UNIT - II

Scanned with CamScanner

UNIT - II

LIMIT STATE OF COLLAPSE (FLEXURE)

Limit State method of Design (LSD)

We have seen that ~~the~~ while the working stress method gives satisfactory performance of the structure at working loads, it is unrealistic at ultimate state of collapse.

Similarly while the ultimate load method provides realistic assessment of safety, it does not guarantee the satisfactory ~~service~~ serviceability requirements at service load.

An ideal method is the one which takes into account not only the ultimate strength of the structure but also the serviceability and durability requirements. The "Limit State method" of design is oriented towards the simultaneous satisfaction of all these requirements.

This new method makes a judicious combination of working stress philosophy as well as the ultimate load philosophy, thus avoiding the demerits of both.

→ The acceptable limit of safety and serviceability requirements before failure occurs is called a LIMIT STATE.

SHAISTA BEGUM
M.Tech (Structure)
Associate Professor

Scanned with CamScanner

In design procedure three types of limit states are considered –

1. Limit state of collapse
2. Limit state of serviceability
3. Limit state of durability (other limit states)

(1) ~~Limit State of collapse~~ (already discussed in UNIT-II)

(i) Moment of Resistance of Singly Reinforced Beam (M_u ...)

The total compressive force C_u in a rectangular beam, singly reinforced is

$C_u = 0.36 f_{ck} \cdot b \cdot x_u$ while the Total tensile force T_u in Reinforcement will be equal to $0.87 f_y \cdot A_{st}$

∴ For equilibrium of forces $C_u = T_u$

$\Rightarrow 0.36 f_{ck} \cdot b \cdot x_u = 0.87 f_y \cdot A_{st}$

$$\Rightarrow \frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} \cdot bd} = 2.417 p_t \cdot \frac{f_y}{f_{ck}}$$

where $p_t = \frac{A_{st}}{bd}$ = reinforcement Ratio

** PROCEDURE FOR FINDING M.R

The M.R of rectangular section without compression reinforcement (SR) should be obtained as follows (IS 456-2000):

ⓐ Determine the depth of NA using the Eqn

$$\because \frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b d} = 2.417 \cdot \frac{f_y}{f_{ck}} \cdot \frac{A_{st}}{bd}.$$

ⓑ If the value of (x_u/d) is less than the limiting value (w.r.t IS 456), calculate the M.R using the equation

$$M_u = 0.87 f_y A_{st} d\left(1 - \frac{A_{st}}{bd} \cdot \frac{f_y}{f_{ck}}\right) \text{ or } 0.87 f_y A_{st} d(d - 0.42 x_u) \quad ②$$

(w.r.t steel) = Tensile force × L.A (T.F × L.A)

ⓒ If the value of x_u/d is equal to the limiting value, the M.R of the section is given by ~~$M_{u,lim} = 0.36 f_{ck} b \frac{x_{umax}}{d}\left[1 - 0.416 \frac{x_{umax}}{d}\right]$~~

$$M_{u\,lim} = 0.36 f_{ck} b x_u (d - 0.42 x_u) \quad — ①$$

(w.r.t conc.) = comp force × Leverarm

Note :- → Eqn ① is used to find eff depth 'd' of flexure member i.e beams & slabs etc.,

→ Eqn ② is used to find area of steel reqd, Ast.

ⓓ If x_u/d is greater than the limiting value, the section is over Reinforced. The M.R of such a section is limited to $M_{u\,lim}$ given by eqn ①

NOTE :- In the case of an over reinforced section,

Scanned with CamScanner

not only that there is a wastage of excess steel, the concrete reaches its ultimate capacity before steel yields resulting in a sudden failure. The code therefore recommends that such a section should be redesigned.

Doubly Reinforced beams:

The SR section has a limiting value of M.R, corresponding to limiting value of steel reinforcement. However if the ultimate moment due to applied loads Mu is larger than Mulim, two alternatives will be available: $M_{u,app} > M_{ulim}$

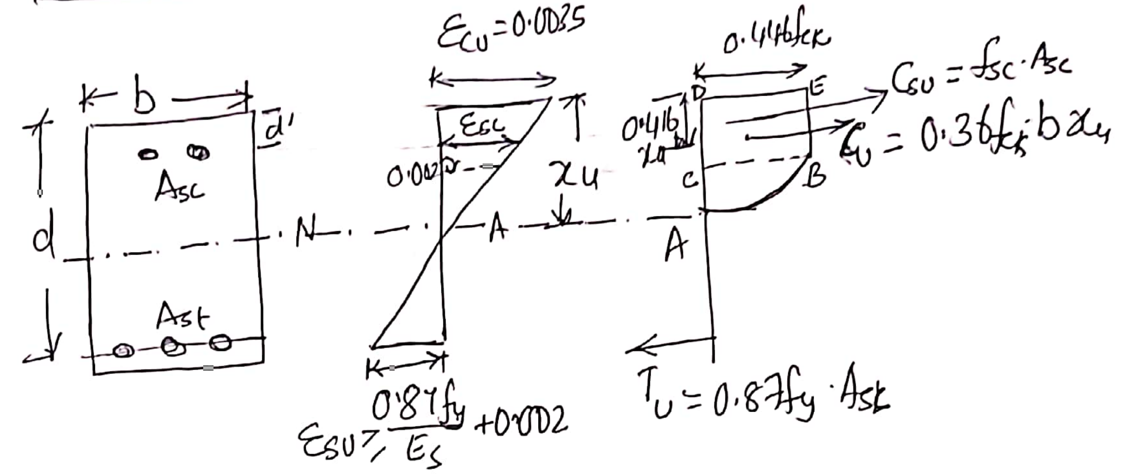
i) To increase the depth of section
ii) To provide compression reinforcement

In many cases the max. value of depth is restricted from architectural considerations. In that case only alternative is to provide reinforcement in the compression zone giving-rise to "Doubly Reinforced section.

SHAISTA BEGUM
M.Tech (Structure)
Associate Professor

Scanned with CamScanner

STRESS BLOCK AND LOCATION OF NA



DRB

Total Comp. force $C_u = C_{cu} + C_{su}$

$$= 0.36 f_{ck} b x_u + f_{sc} \cdot A_{sc}$$

Tensile force, $T_u = 0.87\, f_y A_{st}$

In order to locate NA $C_u = T_u$

$$0.36 f_{ck} b \cdot x_u + f_{sc} \cdot A_{sc} = 0.87 f_y \cdot A_{st} \Rightarrow x_u = N.A$$

→ When the limiting MR of beam $M_{u\text{-}lim}$ is less than ($M_u > M_{ulim}$) the ultimate moment M_u due to applied loads, then compression reinforcement (Asc) may be obtained from

$$M_u - M_{u\text{-}lim} = f_{sc} \cdot A_{sc} \, (d - d')$$

where f_{sc} = stress in compn steel corresponding to a strain of $\frac{0.0035 (x_{umax} - d')}{x_{u\text{-}max}}$

SHAISTA BEGUM
M.Tech (Structure)
Associate Professor

Scanned with CamScanner

d' = cover to compression steel

→ Additional tensile steel to balance the compression steel is obtained by equating compressive force due to compression steel A_{sc} and tensile force due to additional tensile steel A_{st2}

$$0.87 f_y \cdot A_{st-2} = f_{sc} \cdot A_{sc}$$

or

$$A_{st-2} = \frac{f_{sc} A_{sc}}{0.87 f_y}$$

(A/N)

FLANGED BEAM OR T-BEAM :

→ When $\frac{D_f}{d} \leq 0.2$, mom. of resistance M_u may be determined from the eqn

$$M_u = 0.36 f_{ck} \cdot b_w x_u (d - 0.42 x_u) + 0.45 f_{ck} (b_f - b_w) D_f (d - D_f/2)$$

When $\frac{D_f}{d} > 0.2$ M.R, M_u may be determined from eqn

$$M_u = 0.36 f_{ck} \cdot b_w x_u (d - 0.42 x_u) + 0.45 f_{ck} (b_f - b_w) y_f (d - \frac{y_f}{2})$$

where $y_f = 0.15 x_u + 0.65 D_f \not> D_f$

Scanned with CamScanner

d' = cover to Compression steel

→ Additional tensile steel to balance the compression steel is obtained by Equating compressive force due to compn steel 'A_{sc}' and tensile force due to addl tensile steel A_{st2}

$$\Rightarrow 0.87 f_y \cdot A_{st2} = f_{sc} \cdot A_{sc}$$

or

$$A_{st2} = \frac{f_{sc} A_{sc}}{0.87 f_y}$$

SHAISTA BEGUM
M.Tech (Structure)
Associate Professor

Scanned with CamScanner

Design and Analysis of Singly Reinforced Beams

Steps:

1) given span of beam 'l', load acting on beam w, grades of steel & concrete - f_y & f_{ck} respectively

2) Find BM, $M_u = 1.5 \frac{wl^2}{8}$

3) Adopt width of beam, b = 230mm to 300mm.

4) Find the depth of beam 'd', using

$$M_u = 0.36 f_{ck} b \cdot x_u (d - 0.42 x_u) \quad \text{—ⓐ}$$

using M_{20} Concrete $f_{ck} = 20 N/mm^2$ &

Fe415 steel $f_y = 415 N/mm^2$

$$\Rightarrow \boxed{x_u = 0.48d}$$

substituting in the eqn ⓐ

$$M_u = 0.36 \times 20 \times b \times 0.48d (d - 0.42 \times 0.48d)$$

$$\Rightarrow M_u = 2.76\ bd^2$$

$$\text{or } d = \sqrt{\frac{M_u}{2.76 b}}$$

5) Find the area of steel A_{st} using the Relation

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$\Rightarrow \boxed{A_{st} = M_u / 0.87 f_y (d - 0.42 x_u)}$$

SHAISTA BEGUM
M.Tech (Structures)
Associate Professor

Scanned with CamScanner

6) Check for minimum tension steel ($A_{st\,min}$) using

$$\frac{A_{st}}{bd} = \frac{0.85}{f_y}$$ and check for max. steel

using $A_{st\,max} \not> 0.04bD$.

7) Draw the neat sketches showing Reinforcement details.

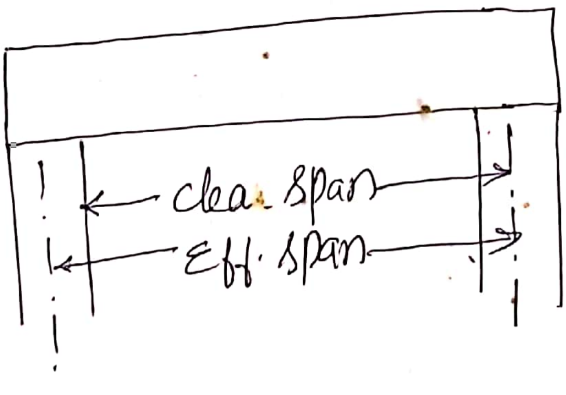
Eff. Span and Clear Span

→ for BM calculation use eff. span.

→ eff. span shall be lesser of

(i) c/c of support

ii) Clear span plus Eff. depth



SHAISTA BEGUM
M.Tech (Structure)
Associate Professor

Scanned with CamScanner

#1) DESIGN a SRB section and find the necessary reinforcement ~~of~~ for an applied moment of 60kN-m. The width of beam is limited to 175mm. use M_{20} & Fe415 grades.

~~sol~~ $M_{s} = 60 \times 10^6$ N-mm.

$M_U = 1.5 \times 60 \times 10^6 = 90 \times 10^6$ N-mm.

~~for a~~ $M_{ULIM} = 0.36 f_{ck} \cdot b \, x_u \, (d - 0.42 \, x_u)$

x_{umax} for f_y 415 = 0.

SHAISTA BEGUM
M.Tech (Structure)
Associate Professor

Scanned with CamScanner

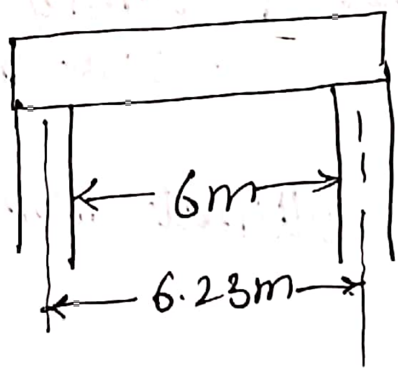
#2) A rectangular beam is to be S.S. on supports of 230mm width. The clear span of the beam is 6m., the beam is to have width of 300mm. The characteristic SI load is 12 kN/m. using M_{20} Concr. and Fe415 steel, design the beam.

Sol step 1

Given; $l = 6m$; $b = 300mm$; $w = 12 kN/m$

$f_{ck} = 20 N/mm^2$; $f_y = 415 N/mm^2$

→ From stiffness consideration.



$d = \frac{Span}{20}$ (IS 456 Pg: 37)

$= \frac{6.23}{20} = 0.31 m$

eff. span (lesser of two)

i) c/c of support $= 6.23m$

ii) clear span + Eff depth $= 6 + 0.31 = 6.31m$

$\therefore l_{eff} = 6.23m$

Provide an eff. cover of 30mm

$\Rightarrow D = 310 + 30 = 340mm$

BHAIS... Asso... Professor

Scanned with CamScanner

Analysis of a SRB

→ Finding out the moment capacity $M_{u\text{-}lim}$ of a beam is known as Analysis.

Steps for Analysis

1) Given size of beam $b \times d$ and area of Steel A_{st}.
2) Find depth of NA by equating Comp. force and Tensile force. i.e.,

$$0.36 f_{ck} \cdot b \cdot x_u = 0.87 f_y \cdot A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

3) Find limiting depth of N.A i.e x_{umax} (0.48d for Fe415)
4) If $x_u < x_{umax}$, Steel yield first and Moment capacity is given by

$$M_{u\text{-}lim} = 0.87 f_y A_{st} (d - 0.42 x_u)$$

5) If $x_u > x_{umax}$, concrete will crush first and moment capacity is given by

$$M_{u\text{-}lim} = 0.36 f_{ck} \cdot b x_u (d - 0.42 x_u)$$

ϕ of bars – 8mm, 10mm, 12, 16, 20, 22, 25, 30, 32 mm

SHAISTA BEGUM
M.Tech (Struct.)
Associate Professor

Scanned with CamScanner

Problems

Find the M.R (or limiting moment (or) moment capacity) of a SRCB of 200mm width and 400mm Eff. depth, Reinforced with 4 bars of 16mm ϕ of Fe415 Steel. Take M_{20} Concrete, Redesign the beam if necessary. Also find the u.d.l it can carry if the span of the beam is 4.9m

Sol **Step 1:**

Given : $b = 200mm$; $D = 400 + 30 = 430mm$;

$$A_{st} = \frac{4\pi(16)^2}{4} = 804.25\,mm^2$$

$f_{ck} = 20N/mm^2$; $f_y = 415N/mm^2$

Step 2 Depth of Act. N.A (x_u)

$$C_u = T_u$$

$$0.36 f_{ck} \cdot b \cdot x_u = 0.87 f_y \cdot A_{st}$$

$$0.36 \times 20 \times 200 \times x_u = 0.87 \times 415 \times 804.25$$

$$\Rightarrow x_u = \frac{0.87}{0.36} \times \frac{415}{20} \times \frac{804.25}{200} = 201.64\,mm$$

Max. depth of N.A ($x_{u.lim}$) for Fe415

$$x_{umax} = 0.48d = 0.48 \times 400 = 192mm$$

SHAISTA BEGUM
M.Tech (Structure)
Associate Professor

Scanned with CamScanner

Since $x_u > x_{umax}$ ⇒ Concrete Crushes first

Such a beam is over reinforced and hence undesirable. The Code recommends that such a beam be Redesigned. The limiting M.R for such a beam is found using

$$M_{u\text{-}Lim} = 0.36 f_{ck} \cdot \frac{x_{umax}}{d}\left(d - 0.42 x_{umax}\right) b$$

$$= 0.36 \times 20 \times (0.48 \times 400)\,(400 - 0.42(0.48 \times 400))\,200$$

$$= 88.37 \text{ kN-m}.$$

Redesign :- $x_{umax} = 0.48\, d$

$$p_t = \frac{A_{st}}{bd}$$

using Eqn $\frac{x_{umax}}{d} = \frac{0.87}{0.36} \cdot \frac{f_y}{f_{ck}} \cdot \frac{A_{st}}{bd}$

$$\frac{x_{umax}}{d} = 2.417 \cdot p_t \cdot \frac{f_y}{f_{ck}}$$

$$\Rightarrow p_t = \left(\frac{x_{umax}}{d}\right)\frac{f_{ck}}{f_y} \cdot \frac{1}{2.417} = (0.48) \times \frac{20}{415} \times \frac{1}{2.417}$$

$p_t = 9.551 \times 10^{-3}$ ⇒ $A_{st} = p_t \times b \times d$

$A_{st} = 9.551 \times 10^{-3} \times 200 \times 400 = 765.66\ mm^2$

(as against $804.25\ mm^2$)

SHAISTA BEGUM
M.Tech (Structure)
Associate Professor

Scanned with CamScanner

b) To find working w

$M_{u\,lim} = 1.5 \times \frac{wl^2}{8}$

$88.37 \times 10^6 = 1.5 \times \frac{w \times 4.2^2}{8}$

$\Rightarrow \boxed{w = 26.72 \text{ kN/m}}$

Note:- reducing an OR section to A balanced Section is what we have done in this problem

DOUBLY REINFORCED SECTIONS.

→ If the applied moment M_u is greater than $M_{u\text{-}lim}$, two alternatives will be available:

i) To increase the depth of the section (or)

(ii) To provide compression Reinforcement.

In many cases the max. value of the depth of the section may be limited or restricted due to architectural or other considerations. In such cases only alternative available will be to provide reinforcement in the compression zone, giving rise to a doubly reinforced section.

→ Compression Reinforcement 'A_{sc}' may be obtained from the following Equation:

SHAISTA BEGUM
M.Tech (Structures)
Associate Professor

Scanned with CamScanner

$(M_u - M_{ulim}) = f_{sc} \cdot A_{sc} (d - d')$

where f_{sc} = stress in compr steel corresponding to a strain of $0.0035 \frac{(x_{umax} - d')}{x_{umax}}$

d' = Cover to Compr. steel.

→ Additional tensile steel to balance the compression steel is obtained by equating compressive force due to compression steel A_{sc} and tensile force due to additional tensile steel A_{st_2}

$0.87 f_y \cdot A_{st2} = f_{sc} \cdot A_{sc}$

$$\Rightarrow A_{st-2} = \frac{f_{sc} \cdot A_{sc}}{0.87 f_y}$$

Design and Analysis of a Doubly Reinforced Beam.

→ A DRB is that in which steel is provided both in tension and compression zone.

→ A DRB is provided:

i) when the size of the beam is restricted from architectural point of view

ii) In continuous beams where tension is developed at bottom at midspan and on top face

SHAISTA BEGUM
M.Tech (Structure)
Associate Professor

Scanned with CamScanner

near supports.

* In industrial buildings where loads are alternating i.e sometime acting d/w & some times u/w

Steps for design of Doubly Reinforced Beam

1) Given load acting on beam 'w', span 'l', and size of beam b×d.
2) Determine design BM, $M_u = 1.5 \frac{wl^2}{8}$ and also limiting moment $M_{u\text{-}Lim} = 0.36 f_{ck} x_{umax} b(d - 0.42 x_{umax})$
3) If $M_{ud} > M_{uLim}$, the beam is design as DRB
4) find Area of steel 'A_{st1}' for $M_{uLim} = 0.87 f_y A_{st1}(d - 0.42 x_{um})$
5) Find area of comp. steel 'A_{sc}' for remaining moment
   $(M_u - M_{uLim}) = f_{sc} \cdot A_{sc}(d - d')$
   where f_{sc} = stress in comp steel
   d' = cover to comp. steel
6) Find area of additional tensile steel $A_{st\text{-}2}$ using Eqn $0.87 f_y \cdot A_{st2} = f_{sc} \cdot A_{sc}$

$$\Rightarrow \boxed{A_{st2} = \frac{f_{sc} \cdot A_{sc}}{0.87 f_y}}$$

AISTA BEGUM
M.Tech (Structures)
Associate Professor

Scanned with CamScanner

#1 An RCC beam of eff. span 7m is S.S at Ends. The over all size of beam is Restricted to 230x500 mm. The beam is subjected to a S.I load of 20kN/m including its self wt. Design the beam for flexure and draw neat sketches showing reinforcement details.

Sol

Step 1) Given; $l_{eff} = 7m$; $b = 230mm$; $D = 500mm$

$w = 20kN/m$;

→ provide an Eff. cover of 30mm,

$\therefore d = 500 - 30 = 470mm$

Step 2 → Factored B.M, $M_{ud} = 1.5 \times \frac{wl_e^2}{8}$

$= \frac{1.5 \times 20 \times 7^2}{8} = 183.75 \, kN\text{-}m$

→ Limiting Moment;

$M_{ulim} = 0.36 f_{ck} \cdot b \cdot x_{umax}(d - 0.42 x_{umax})$ where $x_{umax} = 0.48d$

$= 0.36 \times 20 \times 230 \times 0.48 \times 470 (470 - 0.42 \times 0.48 \times 470)$

$= 140.22 \times 10^6 \, N\text{-}mm$

Step 3 $M_{ud} > M_{ulim}$

$\therefore$ DRB

Scanned with CamScanner

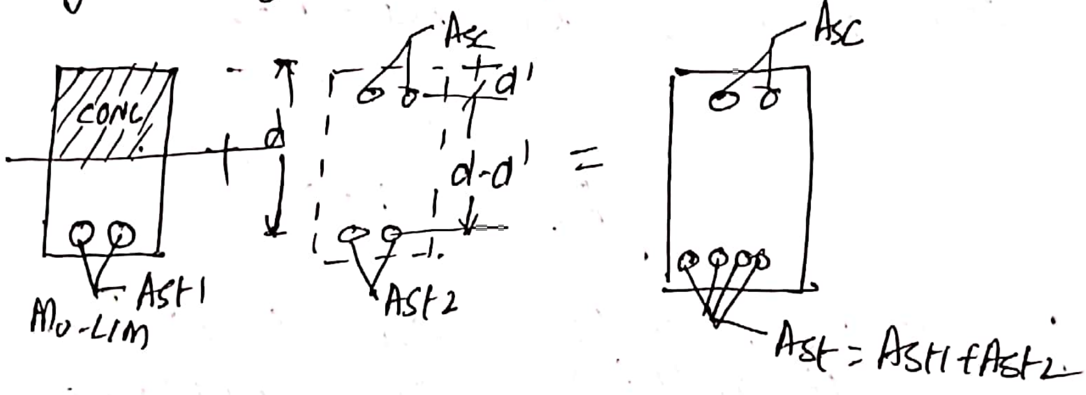
step 4

→ Area of Steel A_{st-1} for Limiting Moment $= M_{u-lim} = 0.87 f_y A_{st1}(d - 0.42 x_u)$

$M_{ud} \Rightarrow A_{st-1} = \frac{140.22 \times 10^6}{0.87 \times 415 (470 - 0.42 \times 0.48 \times 470)}$ (x_{umax})

$\therefore A_{st-1} = 1035 mm^2$

→ Remaining Moment $(M_{ud} - M_{ulim})$ is taken up by Comp. steel and additional tensile steel A_{st2} by forming a lever arm $(d-d')$



step 5 Area of Comp. Steel 'A_{sc}':

$(M_{ud} - M_{ulim}) = f_{sc} \cdot A_{sc}(d - d')$ —— (1)

where f_{sc} corresponding to a strain $\varepsilon = 0.0035 \frac{(x_{umax} - d')}{x_{umax}}$

$\varepsilon = \frac{0.0035(225.6 - 30)}{225.6} = 0.003$

from stress-strain curve IS456 pg 70 $f_{sc} = 0.85 f_y$

subst in eqn (1)

SHAISTA BEGUM
M.Tech (Structure)
Associate Professor

Scanned with CamScanner

$(183.75-140.72)10^6 = 0.85 \times 415\, A_{sc}\,(470-30)$

$\Rightarrow \boxed{A_{sc} = 281\, mm^2}$

use 12mm ϕ bar $\Rightarrow a_{sc} = \frac{\pi (12)^2}{4} = 113.10$

$\therefore$ No. of bars $= \frac{281}{113.1} = 3$ bars

step6 Area of additional Steel Ast-2.

$A_{st-2} = \frac{A_{sc} \cdot f_{sc}}{0.87 f_y} = \frac{0.85 f_y \times A_{sc}}{0.87 f_y} = 274\, mm^2$

step7

$\therefore$ Total tension Steel $A_{st} = A_{st1} + A_{st2}$

$= 1035 + 274$

$\boxed{A_{st} = 1309\, mm^2}$

~~provide 20mm ϕ bars $\Rightarrow a_{st_1} = \frac{\pi}{4} \times (20)^2 = 314.16$~~
~~and 16 mm ϕ bars $\Rightarrow a_{st_2} = \pi (16)^2/4 = 201.06$~~
~~$\therefore$ No. of bars $= n_1 = \frac{1309}{(314.16+201.6)} = 3$ bars~~

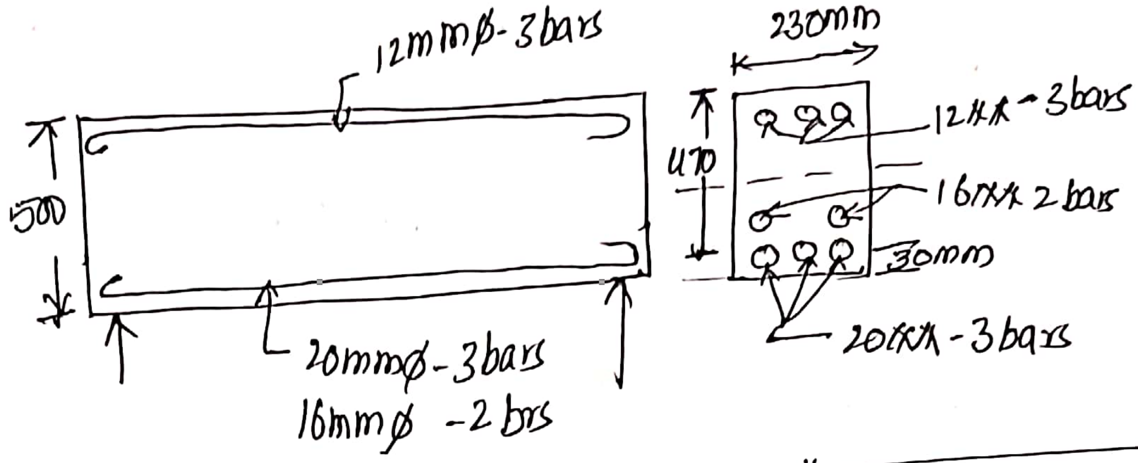
~~$n_2 =$~~

no: of 20mm ϕ bars - 3 $\Rightarrow A_{st} = \frac{3\pi(20)^2}{4} = 942.477^2$

Remaing area of steel $= 1309 - 942.48 = 366.52\, mm^2$

Provide 16mm ϕ bars, $n = \frac{366.52}{\frac{\pi (16)^2}{4}} = 2$ bars

$\therefore$ 20mm ϕ - 3 bars
16mm ϕ - 2 bars } ~~Ans~~



#2. Design a DRB of size 350 mm breadth and 700mm deep to the centre of reinforcement, subjected to a BM of 400kN.m use M_{20} grade concrete and Fe415 grade steel.

Sol step 1 Given: $b = 350mm$; $d = 700mm$; $d' = 30mm$

$M = 400kNm$; $f_{ck} = 20N/mm^2$; $f_y = 415N/mm^2$

Step 2) $M_{ud} = 1.5 \times 400 = 600kN.m$

$M_{ulim} = 0.36 f_{ck} \cdot b \cdot x_{umax} (d - 0.42 x_{umax})$

$\Rightarrow x_{umax} = 0.48 \times 700 = 336mm$

$\therefore M_{ulim} = 0.36 \times 20 \times 350 \times 336 (700 - 0.42 \times 336)$

$= 473.21 kN.m$

Step 3 $M_{ud} > M_{ulim}$ $\therefore$ DRB.

SHAISTA BEGUM
M.Tech (Structure)
Associate Professor

Scanned with CamScanner

Step 5 Area of Steel in tensile zone A_{st_1}

$$A_{st_1} = \frac{M_{ulim}}{0.87 f_y (d - 0.42 x_{umax})} = \frac{473.21 \times 10^6}{0.87 \times 415 (700 - 0.42 \times 336)}$$

$$A_{st_1} = 2345.13 \text{ mm}^2$$

Step 6 Area of Steel in Comp. zone A_{sc}

$$(M_{ud} - M_{ulim}) = \sout{0.6}\ f_{sc} (A_{sc}) (d - d')$$

$$(600 - 473.21) \times 10^6 = 0.85 \times 415 \times A_{sc} (700 - 30)$$

$$\Rightarrow A_{sc} = 536.5 \text{ mm}^2$$

provide 16mm φ bars, $n = \frac{536.5}{\pi (16)^2 / 4} = 3$ bars

3-16mm φ or **5-12mm φ**

Step 7 Area of additional Reinforcement A_{st_2}

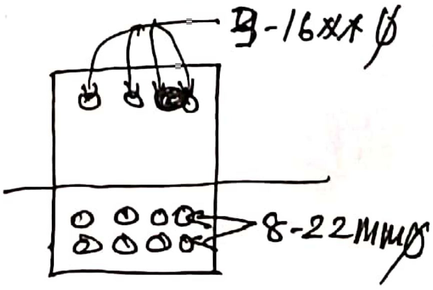
~~$(M_{ud} - M_{ulim})$~~ $= 0.87 f_y A_{st_2} = f_{sc} \cdot A_{sc}$

$$\Rightarrow A_{st_2} = \frac{f_{sc} A_{sc}}{0.87 f_y} = \frac{0.85 \times 415 \times 536.5}{0.87 \times 415}$$

$$A_{st_2} = 524.16 \text{ mm}^2$$

$$A_{st} = 2345.13 + 524.16 = 2869.3 \text{ mm}^2$$

provide 22mm φ bars, $n = \frac{2869.3}{\pi (22)^2 / 4} = 8$ bars



ANALYSIS OF DOUBLY Reinforced Beams:

→ Determining the load carrying capacity or Moment Capacity (M_{ulim}) of the beam is known as Analysis.

Steps for the Analysis of a DRB

1) Given: Size b×d of beam, Area of tensile Steel ($A_{st} = A_{st1} + A_{st2}$) and area of Compression Steel A_{sc}, grade of steel and grade of concrete.

2) Find Area of Steel A_{st2} corresponding to A_{sc} and obtain $A_{st1} = A_{st} - A_{st2}$. ⇒ $0.87 f_y A_{st2} = f_{sc} A_{sc}$

3) Moment Capacity of a DRB = $M_{R1} + M_{R2}$

$$= \underset{M_{ulim}}{0.36 f_{ck} b x_{umax}(d - 0.42 x_{umax})} + \underset{(M_u - M_{ulim})}{f_{sc} A_{sc}(d - d')}$$

(or) $0.87 f_y A_{st}(d - 0.42 x_{umax})$

4) Find x_u for a given size of beam b×d and A_{st1} and compare with $x_{u\text{-}max}$. ⇒ C = T

5) If $x_u < x_{umax}$ ⇒ $M_{R1} = 0.87 f_y A_{st}(d - 0.42 x_u)$

If $x_u > x_{u\text{-}max}$ ⇒ $M_{R1} = 0.36 f_{ck} b x_u (d - 0.42 x_u)$

6) Substitute M_{R1} as step (3) and obtain Moment Capacity

SHAISTA BEGUM
M.Tech (Structure)
Associate Professor

Scanned with CamScanner

PROBLEMS:

A RCB has a section of 300mm × 600mm. The tension steel consists of 4-20mmφ and compn steel 4-12mmφ. A clear cover of 30mm is provided. Determine the ultimate moment capacity of the section. Use M20 & Fe415.

Sol Step 1: Given: $b = 300mm$; $D = 600mm$; $A_{st} = \frac{4\pi(20)^2}{4} = 1256mm^2$

$A_{sc} = \frac{4\pi(12)^2}{4} = 452mm^2$; $f_{ck} = 20N/mm^2$; $f_y = 415N/mm^2$

→ Eff. cover on tension face $= 30 + \frac{20}{2} = 40mm \Rightarrow d = 600-40 = 560mm$

→ eff. cover on compn face $= 30 + \frac{12}{2} = 36mm$ (d')

Step 2 Ultimate moment capacity of beam $= M_{R1} + M_{R2}$

$= 0.36 f_{ck} b x_u (d - 0.42 x_u) + f_{sc} A_{sc} (d - d')$

or

$0.87 f_y A_{st} (d - 0.42 x_u)$

Step 3: To find A_{st1} & A_{st2}

$$A_{st2} = \frac{0.85 f_y \cdot A_{sc}}{0.87 f_y} = \frac{0.85 \times 415 \times 452}{0.87 \times 415} = 442mm^2$$

Step 4 Area of steel for a balanced section A_{st-1}

$$A_{st1} = A_{st} - A_{st2} = 1256 - 442 = 814mm^2$$

Step 5 Depth of Actual NA

$$x_u = \frac{0.87 f_y A_{st1}}{0.36 f_{ck} \cdot b} = \frac{0.87 \times 415 \times 814}{0.36 \times 20 \times 300} = 136mm$$

Scanned with CamScanner

$\Rightarrow x_{umax} = 0.48d = 0.48 \times 560 = 268.8mm$

$\Rightarrow x_u < x_{umax} \Rightarrow$ Steel yields first

$\Rightarrow M_{ulim} = 0.87 f_y A_{st1} (d - 0.42 x_u) + f_{sc} A_{sc} (d - d')$

$= 0.87 \times 415 \times 814 (560 - 0.42 \times 136) + (0.85 \times 415 \times 452 (560 - 36))$

$= (147.8 + 83.55) \times 10^6$ N-mm

$M_{ulim} = 231.38 \times 10^6$ N-mm $= 231.38$ kN-m

Ass A DRB 230mm wide has a depth of 450mm. It is provided with 4-20mm ϕ as tensile steel and 4 bars of 16mm ϕ as compression steel at an Eff. Cover of 40mm. Find the ultimate MR of the beam section. Assume M20 & Fe415 grades.

Sol $\Rightarrow A_{st2} = 786mm^2$; $A_{st1} = 470mm^2$

$\Rightarrow x_u = 102.47$ mm; $x_{umax} = 196.8$ mm

$x_u < x_{um}$ $\therefore$ Steel yield first

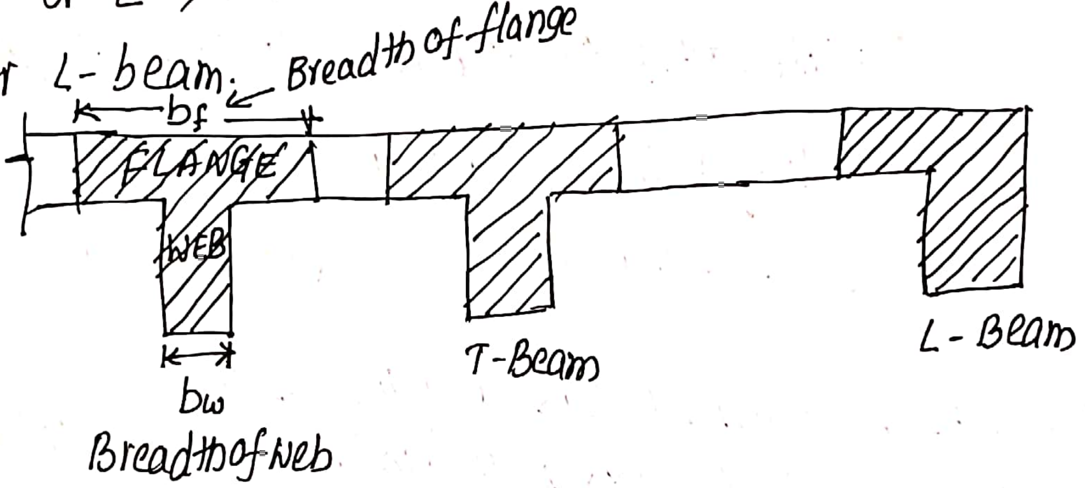
$\Rightarrow$ $M_{ulim} = 167.24$ kN-m

SHAISTA BEGUM
M.Tech (Structure)
Associate Professor

Scanned with CamScanner

T-BEAM and L-BEAM (Analysis & DESIGN)

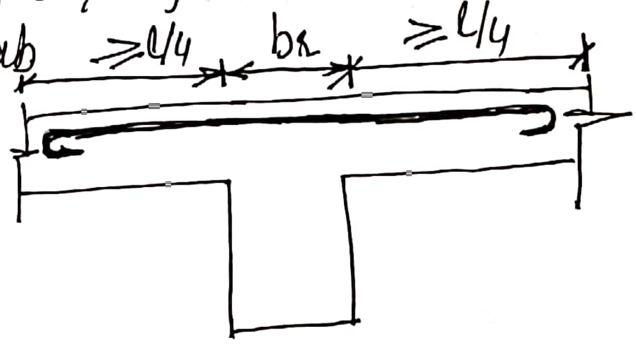
Def: When ever slab and beam are casted monolithi-cally, a part of the slab act along with the beam. The cross-section of the beam is similar to the 'T' or 'L'; such beams are known as T-beams or L-beam.



As per IS 456 (Pg 36), a T-beam or L-beam shall satisfy the following conditions:

@ The slab shall be cast integrally with the web

b) If the main reinforcement of the slab is parallel to the beam, transverse reinforcement shall be provided as shown below. Such reinforcement shall not be less than 60% of the main reinforcement at midspan of the slab



Scanned with CamScanner

Effective Width of Flange :

The Eff. width of flange may be taken as the following, but it should not be greater than the breadth of the web plus half the clear distance to the adjacent beams on either side. ($b_f \ngtr b_w + \frac{l}{2}$)

a) For T-beams, $b_f = \frac{l_0}{6} + b_w + 6D_f$

b) For L-beams, $b_f = \frac{l_0}{12} + b_w + 3D_f$

c) For Isolated beams, the flange width shall be obtained as

T-beam – $b_f = \frac{l_0}{\left(\frac{l_0}{b}\right) + 4} + b_w$

L-beam – $b_f = \frac{0.5\, l}{\left(\frac{l_0}{b}\right) + 4} + b_w$

where b_f = Eff. width of flange

l_0 = dist. b/w points of zero moments

b_w = breadth of web

D_f = thickness of flange and

b = actual width of flange

Note: For continuous beams and frames, 'l_0' may be assumed as 0.7 times the effective span.

SHAISTA BEGUM
M.Tech (Structure)
Associate Professor

Scanned with CamScanner

Moment of Resistance of Flanged Beams

→ When $\frac{D_f}{d} \leq 0.2$, The M.R may be obtained from the following equation

$$M_u = 0.36 \frac{x_{umax}}{d}\left(1 - 0.42 \frac{x_{umax}}{d}\right) f_{ck}\, b_w d^2 + 0.45 f_{ck} (b_f - b_w) D_f \left(d - \frac{D_f}{2}\right)$$

→ When $\frac{D_f}{d} > 0.2$ the M.R may be obtained from the Equation

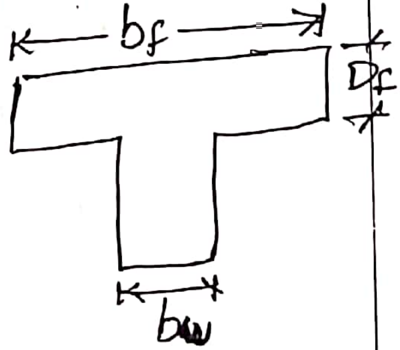
$$M_u = 0.36 \frac{x_{umax}}{d}\left(1 - 0.42 \frac{x_{u\text{-}max}}{d}\right) f_{ck} \cdot b_w \cdot d^2 + 0.45 f_{ck} (b_f - b_w) y_f \left(d - \frac{y_f}{2}\right)$$ where

$$y_f = (0.15 x_u + 0.65 D_f) \not> D_f$$

Note:- Lever arm in T-beam is taken as $\left(d - \frac{D_f}{2}\right)$

problem:

\# Particulars of T-beam are given below.



step 1. Flange width $b_f = 1250$ mm

Effective depth $d = 450$ mm

Rib width $b_w = 230$ mm

Flange thickness $D_f = 110$ mm

Ultimate Moment $M_u = 1000$ kN-m

Find the Steel Reinforcement Required. use M_{20} + Fe415 grades

Sol →
Step 2 $\frac{D_f}{d} = \frac{110}{450} = 0.24 > 0.2$

$\Rightarrow$ $M_{ULIM} = 0.36 \frac{x_{umax}}{d}\left(1 - 0.42\frac{x_{umax}}{d}\right) f_{ck} b_w d^2 + 0.45 f_{ck} (b_f - b_w) y_f \left(d - \frac{y_f}{2}\right)$

where $y_f = 0.15 x_u + 0.65 D_f$

$= 0.15 \times \frac{0.48 \times 450}{d} + 0.65 \times 110$

$= 103.9\,mm \not> D_f\ (=110\,mm)$.

Substituting in $M_{U\text{-}LIM}$

$M_{ULIM} = 0.36 \times 0.48 (1 - 0.42 \times 0.48) \times 20 \times 230 \times 450^2 + 0.45 \times 20 (1250 - 230)\, 103.9 \left(450 - \frac{103.9}{2}\right)$

$M_{ULIM} = 508.17\,kN\text{-}m$

Step 3. Steel required A_{st1} for M_{ULIM}

$M_{U\text{-}LIM} = 0.87 f_y A_{st1} (d - y_f/2)$

$508.17 \times 10^6 = 0.87 \times 415 \times A_{st1} \left(450 - \frac{103.9}{2}\right)$

$\Rightarrow \boxed{A_{st1} = 3536\,mm^2}$

Step 4
Compression Steel A_{sc} for remaining moment

$M_U - M_{ULIM} = f_{sc} A_{sc} (d - d')$

SHAISTA BEGUM
M.Tech (Structure)
Associate Professor

Scanned with CamScanner

$(1000 - 508.17) \times 10^6 = 0.85 \times 415 \times A_{sc}(450-30)$

$\Rightarrow \boxed{A_{sc} = 3320 mm^2}$

Step 5

Additional tensile steel A_{st2} f_{sc}

$A_{st_2} = \frac{f_{sc} A_{sc}}{0.87 f_y} = \frac{0.85 \times 3320 \times 415}{0.87 \times 415}$

$\boxed{A_{st-2} = 3244 mm^2}$

Total tensile steel $A_{st} = A_{st1} + A_{st2}$

$= 3536 + 3244$

$= 6780 mm^2$

→ Provide 30mm ϕ bars

No. of bars on tension face $= \frac{6780}{\frac{\pi (30)^2}{4}} = 10$ bars

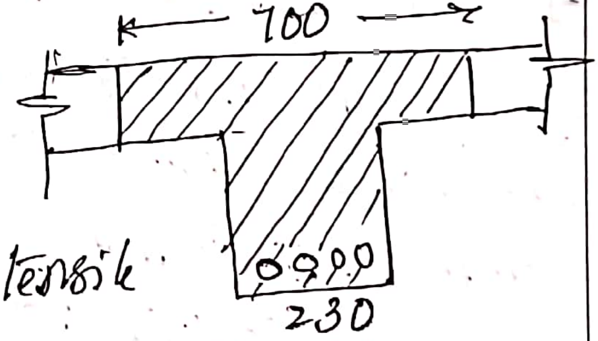
No. of bars on Comp. face $= \frac{3320}{\frac{\pi (30)^2}{4}} = 5$ bars

Scanned with CamScanner

An RCC T-beam of eff. depth 400mm has a flange width of 700mm. The beam is reinforced with 4 bars of 30mm dia. Tor steel bars. width of the web is 230mm and thickness of flange is 80mm. Determine M.R of beam.

sol $d = 400mm$; $b_f = 700mm$; $A_{st} = 4\pi \times \frac{30^2}{4}$
$= 2827 mm^2$

use M_{20} concrete & Fe_{415} steel



step: $\frac{D_f}{d} = \frac{80}{400} = 0.2$

→ Depth of Actual NA x_u:

equaling compressive and tensile forces:

$0.36 f_{ck} \cdot b_w \cdot x_u + 0.45(b_f - b_w) D_f \cdot f_{ck} = 0.87 f_y A_{st}$

$0.36 \times 20 \times 230\, x_u + 0.45(700-230) 80 \times 20 = 0.87 \times 415 \times 2827$

$\Rightarrow \boxed{x_u = 412 mm}$

⇒ Depth of critical N.A $x_{umax} = 0.48d = 0.48 \times 400$

$\boxed{x_{umax} = 192 mm}$

SHAISTA BEGUM
M.Tech (Structure),
Associate Professor

Scanned with CamScanner

→ $x_u > x_{umax}$ ⇒ Concrete will crush earlier than steel.

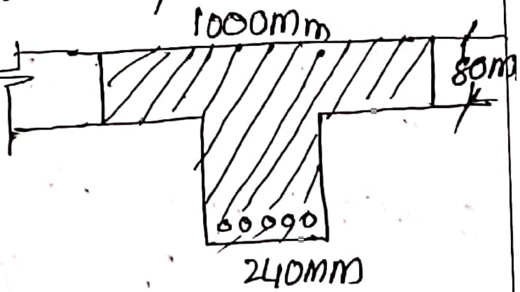
⇒ $M_{ulim} = 2.76\, b_w d^2 + 0.45 f_{ck} (b_f - b_w) D_f \left(d - \frac{D_f}{2}\right)$

$= 2.76 \times 230 \times 400^2 + 0.45 \times 20 (700 - 230) 80 \left(400 - \frac{80}{2}\right)$

$= 101.56 \times 10^6 + 121.52 \times 10^6$

$\therefore M_{ulim} = 223.38 \times 10^6$ N-mm

An isolated T-beam has a flange width 1000mm, flange thickness of 80mm and eff. depth of 400mm. The rib is 240mm wide and reinforced with 5 bars 20mm Ø. Determine the MR of section if M20 grade concrete and Fe415 steel are used. The beam is simply supported over a span of 4m.



Sol: Given:

$b_f = 1000mm$; $D_f = 80mm$

$d = 400mm$; $b_w = 240mm$

$A_{st} = \frac{5\pi(20)^2}{4} = 1570.8 mm^2$

M_{20} concrete & Fe 415 steel; $\frac{D_f}{d} = \frac{80}{400} = 0.2$

→ Depth of Actual N.A x_u

$$C_u = T_u$$

$$0.36 f_{ck} \cdot b_w \cdot x_u + 0.45 f_{ck} (b_f - b_w) D_f = 0.87 f_y A_{st}$$

$$(0.36 \times 20 \times 240 \times x_u) + (0.45 \times 20 (1000-240) 80) = 0.87 \times 415 \times 1570.8$$

$$\Rightarrow \boxed{x_u = -11.37\,mm}$$

→ Limiting depth of NA $x_{u\,max} = 0.48d = 0.48 \times 400$

$$\boxed{x_{u\,max} = 192\,mm}$$

⇒ $x_u < x_{u\,max}$, steel yields ~~first~~, under reinforced

$$M_{u\,lim} = 0.87 f_y A_{st} \left(d - \frac{D_f}{2}\right)$$

$$= 0.87 \times 415 \times 1570 \left(400 - \frac{80}{2}\right) = 204 \times 10^6\ N\text{-}mm$$

∴ ~~$M_{u\,lim}$ = 204240~~

$$\boxed{M_{u\,lim} = 204\ kN\text{-}m}$$

SHAISTA BEGUM
M.Tech (Structure)
Associate Professor

Scanned with CamScanner

I. Moment of Resistance when $x_u < D_f$:

→ If $x_u < D_f$, the M.R can be found by treating the beam to be a DRB of width b_f.

Again if $x_u < x_{umax}$, the section is underreinforced and M.R is given by

$$M_u = 0.36 f_{ck} \cdot b_f \cdot x_u (d - 0.42 x_u) + f_{sc} \cdot A_{sc}(d-d')$$

when ~~$x_u = x_{umax}$~~ $x_u = D_f$, we get

$$M_{u1} = 0.36 f_{ck} b_f D_f (d - 0.42 x_u) + f_{sc} \cdot A_{sc} (d-d')$$

If $x_u = x_{umax}$, we have a balanced section, for which

$$M_{ulim} = 0.36 f_{ck} \cdot x_{umax} b_f (d - 0.42 x_u) + f_{sc} A_{sc} (d-d')$$

If $x_u > x_{u-max}$, the section is over Reinforced and M.R will be limited to M_{ulim} as given above

II M.R when $x_u > D_f$

(a) when $\frac{D_f}{d} \leq 0.2$

$$M_{ulim} = 0.36 \left(\frac{x_{umax}}{d}\right) (1 - 0.42 x_{umax}) f_{ck} b_w d^2 + 0.45 (b_f - b_w) f_{ck} D_f (d - D_f/2) + f_{sc} A_{sc} (d-d')$$

Scanned with CamScanner

b) When $\frac{D_f}{d} > 0.2$

$$M_{u lim} = 0.36\left(\frac{x_{u max}}{d}\right)\left(1 - 0.42\frac{x_{u max}}{d}\right) f_{ck} b_w d^2 + 0.42 f_{ck}(b_f - b_w) y_f \left(d - \frac{y_f}{2}\right) + f_{sc} \cdot A_{sc}(d - d')$$

PART–A

1) What are the limiting values of depth of N-A for different grades of steel.

Ans $\frac{x_{u-lim}}{d} = 0.53 \rightarrow$ M.S

$= 0.48 \rightarrow$ Fe415

$= 0.46 \rightarrow$ Fe500

2) What are the min. & max. percentage of steel to be provided in beam in tension.

sol $\frac{A_{s min}}{bd} = \frac{0.85}{f_y}$ & $A_{st max} = 0.04bD$.

3. What is Critical N.A ?

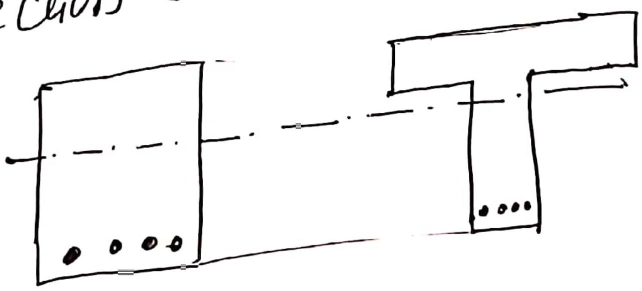
Ans: The depth of N.A of a balanced section is termed as Critical N.A.

4.

Scanned with CamScanner

FLANGED SECTION

INTRODUCTION: ~~We have seen that~~ the concrete below N.A does not resist any BM but simply serves to embed the tensile steel. Also the portion of the concrete above N.A carries only very little compressive force since the intensity of compressive stress there is of a very small magnitude. This suggests that the section of the beam should be such that it has greater width at the top (comp side) in comparision to width below NA. Such section is known as T-section (T-beam)



SHAISTA BEGUM
M.Tech (Structure)
Associate Professor

Scanned with CamScanner