements are minimized.

) The load bearing Capacity of the beam is enhanced.

As part a framed structure, the beam will have greater stiffness & stability.

) No: of anchorages are minimized.

i) The max. deflection is minimized.

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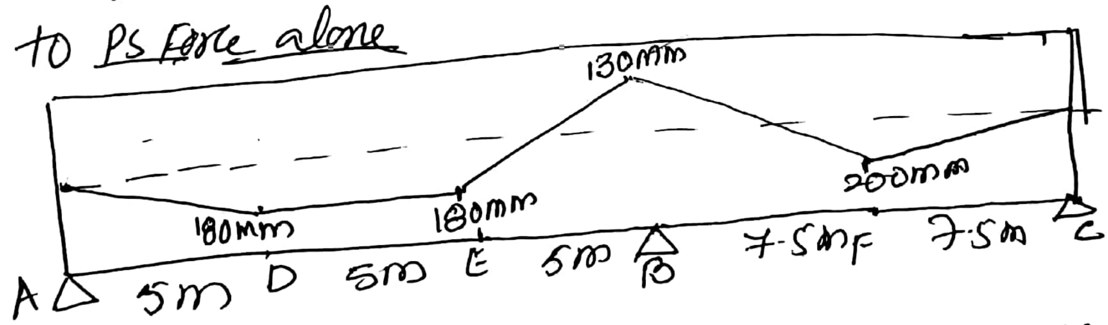
The Cable profile for a two span continuos beam is shown in fig. The PS Force being 1200kN. Locate the pr. line due to PS Force alone

p. line / c-line / T-l



al Step i: determine the primary moments (Pxe) at the various sections of the beam:

Primary moment at A = 0

" " " D = -1200×0.18 = -216 kN-m

" " " E = -1200×0.18 = -216 kN-m

" " " B = +1200×0.13 = +156 kNm

" " " F = -1200×0.2 = -240 kN-m

" " " C = 0 kN-m

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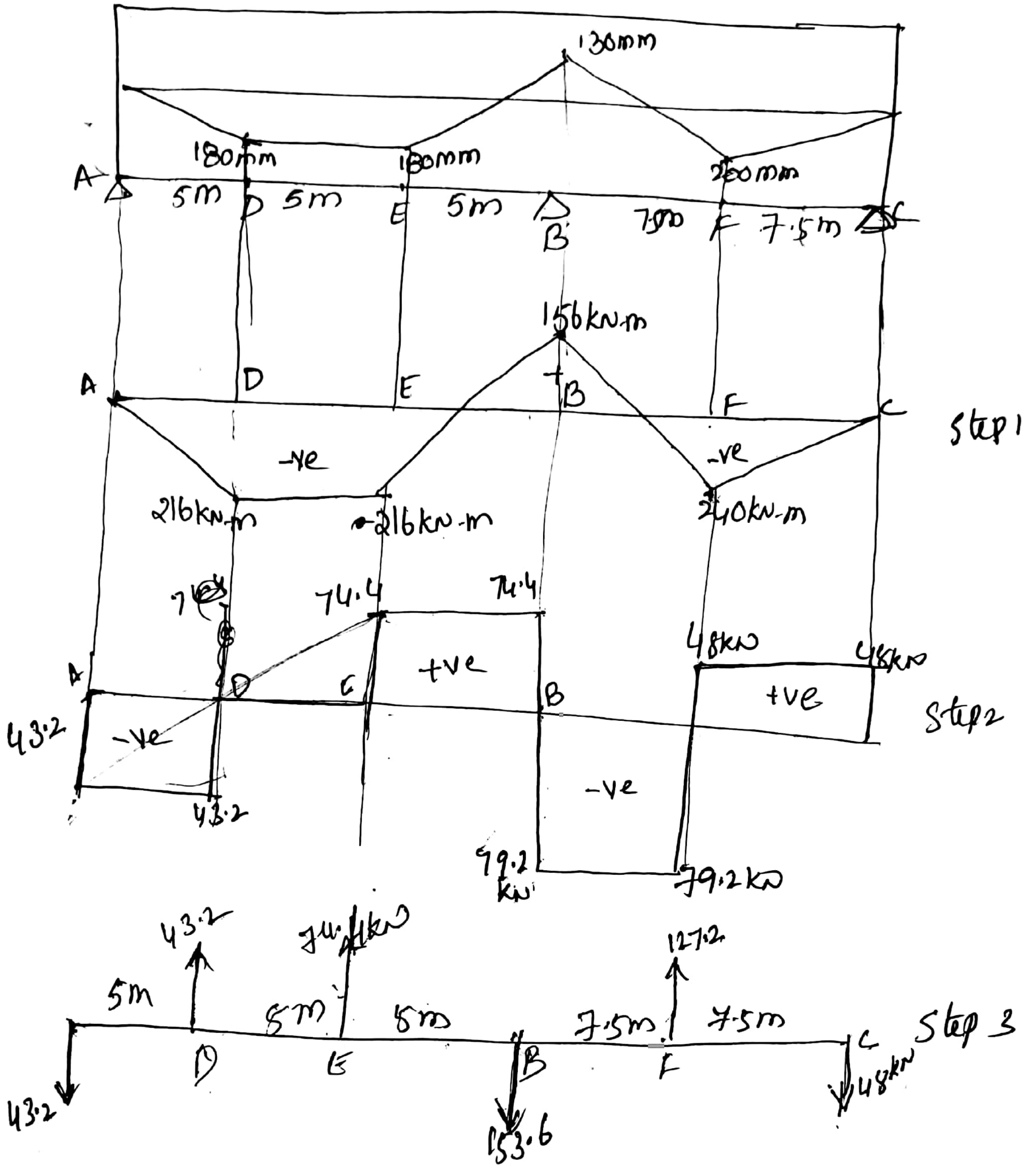

<u>Pressure or thrust line</u> : The pr. line is the locus of the resultant compression at different section of structural member.

Step 2 Primary Shear Analysis

Primary shear b/w A & D = $\frac{-216}{5}$ = −43.20 kN

Primary shear b/w D & E = 0

Primary shear b/w E & B = $+\frac{156+(216)}{5}$ = 74.40 kN (E to B u/w 50 t)

Primary shear b/w B & F = $-\frac{156+(240)}{7.5}$ = 79.20 kN

Primary shear b/w F & C = $+\frac{240}{7.5}$ = 48 kN

(draw primary shear diag)

Step 3 From the primary shear diagram, we determine the primary loading on the beam exerted by the cable

D/w load at A = 43.20 kN ↓

u/w load at D = 43.20 kN ↑

u/w load at E = 74.4 kN ↑

D/w load at B = 74.4 + 79.2 = 153.6 kN ↓

u/w load at F = 79.2 + 48 = 127.2 kN ↑

D/w load at C = 48 kN ↓

(draw loading diag)

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P4 Analysis of Beams by Hardy Cross / moment distribution meth

$$M_{FAB} = +\left(\frac{Wab^2}{l^2} + \frac{Wa^2b}{l^2}\right)$$

$$= +\left(\frac{43.2\times5\times10^2}{15^2} + \frac{74.4\times10\times5^2}{15^2}\right) = +178.67\,kN\text{-}m$$

$$M_{FBA} = -\left(\frac{43.2\times5^2\times10}{15^2} + \frac{74.4\times5\times10^2}{15^2}\right) = -213.33\,kN\text{-}m$$

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$$M_{FBC} = +\frac{127.2 \times 15}{8} = +238.50 \text{ kN-m}$$

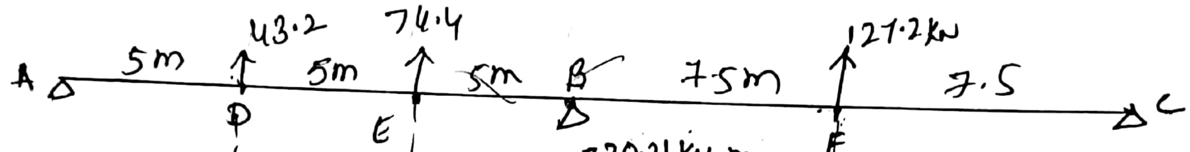
$$M_{FCB} = -\frac{127.2 \times 15}{8} = -238.50 \text{ kN-m}$$

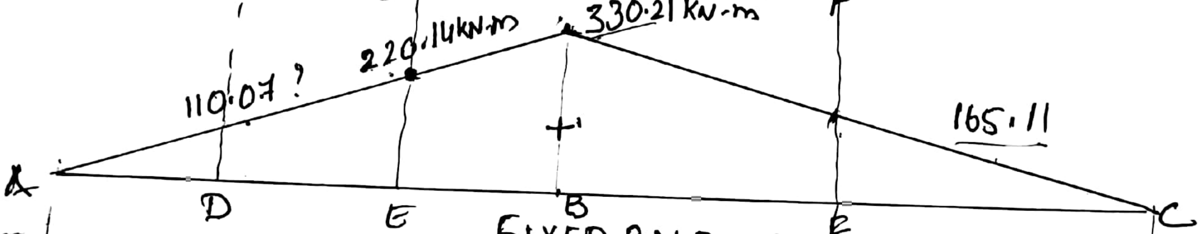
BM $\frac{wl}{4}$

wl $\frac{wl^2}{8}$

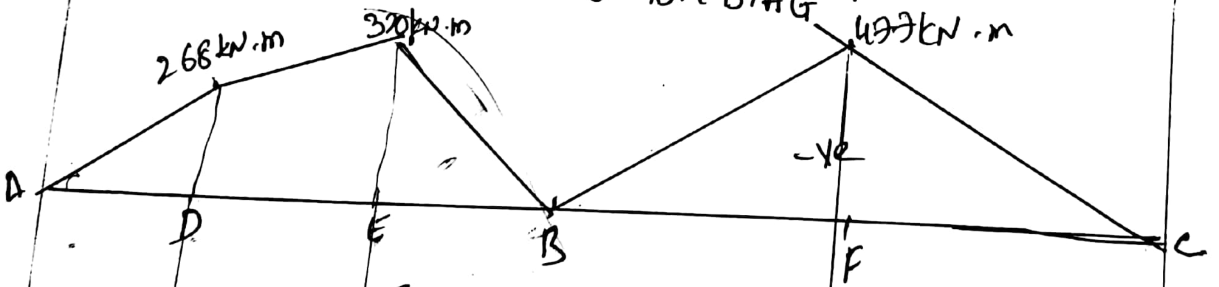
Moment distribution

A	B ($\frac{1}{2}$)	B ($\frac{1}{2}$)	C
+178.62	-213.33	+238.50	-238.50
-178.63	-89.34	+119.25	+238.50
0	-302.67	+357.75	0
	-27.54	-27.54	
0	-330.21	+330.21	0

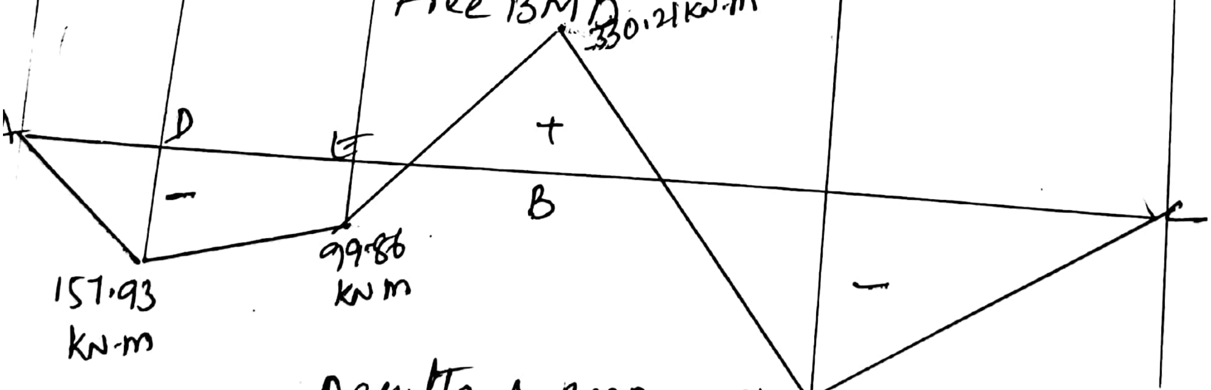




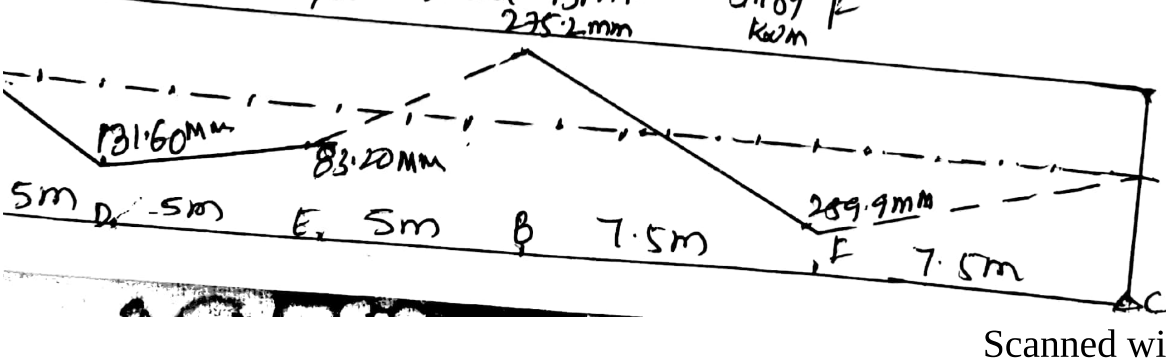
FIXED BM DIAG



Free BMD

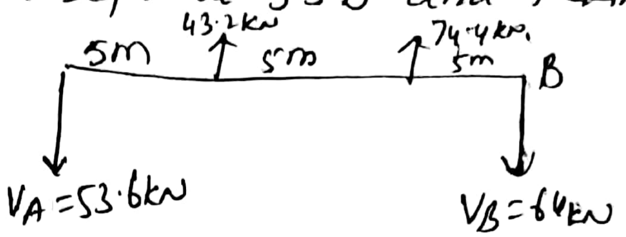


Resultant BMD



FBMD

considering AB as a separate S.S.B and taking moments about A



$\Sigma M_A = 0$

$V_B \times 15 = 43.2 \times 5 + 74.4 \times 10$

$V_B = 64kN$ $\quad V_A + V_B = 43.2 + 74.4$

$V_A = 43.2 + 74.4 - 64 = 53.6kN \downarrow$

Free BM at D = $-53.6 \times 5 = -268kNm$

" " " E = $-64 \times 5 = -320kN\text{-}m$

Similarly

For span BC

FBM at F = $-\frac{127.2 \times 15}{4} = -477 kN\text{-}m$ $\quad \left(\frac{wl}{4}\right)$

By super imposing free & fixed BM diagrams, the resultant BMD is obtained.

Step 6 Determine the P-line profile $\left(\frac{M_R}{P}\right)$

Eccentricity of P-line at any section = $\frac{\text{Resultant BM at that section}}{P}$

Eccentricity of P line at A = 0

" " at D = $\frac{-157.93 \times 1000}{1200} = -131.60mm$

" " at E = $\frac{-98.86 \times 1000}{1200} = -83.20mm$

" " at B = $+\frac{330.21 \times 1000}{1200} = +275.20 mm$

" " at F = $-\frac{311.89 \times 1000}{1200} = -259.90MM$

" " at C = 0

Finally draw P-line as shown. (The end)

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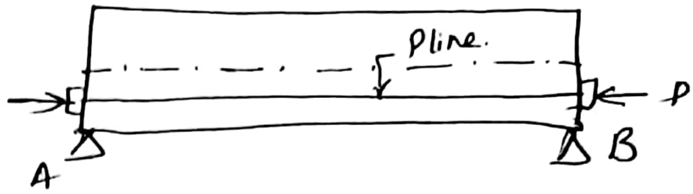
fig shows a S·S·B P
of span l,



Let the beam be provided with a tendon at an eccentricity e'.

If

(Cont..) Net d/w load on the beam $= (w_d + w_L - w')$ per UR

$\therefore$ Max hogging moment over middle support

$$= \frac{(w_d + w_L - w')l^2}{8}$$

Max. Sagg.. moment @ $\frac{3}{8}l$ from each End =

$$\frac{9}{16} \frac{(w_d + w_L - w')l^2}{8}$$

stresses induced due to these moments may be [com]pared with the safe stresses

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The following steps may be adopted in the design of a two span continuous beam subjected to u.d.l.

Data reqd :- Span 'l'; L.L = w/unit run; Safe stresses in concrete & steel.

Steps :-

1) Choose a suitable depth of beam say $\frac{1}{20}$ to $\frac{1}{25}$ of span

2) Choose a suitable max. eccentricity 'e' of the cable for the section at the middle support.
This may be about $\frac{1}{3}$ to $\frac{1}{4}$ of the depth of the beam.
The max. eccentricity below the longitudinal axis $e_o' = \frac{9}{16}e$ at a distance $\frac{3}{8}l$ from the end.

3) The parabolic cable should provide a certain amount of balancing load to counteract the d/w loading i.e balancing load intensity $w' = 0.7$ to 0.9 times the L.L intensity. Let w' be the balancing load intensity chosen.

4) Now determine the P.S force from the relation

$$P = \frac{w' l^2}{8e}$$ and round it off to a convenie

5) Allowing an avg. stress of 8 N/mm², estimate the se area reqd for the beam.

$$A = \frac{\text{PS force}}{\text{Avg. stress of the section}}$$

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Now finalise the sectional dimensions and calculate the dead load intensity w_d per unit length of the beam.

6) Check for the stresses induled

@ Stresses at transfer of prestress.

Allow say a possible loss of PS of 18%

P.S force at the stage of transfer

$$P_t = \frac{P}{0.82}$$

At this stage the beam is subjected to

i) D.L = w_d per u.r

) u/w Pr = $w' = \frac{8P_t e}{l^2}$

∴ Net u/w loading per unit run = $(w' - w_d)$ per ur,

ax Sagg. BM over the middle support = $\frac{(w' - w_d) l^2}{8}$

ax hogg.. BM @ $\frac{3}{8} l$ from each end suppor = $\frac{9}{16} (w' - w_d) \frac{l^2}{8}$

stresses induces due to these moments may be compares with safe stresses

Stresses during the working load conditions

t this stage PS force in the cable is P and

rresponding u/w Pr = $w' = \frac{8Pe}{l^2}$

(cont---)

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Design of Continuous Beam

A post tensioned continuous beam consist of two spans each 20m long. The external loading other than the dead load of the beam is 20kN/m. Design the beam.

Sol (i) Choose a suitable depth of beam equal to about $\frac{1}{20}$ of the span

$\therefore d = \frac{1}{20} \times 20 \times 1000 = 1000mm$

ii) Choose a suitable max. eccentricity 'e' of the cable for the section at the middle support. This may be about $\frac{1}{3}$ to $\frac{1}{4}$ of depth of beam. Max eccentricity below the longitudinal axis $= e_0 = \frac{9}{16} e$ at a dist $\frac{3}{8} l$ from the end.

i.e $e = \frac{1}{3} \times 1000 = 333$ say $320mm$.

$e_0 = \frac{9}{16} \times 320 = 180mm$ @ a dist $\frac{3}{8} \times 20 \times$ = 7.5m from each support.

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Sol iii) The parabolic cable should provide a certain am of balancing load to counteract the d/w loading. (balancing load intensity. This m a certain amount of balancing load intensity be nearly 0.7 to 0.9 times the live load intensity.

ω' be the balancing load intensity.

$= \omega' = 0.9 \times 20 = 18 kN/m$.

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(iv) final PS Force

$$P = \frac{w l^2}{8e} = \frac{18 \times 20^2}{8 \times 0.32} = 2812.5 \text{ kN}$$

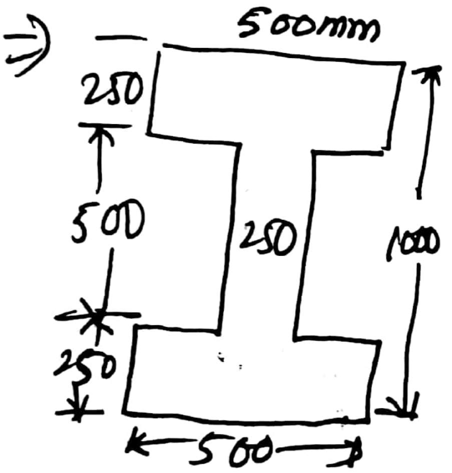
Let us provide a final PS force = 3000 kN

v) ~~Making~~ Allowing an avg stress of 8 N/mm², estimate the sectional area reqd for the beam. i.e

$$A = \frac{\text{PS Force}}{\text{Avg Stress}} = \frac{3000 \times 10^3}{8} = 375000 \text{ mm}^2$$

Now finalise the sectional dimensions of beam and calculate DL intensity w_d per unit length of beam

⇒



is the proposed section of beam.

Actual Area Provided

$$A = [2 \times 500 \times 250] + [250 \times 500]$$
$$= 375000 \text{ mm}^2.$$

$$\text{DL of beam} = \frac{375000}{10^6} \times 24 \text{ kN/m} = 9 \text{ kN/m}$$

$$I = \frac{500 \times 1000^3}{12} - \frac{(500-250)500^3}{12} = 3.90625 \times 10^{10} \text{ mm}^4$$

∴ Section Modulus $Z = \frac{3.90625 \times 10^{10}}{\underset{(d/2)}{500}} = 78125 \times 10^7 \text{ mm}^2$

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VII Check for stresses

(a) Stresses at transfer:

Allow a possible loss of 18% (say)

The initial PS force = $P_t = \frac{P}{0.82} = \frac{3000}{0.82}$ kN

At this stage the beam is subjected to following

~~forces~~ loads

i) DL = 9 kN/m

(ii) u/w pr = $w' = \frac{8 P_t e}{l^2} = \frac{8 \times 3000 \times 0.32}{0.82 \times 20^2} = 23.42$ kN/m

Net u/w loading = 23.42 − 9 = 14.42 kN/m

∴ Max ~~hogging~~ sagging moment at mid support = $\frac{14.42 \times 20^2}{8} = 721$ kNm

∴ Extreme stress for the section = $\frac{P_t}{A} \mp \frac{M}{Z}$

$= \frac{3000 \times 10^3}{0.82 \times 375000} \mp \frac{721 \times 10^6}{7.8125 \times 10^7}$

$= -0.53$ N/mm² & $+18.99$ N/mm²

Safe comp. stress at transfer = 0.5 f_{ck} = 0.5 × 45 = 22.5 N/mm²

(b) Stress during the working load condition

At this stage the loading on the beam consists of

i) DL = 9 kN/m

ii) LL = 20 kN/m

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(iii) u/w pr $= w' = \frac{8Pe}{L^2} = \frac{8 \times 3000 \times 0.32}{20^2} = 19.2$ kN/m

6 ∴ Net d/w load $= 9 + 20 - 19.2 = 9.80$ kN/m

∴ Max hogging moment @ middle support

$= 9.80 \times \frac{20^2}{8} = 490$ kN-m

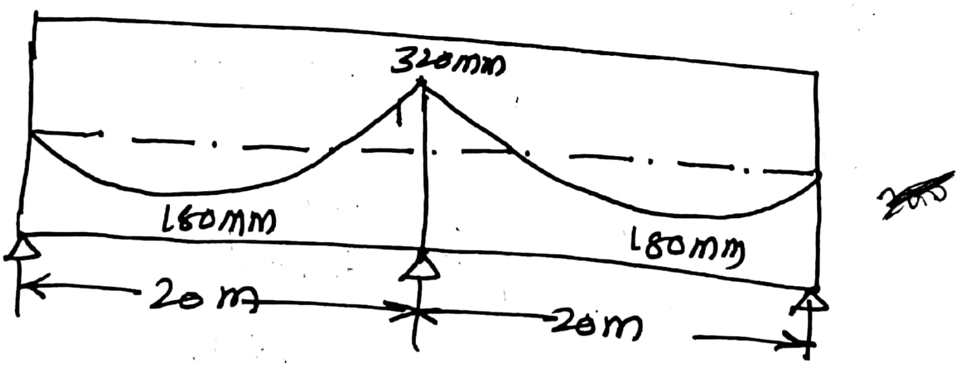
∴ Extreme stresses for the Section

$= \frac{P}{A} \pm \frac{M}{Z} = \frac{3000 \times 10^3}{375000} \pm \frac{490 \times 10^6}{78125 \times 10^3}$

$= 14.27$ N/mm² & 1.73 N/mm²

Safe Comp. Stress at working load condition $= 0.4 f_{ck}$

$= 0.4 \times 45 = 18$ N/mm²



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Beam not subjected to external loading P-line coincides with C-line.

i.e If the beam is not subjected to any external loading then the B.M at any section is Pe. If the Resultant compression at any section be C then P & C act along the same line.

i.e P = C

The C-line is the line of action of the Resultant Comp. force.

If the Beam is subjected to External BM then the C-line will be shifted from the P-line by a distance 'a' equal to the lever arm.

$$a = \text{Shift of C-line from P-line} = \frac{\text{External BM}}{P}$$

Consider a S.S.B of Rectangular section, subjected to end couple M.

At any section, addl BM = M

$$\therefore \text{Lever arm} = a = \frac{M}{P}$$

At any section the stresses produced are entirely due to the Comp force C. If the position of C-line at any section is known, then the extreme stresses for the section are given by $\frac{C}{A} \mp \frac{C \times e}{Z}$

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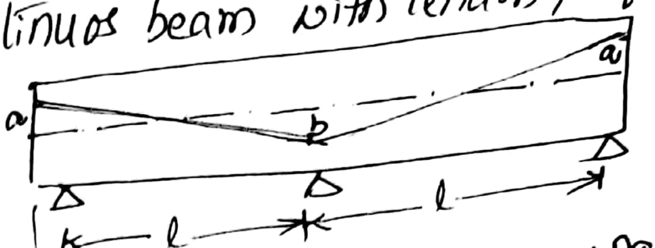
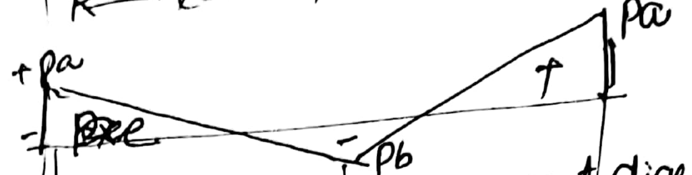
consider a continuos beam with [illegible] in fig below



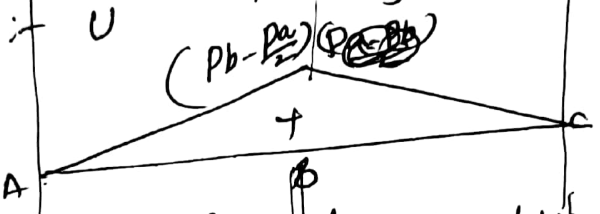
Note:

Primary moment = Pxe



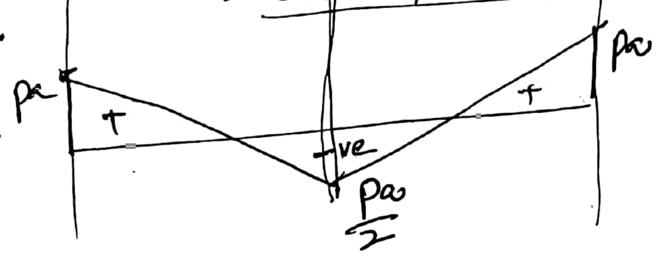
Primary moment dig

Secondary moment: U



Secondary moment dig

under the action of primary moment, the beam deflect. Since the supports do not allow any deflection, reactions



Resultant moment dig

will be induced over the supports. These reactions produce a moment at every section. The BM at any section due to reaction alone caused by primary moment, is called secondary moment. Hence resultant moment at any section is the algebraic sum of primary moment & secondary moment.

It can be determined either by theorem of 3 moments or moment distribution method.

End moment at A = $+Pa$ & @ C = $-Pa$

A	B	B	C
$+Pa$	$+\frac{Pa}{2}$	$-\frac{Pa}{2}$	$-Pa$

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Suppose the cable profile is such that there is no secondary moment for the beam the

$$\cancel{\frac{Pa - Pb}{2}} = \cancel{0} \quad Pb - \frac{Pa}{2} = 0$$

$$\Rightarrow b = \frac{a}{2}$$

If this condition is satisfied, there will be ~~is~~ no secondary moment and P-line, C-line coincides.

The cable profile corresponding to this condition is called a CONCORDANT PROFILE

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