UNIT - IV

II.] LAPLACE TRANSFORM APPLICATIONS FOR NETWORKS *

let $f(t)$ be a fun^c of time 't'

$$\boxed{F(s) = L\{f(t)\} = \int_{0^-}^{\infty} e^{-st} f(t)\, dt} \longrightarrow (1)$$

$$s = \sigma + j\omega$$

σ ↓ Neper freq (Np/sec)

$j\omega$ ↓ radian freq (Rad/sec)

$\omega \rightarrow$ no. of oscillations

$\sigma \rightarrow$ magnitude of oscillations

Conditions for existentence of L.T :-

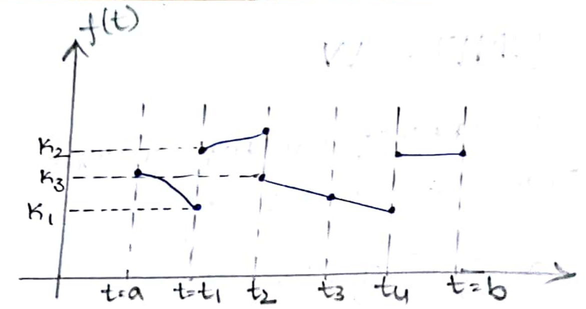
If $f(t)$ is a fun^c of 't'

I, $f(t)$ must be piece-wise continous

II $f(t)$ must be exponential order.

→ let $f(t)$ be a fun^c defined in 't' in the $[a, b]$ such that a, b are divided into sub intervals

→ In each subinterval $f(t)$ must be continous.



fig(a)

2) $f(t)$ is said to be exponential order iff $\boxed{\lim_{t\to\infty} e^{-st} f(t) = 0}$

<u>Concept of t^+ & t^-</u> :-

* → At $t = t_1^-$ (instant before step) $\Rightarrow f(t) = K_1$
  $t = t_1^+$ (instant after step) $\Rightarrow f(t) = K_2$

* → At $t = t_3 \Rightarrow f(t) = K_3$
  $t = t_3^+ = t_3^- \Rightarrow f(t) = K_3$

From eqn ①; $\boxed{t = 0^+ = 0^- = 0}$

→ If $f(t)$ is continuous at $t = 0$ then

→ If $f(t)$ is discontinuous at $t=0$ then $\boxed{t=0^+ \neq t=0^-}$

Some <u>Important</u>

1) $L[K] = \frac{K}{s}$;

2) $L[e^{-at}] = \frac{1}{s+a}$;

3) $L[e^{at}] = \frac{1}{s-a}$

4) $L[\sin at] = \frac{a}{s^2+a^2}$

5) $L[\cos at] = \frac{s}{s^2+a}$

6) $L[t^n] = \frac{\Gamma(n+1)}{s^{n+}}$

If n is an in

i.e $L[t^n] =$

7) $L[\sinh(at)] =$

8) $L[\cosh(at)] =$

9) If $L[f_1(t)] =$

$L[K_1 f_1(t)$.

10) If $L[f(t)]$

$L[e^{at} f($

11) If $L[f(t)]$.

$L[a$

Scanned by CamScanner

Important Formulae:-

1) $L[k] = \frac{k}{s}$; $s > 0$

2) $L[e^{-at}] = \frac{1}{s+a}$; $s > -a$

3) $L[e^{at}] = \frac{1}{s-a}$; $s > a$

4) $L[\sin at] = \frac{a}{s^2+a^2}$; $s > 0$

5) $L[\cos at] = \frac{s}{s^2+a^2}$; $s > 0$

6) $L[t^n] = \frac{\Gamma(n+1)}{s^{n+1}}$; $s > 0$

If n is an integer, $\Gamma(n+1) = n!$

ie $L[t^n] = \frac{n!}{s^{n+1}}$; $s > 0$

7) $L[\sinh(at)] = \frac{a}{s^2-a^2}$; $s > |a|$

8) $L[\cosh(at)] = \frac{s}{s^2-a^2}$; $s > |a|$

9) If $L[f_1(t)] = F_1(s)$ & $L[f_2(t)] = F_2(s)$ then

$L[k_1 f_1(t) + k_2 f_2(t)] = k_1 F_1(s) + k_2 F_2(s)$

10) If $L[f(t)] = F(s)$ then

$L[e^{at} f(t)] = F(s-a)$

12.) If $L[f(t)] = F(s)$

$$L[f(at)] = \frac{1}{|a|} F\left(\frac{s}{a}\right)$$

***

13.) $L[t^n f(t)] = (-1)^n \frac{d}{ds^n} F(s)$

14.) $f(0^+) = \underset{s\to\infty}{Lt}\ sF(s)$ → initial value th.

***

15.) $L\left[\frac{d}{dt} f(t)\right] = sF(s) - f(0^-)$

16.) If $f(t) = f(t+T)$ → periodic signal

$$L[f(t)] = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t) \cdot dt$$

17.) If $L[f(t)] = F(s)$

$f(\infty) = \underset{s\to 0}{Lt}\ sF(s)$ → Final value th.

***

18.) If $f^n(t) = \frac{d^n}{dt^n} f(t)$ then;

$$L[f^n(t)] = s^n F(s) - s^{n-1} f(0^-) - s^{n-2} f'(0^-) - \cdots$$
$$\cdots - sf'(0^-) - f(0^-)$$

19.) If $L[f(t)] = F(s)$ then

$$L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s) \cdot ds$$

***

20.) If $L[f(t)] = F(s)$ then

$$L\left[\int_0^t f(t) \cdot dt\right] = \frac{F(s)}{s} + \frac{f^{-1}(0^-)}{s}$$

where f^{-1} → integral of $f(t)$

Scanned by CamScanner

Find the L.T of $\cosh at \cdot \cos bt$.

$L[\cosh at \cdot \cos bt]$

Sol: $= L\left[\frac{e^{at}+e^{-at}}{2} \cdot \cos bt\right]$

$= \frac{1}{2} L[(e^{at}+e^{-at}) \cos bt]$

$= \frac{1}{2} L[e^{at} \cos bt] + \frac{1}{2} L[e^{-at} \cos bt]$

$= \frac{1}{2}\left[\frac{(s-a)}{(s-a)^2+b^2}\right] + \frac{1}{2}\left[\frac{(s+a)}{(s+a)^2+b^2}\right]$

Find the L.T of $t \cdot e^{-2t} \cdot \sin 3t$

Sol: $L[t \cdot e^{-2t} \cdot \sin 3t]$

$L[f(t)] = L[\sin 3t] = \frac{3}{s^2+9} \Rightarrow F(s)$

$L[t^1 \cdot \sin 3t] = (-1)^1 \frac{d}{ds} F(s)$

$= -1 \frac{d}{ds}\left(\frac{3}{s^2+3^2}\right)$

$= -3 \frac{[(s^2+9)(0) - 1(2s)]}{(s^2+9)^2}$

$= \frac{6s}{(s^2+9)^2}$

$L[e^{-2t} \cdot (t \sin 3t)] = \frac{6(s+2)}{[(s+2)^2+9]^2}$

INVERSE LAPLACE TRANSFORMS [ILT]

If the L.T of a func $f(t)$ is $F(s)$ then I.L.T is applied to convert from frequency domain to time domain & is given as:-

$$L[f(t)] = F(s)$$

$$\boxed{f(t) = L^{-1}[F(s)]}$$

$L^{-1} \to$ ILT operator

Properties:-

① $L^{-1}[k_1 F_1(s) + k_2 F_2(s)] = k_1 f_1(t) + k_2 f_2(t)$

② If $L^{-1}[F(s)] = f(t)$ then $L^{-1}[F(s-a)] = e^{at} f(t)$

③ If $L^{-1}[F(s)] = f(t)$ then $L^{-1}[F(as)] = \frac{1}{a} f\left(\frac{t}{a}\right)$

PARTIAL FRACTIONS

* → $Q(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0$

where $a_n, a_{n-1}, \ldots, a_1, a_0 \to$ real constants

If $\boxed{a_n \neq 0}$ then $Q(s) \to$ polynomial of degree 'n'.

→ If $P(s) = \frac{N(s)}{D(s)}$ $\begin{matrix} \to \text{polynomial of degree 'n'} \\ \to \quad " \quad " \quad \text{'m'} \end{matrix}$

↓

Rational polynomial

proper rational p...

improper Rational ...

→ Every improper ...

can be expresse...

& a proper pol...

→ consider $P(s$...

case-i;- If roots ...

i.e $D(s) = (s-\lambda$...

$\frac{N(s)}{D(s)} = \frac{K_1}{(s-\lambda_1)} + \frac{K}{(s}$...

if $i = 1, 2, \ldots$

$K_i = \frac{N(s)}{D(s}$...

case-ii;- If r...

i.e $D(s) = (s-$...

$\frac{N(s)}{D(s)} = \frac{a_0}{(s-\alpha)^r} + \ldots$

where

$a_j = \frac{1}{j!}$...

→ Every improper rational function can be expressed as the sum of polynomial & a proper polynomial $(n<m)$ → $P(s) = Q + \frac{R}{D}$.

→ consider $P(s) = \frac{N(s)}{D(s)}$ $N(s)$ → 'n' degree, $D(s)$ → 'm' degree $(n<m)$

case-i:- If roots of D(s) are real & distinct:

i.e $D(s) = (s-\lambda_1)(s-\lambda_2)\ldots\ldots(s-\lambda_m)$

$$\frac{N(s)}{D(s)} = \frac{K_1}{(s-\lambda_1)} + \frac{K_2}{(s-\lambda_2)} + \cdots\cdots + \frac{K_m}{(s-\lambda_m)}$$

if $i = 1, 2, \ldots\ldots m$ then

$$\boxed{K_i = \frac{N(s)}{D(s)}.(s-\lambda_i)\Big|_{\text{at } s=\lambda_i}}$$

case-ii:- If roots are real & repeated:

i.e $D(s) = (s-\alpha)^r(s-\lambda_1)(s-\lambda_2)\ldots\ldots(s-\lambda_n)$

$$\frac{N(s)}{D(s)} = \frac{a_0}{(s-\alpha)^r} + \frac{a_1}{(s-\alpha)^{r-1}} + \cdots\cdots + \frac{a_{r-1}}{(s-\alpha)} + \frac{K_1}{(s-\lambda_1)} + \cdots + \frac{K_l}{(s-\lambda_l)}$$

where

$$\boxed{a_j = \frac{1}{j!}\frac{d^j}{ds^j}\left[(s-\alpha)^r P(s)\right]\Big|_{s=\alpha} \quad j = 0,1,2,\ldots r-1}$$

$$\boxed{K_i = P(s)\cdot(s-\lambda_i)\Big|_{s=\lambda_i}}$$

$i = 1, 2, 3 \ldots\ldots c$

Case-iii):- If roots are quadratic in nature & not repeating.

i.e $D(s) = (a_1 s^2 + b_1 s + c_1)(a_2 s^2 + b_2 s + c_2)$

$$\frac{N(s)}{D(s)} = \frac{K_1 s + K_2}{a_1 s^2 + b_2 s + c_1} + \frac{K_3 s + K_4}{a_2 s^2 + b_2 s + c_2}$$

Case-iv):- If roots are quadratic & Repeated

i.e $D(s) = (a_1 s^2 + b_1 s + c_1)^r (a_2 s^2 + b_2 s + c_2)$

$$\frac{N(s)}{D(s)} = \frac{\alpha_r s + \beta_r}{(a_1 s^2 + b_1 s + c_1)^r} + \frac{\alpha_{r-1} s + \beta_{r-1}}{(a_1 s^2 + b_1 s + c_1)^{r-1}} + \ldots\ldots$$

$$\ldots\ldots + \frac{\alpha_1 s + \beta_1}{(a_1 s^2 + b_1 s + c_1)} + \frac{K_1 s + K_2}{(a_2 s^2 + b_2 s + c_2)}$$

Case-v):- If Roots are complex-conjugate

i.e $D(s) = [s-(\alpha_1 + j\beta_1)][s-(\alpha_1 - j\beta_1)][s-(\alpha_2 + j\beta_2)][s-(\alpha_2 - j\beta_2)]$

$$\frac{N(s)}{D(s)} = \frac{K_1}{[s-(\alpha_1 + j\beta_1)]} + \frac{K_1^*}{[s-(\alpha_1 - j\beta_1)]} + \frac{K_2}{[s-(\alpha_2 + j\beta_2)]} + \frac{K_2^*}{[s-(\alpha_2 - j\beta_2)]}$$

where $K_1^* \rightarrow$ complex conjugate of K_1

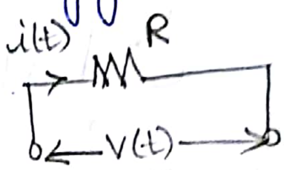
Laplace Transform Application to circuit elements:-

I] Resistor:-

Resistor dissipates the energy

$v(t)$ voltage across 'R'

$i(t)$ current through 'R'



time domain equivalent ckt

According to ohm's law

$$v(t) = R\,i(t)$$

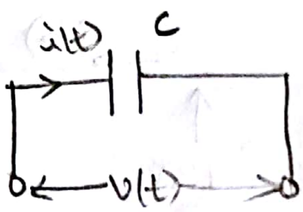
$$L[v(t)] = L[R\,i(t)]$$

$$\boxed{V(S) = R\,I(S)}$$

where $V(s)$ is L.T of $v(t)$

& $I(s)$ is L.T of $i(t)$

II] Capacitor:-



$$i(t) = C\frac{dv(t)}{dt}$$

Apply LT

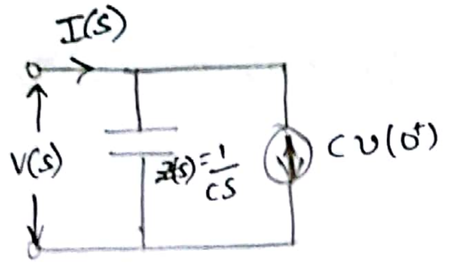
$$I(S) = C\{SV(S) - v(0^+)\}$$

$$I(S) = CSV(S) - Cv(0^+)$$

$$I(S) = \frac{V(S)}{Z(S)} + [-Cv(0^+)]$$

where $Z(S) = \frac{1}{CS}$

Kap



Norton's equivalent ckt in freq. domain

$$v(t) = \frac{1}{C}\int_0^t i(t)\,dt$$

$$i^{-1}(t) = \int_0^t i(t)\,dt = C v(t)$$

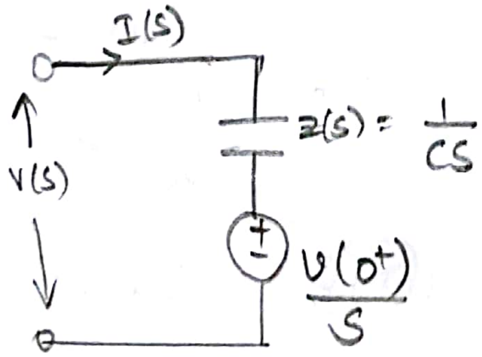
Apply L.T

$$V(s) = \frac{1}{C}\left\{\frac{I(s)}{s} + \frac{i^{-1}(0^+)}{s}\right\}$$

$$V(s) = \frac{I(s)}{Cs} + \frac{\cancel{C}\,v(0^+)}{\cancel{C}s}$$

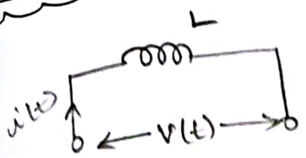
$$V(s) = \underbrace{\boxed{Z(s)\cdot I(s)}}_{V_1} + \boxed{\frac{v(0^+)}{s}}\ V_2$$

where $Z(s) = \frac{1}{Cs}$



Thevenin's equivalent in freq. domain

III. Inductor:-



$$v(t) = L\frac{di(t)}{dt}$$

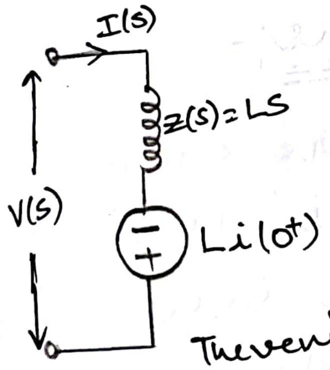
Apply L.T

$$V(s) = L\{sI(s) - i(0^+)\}$$

$$V(s) = sLI(s) - Li(0^+)$$

$$V(s) = \underbrace{Z(s)\cdot I(s)}_{V_1} + \underbrace{[-Li(0^+)]}_{V_2}$$

where $Z(s) = Ls$



Thevenin's equivalent in freq. domain.

Now,

$$i(t) = \frac{1}{L}\left\{\int_0^t v(t)\,dt\right\}$$

$$v^{-1}(t) = \int_0^t v(t)\,dt = Li(t)$$

$$I(s) = \frac{1}{L}\left\{\frac{V(s)}{s} + \frac{v^{-1}(0^+)}{s}\right\}$$

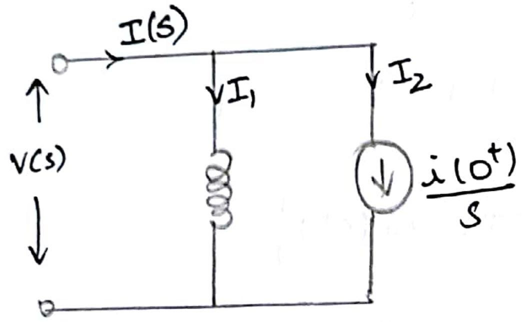
$$I(s) = \frac{V(s)}{sL} + \frac{\cancel{L}i(0^+)}{\cancel{L}s}$$

$$I(s) = \frac{V(s)}{sL} + \frac{i(0^+)}{s}$$

$$I(s) = \underbrace{Y(s) \cdot V(s)}_{I_1} + \underbrace{\frac{i(0^+)}{s}}_{I_2}$$

where

$$Y(s) = \frac{1}{sL} \Rightarrow Z(s) = sL$$



Norton's equivalent in freq. domain.

Initial Conditions:-

* Inductor does not allow sudden changes in current i.e if 'i_0' is initial current through the inductor before the application of voltage source $v(t)$ at $t=0$ i.e $i(0^-) = i_0$. Then, after applying voltage source $v(t)$ at $t=0$ is $\boxed{i(0^+) = i(0^-) = i_0}$

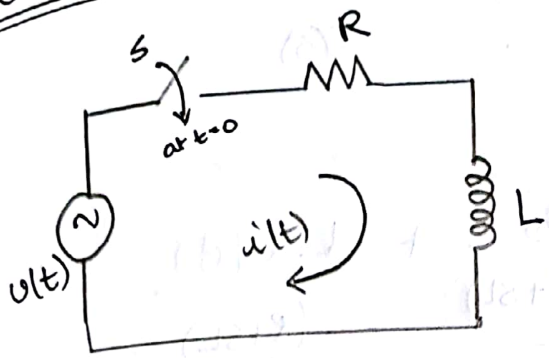
* Capacitor does not allow sudden changes in voltage i.e $\boxed{v(0^+) = v(0^-) = v_0}$

where 'v_0' is initial voltage across capacitor before $v(t)$ is applied.

L.T of $u(t)$ i.e $L[u(t)] = \frac{1}{S}$

" " $\delta(t)$ i.e $L[\delta(t)] = 1$

RL - Series Circuit Analysis :-



let $i(0^-)$ is initial current through inductor

Apply KVL;

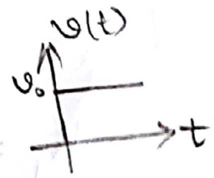
$$v(t) = Ri(t) + L\frac{di}{dt}$$

$$\boxed{V(S) = RI(S) + SLI(S) - i(0^-)L} \rightarrow ②$$

since for inductor: $i(0^+) = i(0^-)$

$$V(S) = I(S)[R+SL] - Li(0^-) \longrightarrow ②$$

A) considering step input :-



$$v(t) = V_0 u(t)$$

$$L[v(t)] = V_0 \{u(t)\}$$

$$\boxed{V(S) = \frac{V_0}{S}}$$

subn in eqn ②

$$\frac{V_0}{S} = I(s)(R+SL) - Li(0^+)$$

$$\frac{\left(\frac{V_0}{S}\right) + Li(0^+)}{R+SL} = I(s)$$

$$I(s) = \frac{V_0}{s(R+SL)} + \frac{Li(0^+)}{(R+SL)}$$

$$I(s) = \frac{(V_0/L)}{s(s+\frac{R}{L})} + \frac{i(0^+)}{(s+\frac{R}{L})} \longrightarrow ③$$

Now consider,

$$\frac{\left(\frac{V_0}{L}\right)}{s(s+\frac{R}{L})} = \frac{K_1}{S} + \frac{K_2}{S+\frac{R}{L}}$$

$$K_1 = \frac{\left(\frac{V_0}{L}\right)}{\cancel{s}(s+\frac{R}{L})} \cdot (\cancel{s}) \Big|_{s=0}$$

$$K_1 = \frac{\left(\frac{V_0}{\cancel{L}}\right)}{\left(\frac{R}{\cancel{L}}\right)} = \frac{V_0}{R}$$

$$K_2 = \frac{\left(\frac{V_0}{L}\right)}{s\cancel{(s+\frac{R}{L})}} \cancel{(s+\frac{R}{L})} \Big|_{s=-\frac{R}{L}} \Rightarrow K_2 = \frac{-V_0}{R}$$

From eqn ③

$$I(s) = \frac{\left(\frac{V_0}{R}\right)}{s} + \frac{\left(-\frac{V_0}{R}\right)}{s+\frac{R}{L}} + \frac{i(0^+)}{s+\frac{R}{L}}$$

Apply ILT

$$i(t) = \frac{V_0}{R} u(t) - \frac{V_0}{R} e^{-\frac{R}{L}t} u(t) + i(0^+) e^{-\frac{R}{L}t}$$

if $i(0^+) = 0$

$$i(t) = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right) u(t)$$

let $\tau = \frac{L}{R}$ → called time constant of RL series N/w

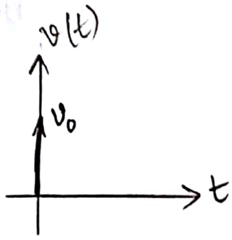
$$i(t) = \frac{V_0}{R}\left(1 - e^{-\frac{t}{\tau}}\right) u(t)$$

if $t = \tau$

$$i(t) = \frac{V_0}{R}\left(1 - e^{-1}\right) u(t)$$

$$\boxed{i(t) = 0.633\,\frac{V_0}{R}}$$

B) Considering Impulse Input :-



$$V(t) = V_0\,\delta(t)$$

$$L[V(t)] = V_0\,L\{\delta(t)\}$$

$$\boxed{V(s) = v_0}$$

sub^n in eq^n 2

$$v_0 = I(s)(R+sL) - Li(0^+)$$

$$I(s) = \frac{[v_0 + Li(0^+)]}{R+sL}$$

$$I(s) = \frac{v_0 + Li(0^+)}{L\left(s+\frac{R}{L}\right)}$$

$$I(s) = \frac{\left[\frac{v_0}{L} + i(0^+)\right]}{\left(s+\frac{R}{L}\right)}$$

Apply ILT

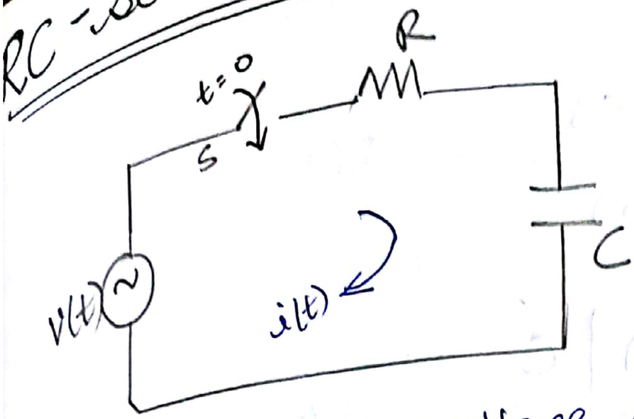
$$i(t) = \left[\frac{v_0}{L} + i(0^+)\right] e^{-\frac{R}{L}t} \cdot u(t)$$

if $i(0^+) = 0$

$$\boxed{i(t) = \frac{V_0}{L} e^{-\frac{t}{\tau}} \cdot u(t)}$$

where $\boxed{\tau = \frac{L}{R}}$ (time const)

RC-Series Circuit :-



$v(0^+)$ is initial voltage across capacitor

Apply KVL

$$v(t) = R\,i(t) + \frac{1}{C}\int_0^t i(t)\,dt$$

Apply LT

$$V(S) = R\,I(S) + \frac{1}{C}\left[\frac{I(S)}{S} + \frac{i^{-1}(0^+)}{S}\right]$$

$$V(S) = R\,I(S) + \frac{I(S)}{CS} + \frac{\cancel{C}\,v(0^+)}{\cancel{C}S}$$

$$V(S) = I(S)\left[R + \frac{1}{CS}\right] + \frac{v(0^+)}{S}$$

$$\left[V(S) - \frac{v(0^+)}{S}\right] = I(S)\left[R + \frac{1}{CS}\right] \longrightarrow 4$$

considering step-Input :-

$$v(t) = V_0\,u(t) \Rightarrow V(S) = \frac{V_0}{S}$$

sub^n in eqn 4

$$\frac{V_0}{s} - \frac{v(0^+)}{s} = I(s)\left[\frac{1+RCs}{Cs}\right]$$

$$\frac{[V_0 - v(0^+)]\,C\cancel{s}}{\cancel{s}(1+RCs)} = I(s)$$

$$\frac{[V_0 - v(0^+)]\,\cancel{C}}{R\cancel{C}\left[s+\frac{1}{RC}\right]} = I(s)$$

$$\therefore I(s) = \left[\frac{V_0 - v(0^+)}{R}\right] \cdot \frac{1}{\left(s+\frac{1}{RC}\right)}$$

Apply ILT

$$i(t) = \left[\frac{V_0 - v(0^+)}{R}\right] e^{-\frac{t}{RC}} \cdot u(t)$$

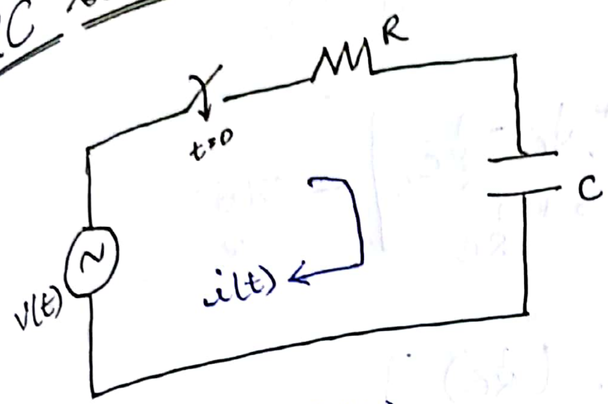
$$i(t) = \left[\frac{V_0 - v(0^+)}{R}\right] \cdot e^{-\frac{t}{\tau}} \cdot u(t)$$

where $\boxed{\tau = RC}$ (time constant)

if $v(0^+) = v(0^-) = 0$

$$\boxed{\therefore i(t) = \frac{V_0}{R} \cdot e^{-t/\tau}\, u(t)}$$

B] RC Series circuit with Impulse Input :-



let $v(t) = V_0 \, \delta(t)$

Apply L.T

$$\boxed{V(s) = V_0} \qquad [\because L\{\delta(t)\} = 1]$$

from eqn ④

$$\left[V(s) - \frac{v(0^+)}{s}\right] = I(s)\left[R + \frac{1}{Cs}\right]$$

$$I(s) = \frac{\left[V_0 - \frac{v(0^+)}{s}\right]}{R + \frac{1}{Cs}}$$

$$I(s) = \frac{V_0}{\left(R + \frac{1}{Cs}\right)} - \frac{v(0^+)}{s\left(R + \frac{1}{Cs}\right)}$$

$$I(s) = \frac{V_0 Cs}{(RCs + 1)} - \frac{v(0^+)\, C\cancel{s}}{\cancel{s}(1 + RCs)}$$

$$I(s) = \frac{V_0 \cancel{C} s}{R\cancel{C}\left[s + \frac{1}{RC}\right]} - \frac{\cancel{C}\, v(0^+)}{R\cancel{C}\left[s + \frac{1}{RC}\right]}$$

$$I(s) = \frac{v_0 s}{R\left(s+\frac{1}{RC}\right)} - \frac{v(0^+)}{R\left(s+\frac{1}{RC}\right)}$$

$$I(s) = \frac{V_0}{R}\left[\frac{s+\frac{1}{RC}-\frac{1}{RC}}{s+\frac{1}{RC}}\right] - \frac{v(0^+)}{R}\cdot\frac{1}{\left(s+\frac{1}{RC}\right)}$$

$$I(s) = \frac{V_0}{R}\left[1 - \frac{\left(\frac{1}{RC}\right)}{\left(s+\frac{1}{RC}\right)}\right] - \frac{v(0^+)}{R}\cdot\frac{1}{\left(s+\frac{1}{RC}\right)}$$

Apply ILT

$$\boxed{i(t) = \frac{V_0}{R}\left[\delta(t) - \frac{1}{RC}e^{-\frac{1}{RC}\cdot t}\right] - \frac{v(0^+)}{R}e^{-\frac{1}{RC}\cdot t}}$$

if $v(0^+) = v(0^-) = 0$

$$\boxed{\therefore i(t) = \frac{V_0}{R}\left[\delta(t) - \frac{1}{RC}e^{-\frac{1}{RC}\cdot t}\right]}$$

Imp

NOTE:-

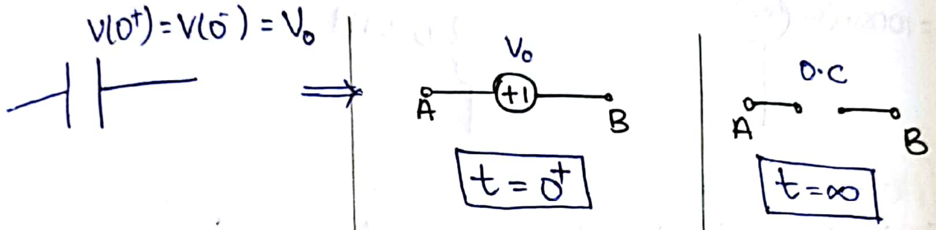
→ Assume if switch 's' is moved from position ① to position ② at $t=0$ then the conditions exist [behaviour of inductor or capacitor] at $t=0^+$ → (instant after switch 's' is moved) are called as "Initial Conditions"

→ One switch is moved the conditions

exist at $t = \infty$ is called "final conditions" (or) "steady state conditions"

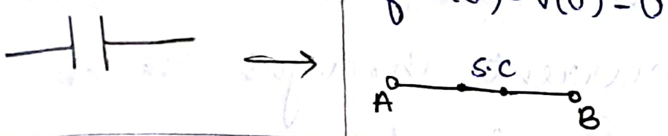
- Capacitor [C]:

$V(0^+) = V(0^-) = V_0$



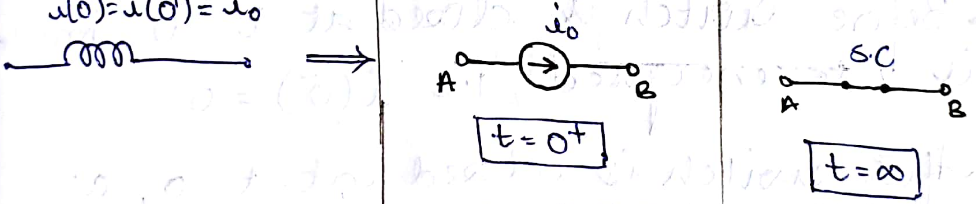
$V(0^+) = V(0^-) = 0$

if $V(0^+) = V(0^-) = 0$



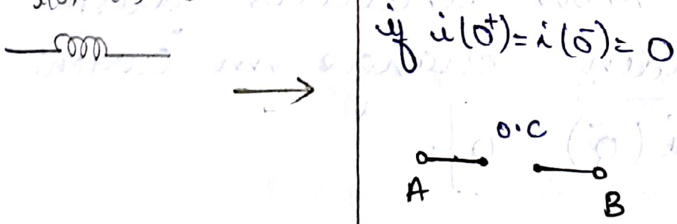
Inductor [L]:-

$i(0^-) = i(0^+) = i_0$



$i(0^-) = i(0^+) = 0$

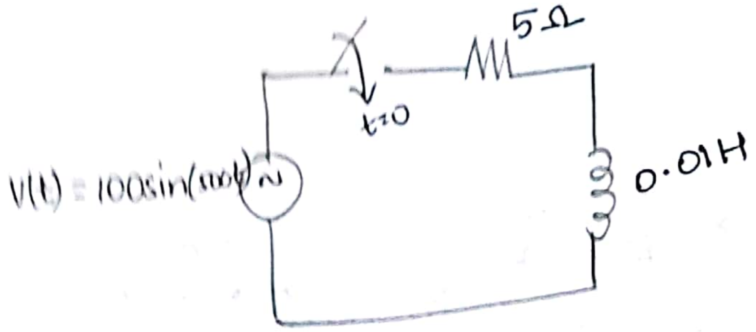
if $i(0^+) = i(0^-) = 0$



- Resistor [R]:-

Problems:

Q1) Consider the RL ckt shown below



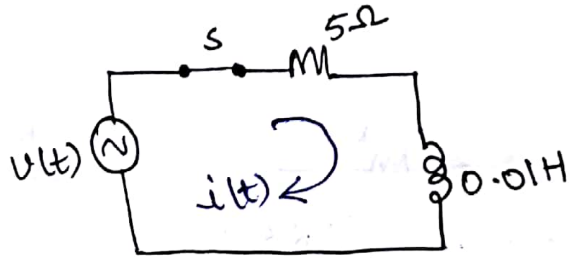
Initially inductor is deenergised & switch 'S' is closed at t=0. Then, calculate current through the ckt after switch is closed.

Sol:-
→ Before switch is closed at t = 0, inductor is de-energised i.e $i(0^-) = 0$

→ After switch is closed at t=0, as already we know inductor doesn't allow sudden changes in "current"

i.e $\boxed{i(0^+) = i(0^-) = 0}$.

→ After switch is closed (the ckt becomes) some current i(t) is flowing



Apply KVL ;

$$v(t) = 5i(t) + L\frac{di(t)}{dt}$$

$$100\sin(500t) = 5i(t) + 0.01\frac{di(t)}{dt}$$

Apply L.T

$$100\left[\frac{500}{s^2+500^2}\right] = 5I(s) + 0.01\left[sI(s) - i(0^+)\right]$$

$$\frac{100\times 500}{s^2+500^2} = 5I(s) + 0.01\,sI(s) - 0$$

$$I(s)\left[0.01s+5\right] = \frac{5\times 10^4}{s^2+500^2}$$

$$I(s) = \frac{5\times 10^4}{(s^2+500^2)(0.01s+5)}$$

0.01 common.

$$I(s) = \frac{5\times 10^4\times 10^2}{(s^2+500^2)(s+500)}$$

$s^2+500^2=0$

$s^2=-500^2$

$s=\sqrt{-500^2}$

$\boxed{s=\pm j500}$

$$I(s) = \frac{5\times 10^6}{(s+500)(s+j500)(s-j500)}$$

P.F

$$\therefore I(s) = \frac{k_1}{(s+500)} + \frac{k_2}{(s+j500)} + \frac{k_2^*}{(s-j500)}$$

using case-① ($\alpha=0$)

↳ case-⑤

$$\left.\frac{N(s)}{D(s)}(s+500)\right|_{s=-500}$$

$$K_1 = \frac{5\times10^6}{\cancel{(s+500)}(s^2+500^2)} \times \cancel{(s+500)} \Bigg|_{s=-500}$$

$$K_1 = \frac{5\times10^6}{(500^2+500^2)}$$

$$\boxed{K_1 = 10}$$

Now; $K_2 = \dfrac{5\times10^6}{(s+500)(s-j500)} \Bigg|_{s=-j500}$

$$= \frac{5\times10^6}{(500-j500)(-2j\times500)}$$

$$= \frac{5\times10^6}{500(1-j)\times(-1000j)} \qquad -\frac{1}{j} = j$$

$$= \frac{j\,5\times10^6}{500(1-j)\times1000}$$

$$= \frac{j5\times10^6}{5\times10^5(1-j)} \times \frac{1+j}{1+j}$$

$$\boxed{K_2 = -5+5j}$$

$$\therefore \boxed{K_2^* = -5-5j}$$

Now, using ILT

$$\rightarrow I(s) = \frac{10}{s+500} + \frac{-5+5j}{(s+j500)} + \frac{-5-5j}{s-j500}$$

Apply ILT

$$i(t) = 10e^{-500t} + (-5+5j)\,e^{-j500t} + (-5-5j)\,e^{j500t}$$

MRD by '2' MRD by 2j

$$i(t) = 10e^{-500t} - 2\times5\left[\frac{e^{-j500t} + e^{j500t}}{2}\right] - 2j\times5j\left[\frac{e^{j500t} - e^{-j500t}}{2j}\right]$$

$$\boxed{i(t) = 10e^{-500t} - 10\cos 500t + 10\sin 500t}$$

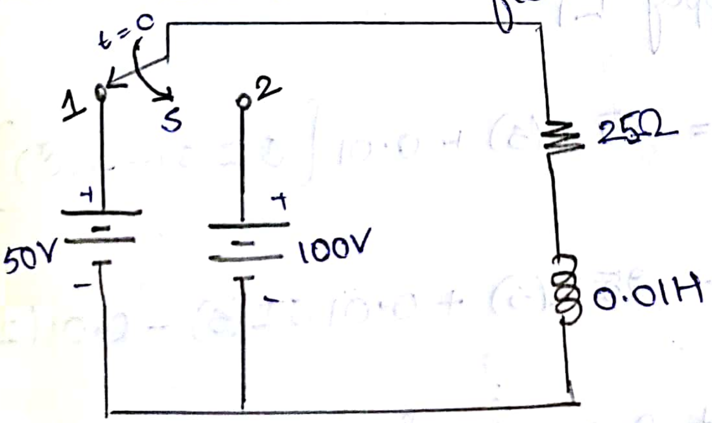
$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$

$\sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}$

$e^{j\theta} = \cos\theta + j\sin\theta$

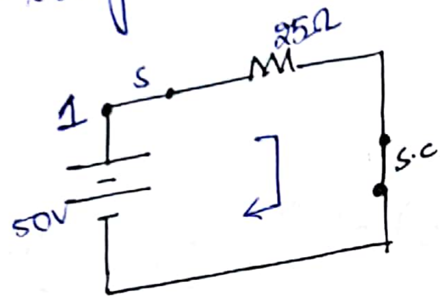
$e^{-j\theta} = \cos\theta - j\sin\theta$

2.) In the series RL ckt shown below, switch 'S' is at position '1' for a long time to establish steady state condition & at t=0 switch is moved from position ① to pos. ②. Find the resultant current after switch is moved.



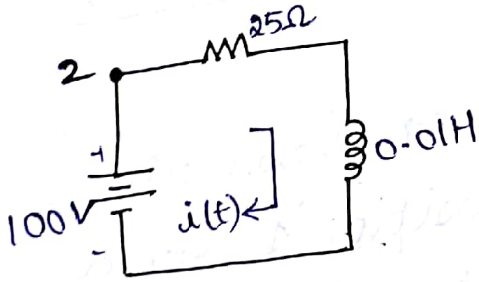
Sol:-

→ If the switch is at position '1' for a long time then the ckt is:



$i(0^-) = \frac{\cancel{50}}{\cancel{25}} \Rightarrow 2A$

→ After switch is moved to position '2' then
$i(0^+) = i(0^-) = 2A$



Apply KVL;

$$100 = 25\,i(t) + 0.01\,\frac{di(t)}{dt}$$

Apply LT

$$\frac{100}{s} = 25\,I(s) + 0.01\left[sI(s) - i(0^+)\right]$$

$$\frac{100}{s} = 25\,I(s) + 0.01\,sI(s) - 0.01(2)$$

$$\frac{100}{s} + 0.02 = \left[25 + 0.01s\right] I(s)$$

Scanned by CamScanner

$$I(s) = \frac{100\times 10^2}{s(s+2500)} + \frac{2}{(s+2500)} \quad \text{---} (*)$$

P.F

consider

$$\frac{10^4}{s(s+2500)} = \frac{K_1}{s} + \frac{K_2}{s+2500}$$

Now:- $K_1 = \left.\frac{10^4}{\cancel{s}(s+2500)} \times \cancel{s}\right|_{s=0}$

$$= \frac{\cancel{10^2}\,{}^4 \times \cancel{10^2}}{\cancel{25}\times\cancel{10^2}}$$

$$\boxed{K_1 = 4}$$

Now:- $K_2 = \left.\frac{10^4}{s\cancel{(s+2500)}} \times \cancel{(s+2500)}\right|_{s=-2500}$

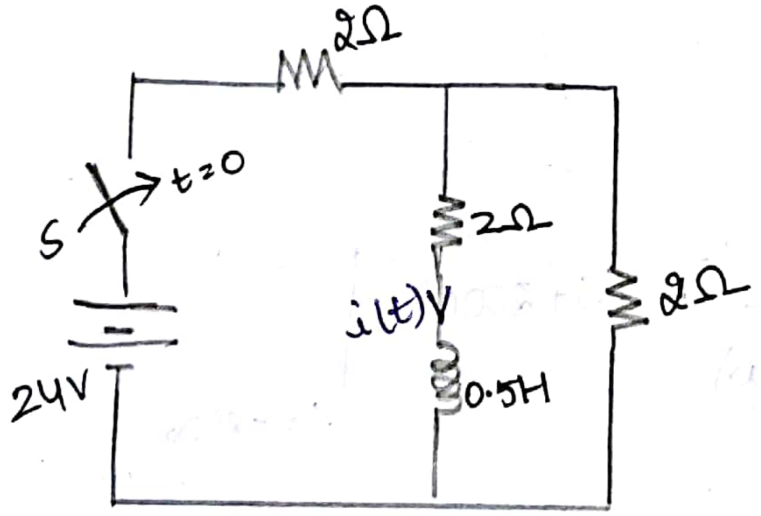
$$K_2 = \left.\frac{10^4}{s}\right|_{s=-2500}$$

$$= -\frac{\cancel{10^2}\times\cancel{10^2}}{\cancel{25}\times\cancel{10^2}}$$

$$\boxed{K_2 = -4}$$

$$\boxed{i(t) = 4 - 2e^{-2500t}}$$

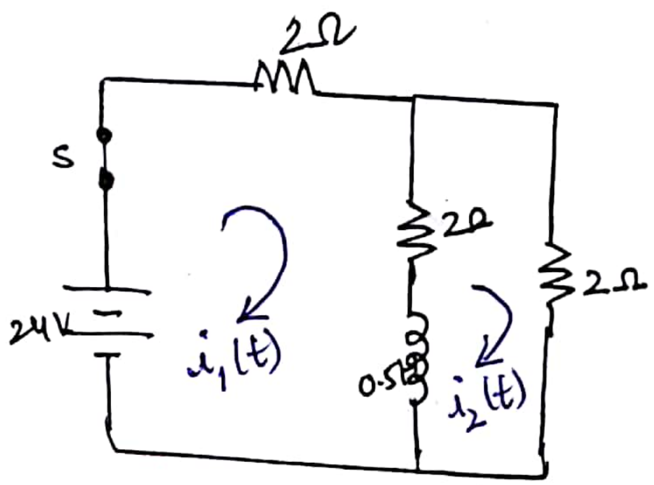
Q.) Calculate steady state current $i(t)$ for the network shown below by assuming initial zero conditions.



Sol:-

→ Before switch is closed $i(0^-) = 0$

→ After switch 's' is closed



KVL, for loop ①

$$24 = 2i_1(t) + 2[i_1(t) - i_2(t)] + 0.5\frac{d}{dt}[i_1(t) - i_2(t)]$$

$$24 = 2i_1(t) + 2i_1(t) - 2i_2(t) + 0.5\frac{di_1(t)}{dt} - 0.5\frac{di_2(t)}{dt}$$

Apply L.T

$$\frac{24}{s} = 4I_1(s) - 2I_2(s) + 0.5[sI_1(s) - i_1(0^+)] - 0.5[sI_2(s) - i_2(0^+)]$$

$$\frac{24}{s} = 4I_1(s) - 2I_2(s) + 0.5[sI_1(s)] + 0.5[sI_2(s)]$$

$$\frac{24}{s} = I_1(s)[4 + 0.5s] - I_2(s)[2 + 0.5s]$$

Mul by '2' on b.s to get rid of 0.5

$$\frac{48}{s} = (8+s)I_1(s) - (4+s)I_2(s) \longrightarrow ①$$

Now, KVL for loop ②

$$2[i_2(t) - i_1(t)] + 2i_2(t) + 0.5\frac{d}{dt}[i_2(t) - i_1(t)] = 0$$

$$-2i_1(t) + 4i_2(t) - 0.5\frac{di_1(t)}{dt} + 0.5\frac{di_2(t)}{dt} = 0$$

Apply L.T

$$-2I_1(s)+4I_2(s)-0.5\,sI_1(s)+0.5\,sI_2(s)=0$$

$$-(2+0.5s)\,I_1(s)+(4+0.5s)\,I_2(s)=0$$

mult by '2'

$$-(4+s)I_1(s)+(8+s)I_2(s)=0 \longrightarrow (2)$$

on solving (1) & (2) Solving using "Cramer's Rule."

$$\begin{bmatrix}(s+8) & -(s+4)\\ -(s+4) & (s+8)\end{bmatrix}\begin{bmatrix}I_1(s)\\ I_2(s)\end{bmatrix}=\begin{bmatrix}\frac{48}{s}\\ 0\end{bmatrix}$$

$$X \qquad I \qquad Y$$

$$|X| = (s+8)^2-(s+4)^2$$

$$= \cancel{s^2}+64+16s-\cancel{s^2}-16-8s$$

$$= 8s+48$$

$$\boxed{|X| \Rightarrow 8(s+6)}$$

Replace const in 1st row.

Now $|X_1| = \begin{vmatrix}\frac{48}{s} & -(s+4)\\ 0 & (s+8)\end{vmatrix}$

$$\boxed{|X_1| = \frac{48}{s}(s+8)+0}$$

$$= 48+\frac{48\times 8}{s}$$

$$\boxed{|X_1| \Rightarrow 48+\frac{384}{s}}$$

now $|X_2| = \begin{vmatrix} s+8 & \frac{48}{s} \\ -(s+4) & 0 \end{vmatrix}$

$$\boxed{|X_2| = 0 + (s+4)\frac{48}{s}}$$

$$\boxed{|X_2| = 48 + \frac{192}{s}}$$

Now $I_1(s) = \frac{|X_1|}{|X|} \Rightarrow \cancel{\frac{48 + \frac{384}{s}}{8(s+6)}} \Rightarrow \cancel{48s+384}$

$$\Rightarrow \frac{\frac{\cancel{48}^{6}}{s}(s+8)}{\cancel{8}(s+6)} \Rightarrow \frac{6(s+8)}{s(s+6)}$$

& $I_2(s) = \frac{|X_2|}{|X|} \Rightarrow \frac{\frac{\cancel{48}^{6}}{s}(s+4)}{\cancel{8}(s+6)} = \frac{6(s+4)}{s(s+6)}$

$$I(s) = I_1(s) - I_2(s)$$

$$= \frac{6(s+8)}{s(s+6)} - \frac{6(s+4)}{s(s+6)}$$

$$= \frac{\cancel{6s} + 48 - \cancel{6s} - 24}{s(s+6)}$$

$$\boxed{I(s) = \frac{24}{s(s+6)}}$$

To get steady state condition apply ILT to get $i(t)$ then sub^n $\boxed{t = \infty}$

(Or)

Applying Final Value Th.

$\therefore$ steady state current :

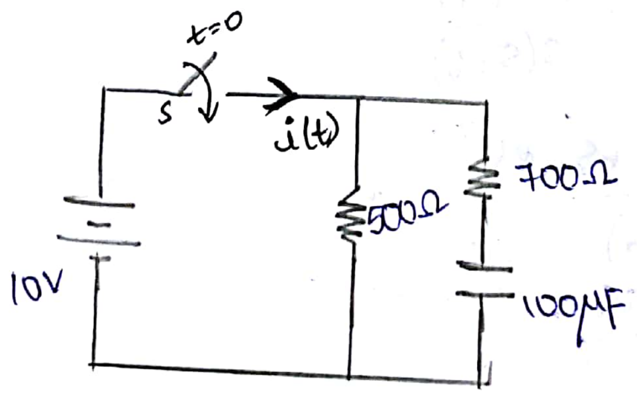
$$i(\infty) = \lim_{s \to 0} sI(s)$$

$$= \lim_{s \to 0} \cancel{s} \left[\frac{24}{\cancel{s}(s+6)} \right]$$

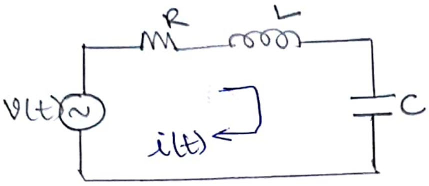
$$i(\infty) = \frac{\cancel{24}}{\cancel{6}}$$

$$\boxed{i(\infty) = 4A.}$$

Q.) By considering zero initial conditions for the ckt shown below, supply voltage is applied at t=0. Then, how much time it will take to reach 25mA.



I] LT Analysis of series RLC circuit



consider zero initial conditions

i.e $v(0^+) = 0 \longrightarrow$ Initial voltage across capacitor

$i(0^+) = 0 \longrightarrow$ Initial current through inductor

Apply KVL

$$V(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(t)\,dt$$

$$V(s) = RI(s) + L\left[sI(s) - i(0^+)\right] + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{v(0^+)}{s}\right]$$

$$\Rightarrow V(s) = \left[R + sL + \frac{1}{Cs}\right]$$

$$I(s) = \frac{V(s)}{R + sL + \frac{1}{Cs}}$$

$$I(s) = \frac{V(s)\,Cs}{1 + s^2LC + RCs}$$

$$I(s) = \frac{\ldots}{L\left[s^2+\frac{R}{L}s+\frac{1}{LC}\right]}$$

Also W.K.T for series resonance ckt

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

A) Series RLC for "step-input":-

let $v(t) = V_0 u(t) \Rightarrow V(s) = \frac{V_0}{s}$

subn in eqn (5)

$$I(s) = \frac{\left(\frac{V_0}{\cancel{s}}\right)\cancel{s}}{L\left[s^2+\frac{R}{L}s+\frac{1}{LC}\right]}$$

$$\boxed{I(s) = \frac{\left(\frac{V_0}{L}\right)}{\left[s^2+\frac{R}{L}s+\frac{1}{LC}\right]}} \longrightarrow (6)$$

Roots:-

$$s = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4\left(\frac{1}{LC}\right)}}{2}$$

$$S = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{\cancel{4}}{\cancel{4}LC}}$$

let

$$S_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \quad \& \quad S_2 = \frac{-R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Now consider a parameter 'ζ' such that

$$\zeta \omega_0 = \frac{R}{2L} \Rightarrow \zeta = \frac{R}{2L} \cdot \frac{1}{\omega_0}$$

$$\zeta = \frac{R}{2L} \cdot \sqrt{LC}$$

$$\boxed{\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}}$$ damping Ratio

$$S_1 = -\zeta\omega_0 + \sqrt{(\zeta\omega_0)^2 - \omega_0^2} \; ; \; S_2 = -\zeta\omega_0 - \sqrt{(\zeta\omega_0)^2 - \omega_0^2}$$

$$\boxed{S_1 = -\zeta\omega_0 + \omega_0\sqrt{\zeta^2 - 1} \; ; \; S_2 = -\zeta\omega_0 - \omega_0\sqrt{\zeta^2 - 1}}$$

From eqn (6)

$$I(s) = \frac{\left(\frac{V_0}{L}\right)}{(s-s_1)(s-s_2)} = \frac{K_1}{(s-s_1)} + \frac{K_2}{(s-s_2)}$$

$$= K_1(s-s_2) + K_2(s-s_1)$$

$K_1 (s_1 - s_2)$

$$K_1 = \frac{V_0}{2\omega_0 L\sqrt{\zeta^2-1}}$$

$$K_2 = \frac{-V_0}{2\omega_0 L\sqrt{\zeta^2-1}}$$

sub^n K_1 & K_2 in $I(s)$

$$\therefore I(s) = \frac{V_0}{2\omega_0 L\sqrt{\zeta^2-1}}\left[\frac{1}{s-s_1} - \frac{1}{s-s_2}\right]$$

Apply ILT

$$i(t) = \frac{V_0}{2\omega_0 L\sqrt{\zeta^2-1}}\left[e^{s_1 t} - e^{s_2 t}\right]$$

$$i(t) = \frac{V_0}{2\omega_0 L\sqrt{\zeta^2-1}}\left[e^{-(\zeta\omega_0 + \omega_0\sqrt{\zeta^2-1})t} - e^{(-\zeta\omega_0 - \omega_0\sqrt{\zeta^2-1})t}\right]$$

$$\boxed{i(t) = \frac{V_0 e^{-\zeta\omega_0 t}}{2\omega_0 L\sqrt{\zeta^2-1}}\left[e^{(\omega_0\sqrt{\zeta^2-1})t} - e^{-(\omega_0\sqrt{\zeta^2-1})t}\right]}$$

$\to$ (7)

$$S = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{\cancel{4}}{\cancel{4}LC}}$$

let

$$S_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \quad \& \quad S_2 = \frac{-R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Now consider a parameter 'ζ' such that

$$\zeta\omega_0 = \frac{R}{2L} \Rightarrow \zeta = \frac{R}{2L} \cdot \frac{1}{\omega_0}$$

$$\zeta = \frac{R}{2L} \cdot \sqrt{LC}$$

$$\boxed{\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}}$$ damping Ratio

$$S_1 = -\zeta\omega_0 + \sqrt{(\zeta\omega_0)^2 - \omega_0^2} \; ; \; S_2 = -\zeta\omega_0 - \sqrt{(\zeta\omega_0)^2 - \omega_0^2}$$

$$\frac{V_0}{L} = K_1(s_1 - s_2)$$

$$K_1 = \frac{V_0}{2\omega_0 L\sqrt{\zeta^2-1}}$$

& $$K_2 = \frac{-V_0}{2\omega_0 L\sqrt{\zeta^2-1}}$$

sub^n K_1 & K_2 in $I(s)$

$$\therefore I(s) = \frac{V_0}{2\omega_0 L\sqrt{\zeta^2-1}}\left[\frac{1}{s-s_1} - \frac{1}{s-s_2}\right]$$

Apply ILT

$$i(t) = \frac{V_0}{2\omega_0 L\sqrt{\zeta^2-1}}\left[e^{s_1 t} - e^{s_2 t}\right]$$

$$i(t) = \frac{V_0}{2\omega_0 L\sqrt{\zeta^2-1}}\left[e^{-(\zeta\omega_0+\omega_0\sqrt{\zeta^2-1})t} - e^{(-\zeta\omega_0-\omega_0\sqrt{\zeta^2-1})t}\right]$$

$$\boxed{i(t) = \frac{V_0 e^{-\zeta\omega_0 t}}{2\omega_0 L\sqrt{\zeta^2-1}}\left[e^{(\omega_0\sqrt{\zeta^2-1})t} - e^{-(\omega_0\sqrt{\zeta^2-1})t}\right]}$$

case-i):-

$b^2 - \omega_0 c > 1$

$\left(\frac{R}{2L}\right) \cdot \frac{1}{\omega_0} > 1$

$\frac{R}{2L} > \frac{1}{\sqrt{LC}}$

If $\zeta > 1$ (or) $\frac{R}{2L} > \frac{1}{\sqrt{LC}}$

[over damped conditions]

eq'n (7) becomes

$$\boxed{i(t) = \frac{V_0 e^{-\zeta\omega_0 t}}{\omega_0 L\sqrt{\zeta^2-1}} \sinh(\omega_0\sqrt{\zeta^2-1})t}$$

case-ii):-

$\zeta < 1$ (or) $\frac{R}{2L} < \frac{1}{\sqrt{LC}}$ [under damped]

$$\sqrt{\zeta^2-1} = j\sqrt{1-\zeta^2}$$

from eq'n (7)

$$i(t) = \frac{V_0 e^{-\zeta\omega_0 t}}{\omega_0 L\sqrt{1-\zeta^2}} \left\{ \frac{e^{j(\omega_0\sqrt{1-\zeta^2})t} - e^{-j(\omega_0\sqrt{1-\zeta^2})t}}{2j} \right\}$$

$$\boxed{i(t) = \frac{V_0 e^{-\zeta\omega_0 t}}{\omega_0 L\sqrt{1-\zeta^2}} \sin\left[\omega_0\sqrt{1-\zeta^2}\right]t}$$

case-iii):-

$\zeta = 1$ (or) $\frac{R}{2L} = \frac{1}{\sqrt{LC}}$ [critically damped]

from eq'n (6)

$\frac{R}{2L} = \frac{1}{\sqrt{LC}}$

$\frac{R}{L} = \frac{2}{\sqrt{LC}} \Rightarrow 2\omega_0$

$$I(S) = \frac{\left(\frac{V_0}{L}\right)}{S^2 + \frac{R}{L}S + \omega_0^2}$$

$$= \frac{\left(\frac{V_0}{L}\right)}{S^2 + 2\omega_0 S + \omega_0^2}$$

$$I(S) = \frac{\left(\frac{V_0}{L}\right)}{(S+\omega_0)^2}$$

Apply ILT

$$\boxed{i(t) = \frac{V_0}{L} t \cdot e^{-\omega_0 t}}$$

B) Series RLC for "Impulse-Input":-

let $v(t) = V_0 \delta(t) \Rightarrow V(S) = V_0$

from eqn ⑤

$$I(S) = \frac{V_0 S}{L\left[S^2 + \frac{R}{L}S + \frac{1}{LC}\right]} \longrightarrow ⑧$$

$$I(S) = \frac{\left(\frac{V_0}{L}\right) S}{(S - S_1)(S - S_2)}$$

where

$S_1 = -\zeta\omega_0 + \omega_0\sqrt{\zeta^2 - 1}$; $S_2 = -\zeta\omega_0 - \omega_0\sqrt{\zeta^2 - 1}$

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$w_0 = \frac{1}{\sqrt{LC}}$$

$$I(s) = \frac{K_1}{(s-s_1)} + \frac{K_2}{(s-s_2)}$$

$$K_1 = \left.\frac{\left(\frac{V_0}{L}\right)s}{s-s_2}\right|_{s=s_1}$$

$$K_1 = \frac{\left(\frac{V_0}{L}\right)s_1}{2w_0L\sqrt{\zeta^2-1}}$$

$$K_2 = \left.\frac{\left(\frac{V_0}{L}\right)s}{s-s_1}\right|_{s=s_2}$$

$$K_2 = \frac{-\left(\frac{V_0}{L}\right)s_2}{2w_0L\sqrt{\zeta^2-1}}$$

sub^n K_1 & K_2 in $I(s)$

$$\therefore\ I(s) = \frac{V_0}{2w_0L\sqrt{\zeta^2-1}}\left[\frac{s_1}{s-s_1} - \frac{s_2}{s-s_2}\right]$$

$$i(t) = \frac{V_0}{2w_0L\sqrt{\zeta^2-1}}\left[s_1e^{s_1t} - s_2e^{s_2t}\right]$$

$$\cancel{i(t) = \frac{V_0}{2w_0L\sqrt{\zeta^2-1}}\left\{(-\zeta w_0 e^{s_1t} + w_0\sqrt{\zeta^2-1}\,e^{s_1t}\right.}$$

$$i(t) = \frac{V_0}{2w_0L\sqrt{\zeta^2-1}}\left\{\left(-\zeta w_0 + w_0\sqrt{\zeta^2-1}\right)e^{-\zeta w_0t}\cdot e^{w_0\sqrt{\zeta^2-1}\,t}\right.$$
$$\left. + \left(\zeta w_0 + w_0\sqrt{\zeta^2-1}\right)e^{-\zeta w_0t}\cdot e^{-(w_0\sqrt{\zeta^2-1})}\right.$$

$$i(t) = \frac{V_0 e^{-\zeta w_0 t}}{2 w_0 L \sqrt{\zeta^2-1}} \left\{ -\zeta w_0 \left[e^{(w_0\sqrt{\zeta^2-1})t} - e^{-(w_0\sqrt{\zeta^2-1})t} \right] + w_0\sqrt{\zeta^2-1} \left[e^{(w_0\sqrt{\zeta^2-1})t} + e^{-(w_0\sqrt{\zeta^2-1})t} \right] \right\}$$

$\rightarrow$ (9)

case-i):-

$\zeta > 1$ [over damped conditions]

from eqn (9)

$$i(t) = \frac{V_0 e^{-\zeta w_0 t}}{w_0 L \sqrt{\zeta^2-1}} \left\{ -\zeta w_0 \sinh\left[(w_0\sqrt{\zeta^2-1})t\right] + w_0\sqrt{\zeta^2-1} \cosh\left[(w_0\sqrt{\zeta^2-1})t\right] \right\}$$

case-ii):-

$\zeta < 1$ [underdamped conditions]

$$\sqrt{\zeta^2-1} = j\sqrt{1-\zeta^2}$$

from eqn (9)

$$i(t) = \frac{V_0 e^{-\zeta w_0 t}}{2j w_0 L \sqrt{1-\zeta^2}} \left\{ -\zeta w_0 \left[e^{j(w_0\sqrt{1-\zeta^2})t} - e^{-j(w_0\sqrt{1-\zeta^2})t} \right] + j w_0\sqrt{1-\zeta^2} \left[e^{j(w_0\sqrt{1-\zeta^2})t} + e^{-j(w_0\sqrt{1-\zeta^2})t} \right] \right\}$$

$$i(t) = \frac{V_0 e^{-\zeta w_0 t}}{w_0 L \sqrt{1-\zeta^2}} \left\{ -\zeta w_0 \sin\left[(w_0\sqrt{1-\zeta^2})t\right] + w_0\sqrt{1-\zeta^2} \cos\left[(w_0\sqrt{1-\zeta^2})t\right] \right\}$$

Case - (iii) :-

$\zeta = 1$ (or) $\frac{R}{2L} = \frac{1}{\sqrt{LC}}$ [critically damped]

from eqn (8)

$$I(s) = \frac{\left(\frac{V_0}{L}\right)s}{\left[s^2 + \frac{R}{L}s + \frac{1}{LC}\right]}$$

$$I(s) = \frac{\left(\frac{V_0}{L}\right)s}{s^2 + 2w_0 s + w_0^2}$$

$$I(s) = \frac{\left(\frac{V_0}{L}\right)s}{(s+w_0)^2} = \frac{k_1}{(s+w_0)^2} + \frac{k_2}{(s+w_0)}$$

$$k_1 = \left(\frac{V_0}{L}\right)s \Big|_{s=-w_0}$$

$$k_1 = \frac{-w_0 V_0}{L}$$

$$k_2 = \frac{d}{ds}\left[\left(\frac{V_0}{L}\right)s\right] \Big|_{s=-w_0}$$

$$k_2 = \frac{V_0}{L}$$

$$\therefore I(s) = \frac{V_0}{L}\left[\frac{-w_0}{(s+w_0)^2} + \frac{1}{(s+w_0)}\right]$$

$$i(t) = \frac{V_0}{L}\left[e^{-w_0 t} - w_0 t \cdot e^{-w_0 t}\right]$$

$$\boxed{i(t) = \frac{V_0}{L} e^{-w_0 t}\left[1 - w_0 t\right]}$$

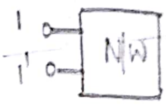
Unit - IV (part - 2) :-

* NETWORK FUNCTIONS *

Network is a interconnection of different electrical elements.
Generally a N/w is represented by a box.

port:-
A pair of terminals across which a device is connected.
Based on ports, N/w's are classified into

→ one-port N/w →



→ Two-port N/w →

→ Multi-port N/w →

Network Function:-

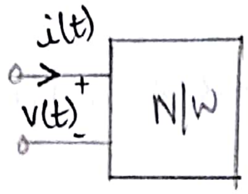
It is defined as a fun^c that relates voltages, currents at diff parts of N/w.
Generally, a N/w fun^c is denoted by:-

$$F(s) = \frac{N(s)}{D(s)}$$

$N(s)$ → Nr polynomial
$D(s)$ → Dr polynomial

Network functions of one-port N/w:-

Consider a one-port N/w with zero initial conditions & "no-independent" internal sources (except controlled sources) as shown in fig. below:-



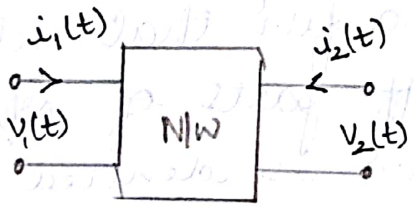
$\frac{L[v(t)]}{L[i(t)]}$

Transform Impedence Func:- $\boxed{Z(s) = \frac{V(s)}{I(s)}}$

Transform Admittance Func:- $\boxed{Y(s) = \frac{I(s)}{V(s)} \Rightarrow \frac{1}{Z(s)}}$

Network functions of two-port N/w's:-

consider a two-port N/w as shown in fig with zero initial conditions & "no-independent internal sources (except controlled sources)



i) **Transfer Functions:-** (defined for two diff ports)

Transfer Function for a two-port N/w is defined as the ratio of L.T of o/p to i/p (or) i/p to o/p.

- **Voltage Transfer Functions :-**

$$G_{12} = \frac{V_1(s)}{V_2(s)} \quad ; \quad G_{21} = \frac{V_2(s)}{V_1(s)}$$

- **Current Transfer Functions:-**

$$\alpha_{12} = \frac{I_1(s)}{I_2(s)} \quad ; \quad \alpha_{21} = \frac{I_2(s)}{I_1(s)}$$

- **Transfer Impedence Functions:-**

$$Z_{12} = \frac{V_1(s)}{I_2(s)} \quad ; \quad Z_{21} = \frac{V_2(s)}{I_1(s)}$$

- **Transfer Admittance Functions:-**

$$Y_{12} = \frac{I_1(s)}{V_2(s)} \quad ; \quad Y_{21} = \frac{I_2(s)}{V_1(s)}$$

(ii) **Driving point Functions:-** (defined for same ports)

- Driving pt. Impedence Functions:-

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)} \quad ; \quad Z_{22}(s) = \frac{V_2(s)}{I_2(s)}$$

- Driving pt. Admittance Functions:-

$$Y_{11}(s) = \frac{I_1(s)}{V_1(s)} \quad ; \quad Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$$

Def: Ratio of L.T of voltage to current / current to volt at same port.

Problems:-

Q1.) Calculate the driving pt Impedance of the following one-port N/wk.

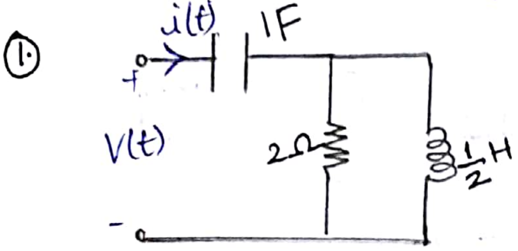
① 

fig (a).

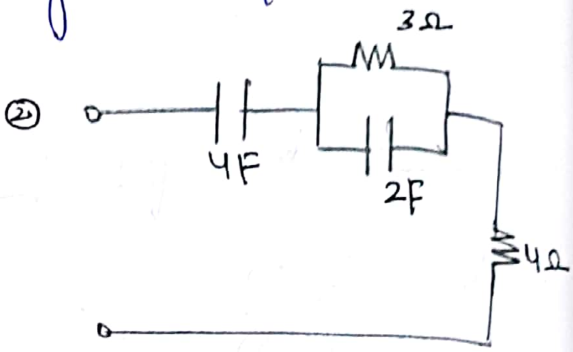
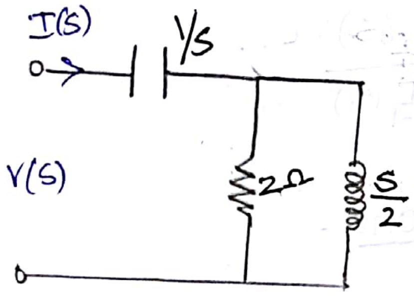
② 

fig (b)

converting into 'freq' domain (i.e Apply L.T)



$$Z(s) = \frac{V(s)}{I(s)} \Rightarrow \frac{1}{s} + (2) \| \left(\frac{s}{2}\right)$$

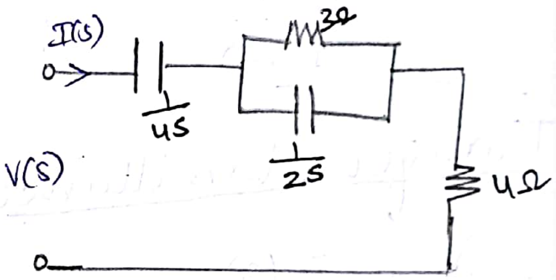
$$Z(s) \Rightarrow \frac{1}{s} + \frac{2\left(\frac{s}{2}\right)}{2+\frac{s}{2}}$$

$$Z(s) = \frac{1}{s} + \frac{2s}{s+4}$$

$$Z(s) = \frac{s+4+2s^2}{s(s+4)}$$

$$Z(s) = \frac{2s^2+s+4}{s^2+4s}$$

converting into 'freq' domain (i.e Apply LT)



$$Z(s) = \frac{V(s)}{I(s)} = \frac{1}{4s} + (3) \| \left(\frac{1}{2s}\right) + 4$$

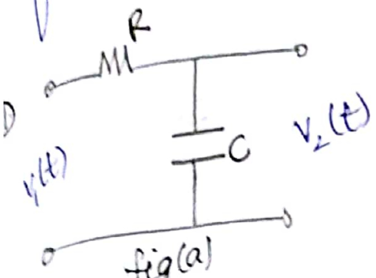
$$Z(s) = \frac{1}{4s} + 4 + \frac{\frac{3}{2s}}{3+\frac{1}{2s}}$$

$$Z(s) = \frac{1}{4s} + 4 + \frac{3}{6s+1}$$

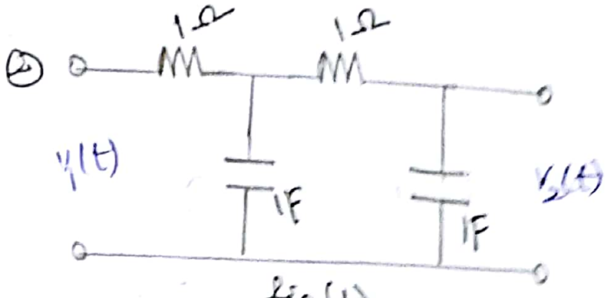
$$Z(s) = \frac{6s+1+4(6s+1)(4s)+12s}{(6s+1)(4s)}$$

$Z(s) =$

Q2) Determine voltage Transfer Ratio $G_{21}(s)$ for the following n/w's.



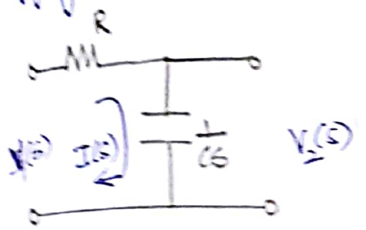
fig(a)



fig(b)

Sol:- voltage Transfer Ratio $G_{21}(s) = \frac{V_2(s)}{V_1(s)}$

Apply LT to fig(a)



$$V_1(s) = RI(s) + \frac{1}{Cs} I(s)$$

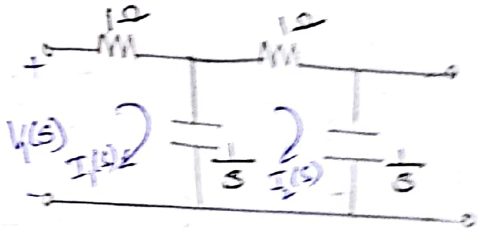
$$V_1(s) = \left(R + \frac{1}{Cs}\right) I(s) \longrightarrow ①$$

$$V_2(s) = \frac{1}{Cs} I(s) \longrightarrow ②$$

$$\therefore \text{Now, } G_{21}(s) = \frac{V_2(s)}{V_1(s)} = \frac{\frac{1}{Cs} I(s)}{\left(R + \frac{1}{Cs}\right) I(s)}$$

$$\boxed{G_{21}(s) = \frac{1}{1 + RCs}}$$

Apply L.T to fig(b)



KVL to ①

$$-V_1(s) + I_1(s)(1) + [I_1(s) - I_2(s)]\frac{1}{s} = 0$$

$$V_1(s) = I_1(s)\left[1 + \frac{1}{s}\right] - I_2(s)\frac{1}{s} \longrightarrow ①$$

KVL to ②

$$\frac{1}{s}[I_2(s) - I_1(s)] + I_2(s) + \frac{1}{s} I_2(s) = 0$$

$$\frac{1}{s} I_2(s) - \frac{1}{s} I_1(s) + I_2(s) + \frac{1}{s} I_2(s) = 0$$

$$I_2(s)\left[1 + \frac{1}{s} + \frac{1}{s}\right] - \frac{I_1(s)}{s} = 0$$

$$I_2(s)\left[\frac{s+2}{s}\right] - \frac{I_1(s)}{s} = 0 \longrightarrow (2)$$

$$V_2(s) = I_2(s) \times \frac{1}{s} \longrightarrow (3)$$

Now, from eqn (2)

$$I_2(s) = \frac{I_1(s)}{\cancel{s}} \times \frac{\cancel{s}}{(s+2)}$$

$$I_2(s) = I_1(s) \times \left(\frac{1}{s+2}\right)$$

from eqn (1)

$$V_1(s) = I_1(s)\left(\frac{s+1}{s}\right) - \frac{1}{s}\left(\frac{I_1(s)}{s+2}\right)$$

$$V_1(s) = \frac{I_1(s)}{s}\left[(s+1) - \frac{1}{s+2}\right]$$

$$= \frac{I_1(s)}{s}\left[\frac{(s+1)(s+2)-1}{s+2}\right]$$

$$V_1(s) = \frac{I_1(s)}{s}\left[\frac{s^2+3s+1}{(s+2)}\right] \longrightarrow (4)$$

Now (3) ÷ (4) we get

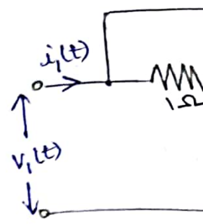
$$\frac{V_2(s)}{V_1(s)} \Rightarrow G_{21}(s) = \frac{\dfrac{\cancel{I_1(s)}}{\cancel{s(s+2)}}}{\dfrac{\cancel{I_1(s)}(s^2+3s+1)}{\cancel{s(s+2)}}}$$

$$\boxed{G_{21}(s) = \frac{1}{s^2+3s+1}}$$

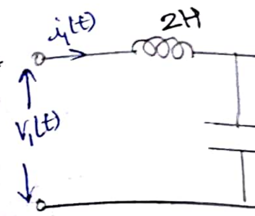
Q3) Calculate ...

for the followi...

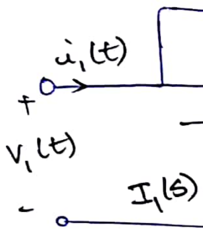
a:)



b:)



sol:) ① apply L.T



KVL to ...

$V_1(s) = [$ ...

$V_1(s) =$ ...

loop ② :-

$V_2(s)$ ...

$V_2($ ...

Q3) Calculate current Transfer Ratio $\alpha_{21} = \frac{I_2(s)}{I_1(s)}$ for the following N/W:

a)

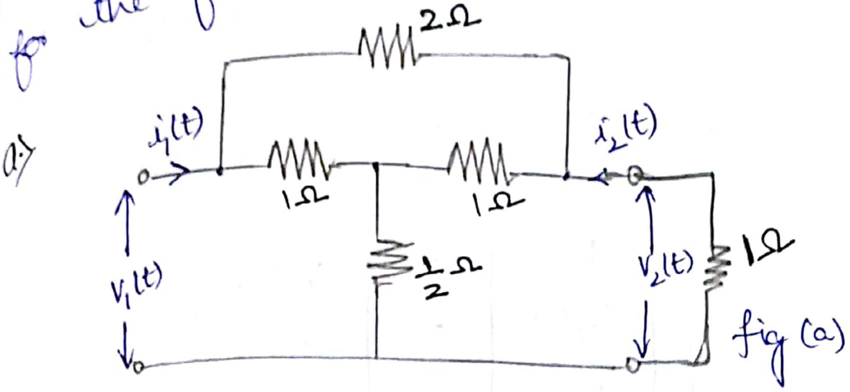


fig (a)

b)

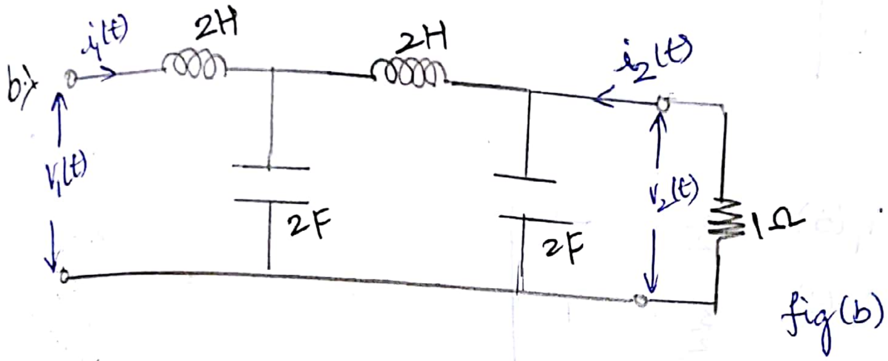
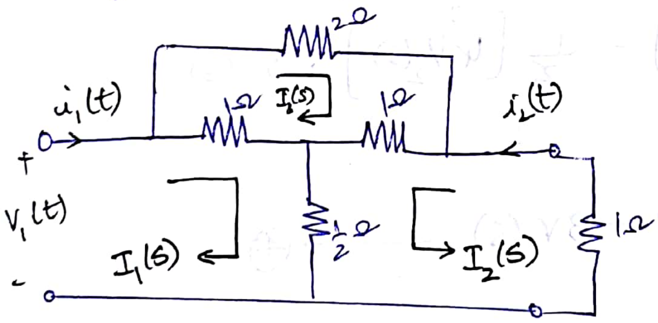
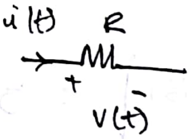


fig (b)

Sol: ① Apply L.T to fig(a)





$$\boxed{V_2(s) = -(1) I_2(s)}$$

KVL to loop ①:-

$$V_1(s) = [I_1(s) - I_3(s)] + \frac{1}{2}[I_1(s) + I_2(s)]$$

$$V_1(s) = \frac{3}{2} I_1(s) + \frac{1}{2} I_2(s) - I_3(s) \longrightarrow ①$$

loop ②:-

$$V_2(s) = 1[I_2(s) + I_3(s)] + \frac{1}{2}[I_2(s) + I_1(s)]$$

$$V_2(s) = \frac{1}{2} I_1(s) + \frac{3}{2} I_2(s) + I_3(s) \longrightarrow ②$$

loop (3) :-

$$I_3(s) - I_1(s) + 2I_3(s) + \left[I_3(s) + I_2(s)\right] = 0$$

$$-I_1(s) + I_2(s) + 4I_3(s) = 0 \longrightarrow (3)$$

$$\begin{bmatrix} \frac{3}{2} & \frac{1}{2} & -1 \\ \frac{1}{2} & \frac{3}{2} & 1 \\ -1 & 1 & 4 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \end{bmatrix} = \begin{bmatrix} V_1(s) \\ V_2(s) \\ 0 \end{bmatrix}$$

$$\boxed{|X| = 4}$$

$$X_1(s) = \begin{vmatrix} V_1(s) & \frac{1}{2} & -1 \\ V_2(s) & \frac{3}{2} & 1 \\ 0 & 1 & 4 \end{vmatrix}$$

$$|X_1(s)| = V_1(s)\left[5\right] - \frac{1}{\cancel{2}}\left[\cancel{4}^{2}V_2(s)\right] - V_2(s)$$

$$|X_1(s)| = 5V_1(s) - 3V_2(s) \longrightarrow (4)$$

$$X_2(s) = \begin{vmatrix} \frac{3}{2} & V_1(s) & -1 \\ \frac{1}{2} & V_2(s) & 1 \\ -1 & 0 & 4 \end{vmatrix}$$

$$|X_2(s)| = \left(\frac{3}{\cancel{2}}\right)\left[\cancel{4}^{2}V_2(s)\right] - V_1(s)\left[2+1\right] - 1\left[V_2(s)\right]$$

$$|X_2(s)| = -3V_1(s) + 5V_2(s) \longrightarrow (5)$$

$$X_3(s) = \begin{vmatrix} \frac{3}{2} & \frac{1}{2} & V_1(s) \\ \frac{1}{2} & \frac{3}{2} & V_2(s) \\ -1 & 1 & 0 \end{vmatrix}$$

$$|X_3(s)| = \frac{3}{2}\left[-V_2(s)\right] - \frac{1}{2}\left[V_2(s)\right] + V_1(s)\left[\frac{1}{2}+\frac{3}{2}\right]$$

$$= -\frac{3V_2(s)}{2} - \frac{V_2(s)}{2} + 2V_1(s)$$

$$= V_2(s) + 2V_1(s)$$

$$I_3(s) = \frac{2V_1(s) \cancel{+2V_2(s)}}{4} \longrightarrow ⑥$$

$$I_1(s) = \frac{X_1(s)}{|X|} \Rightarrow \frac{5V_1(s) - 3V_2(s)}{4} \longrightarrow ①$$

$$I_2(s) = \frac{X_2(s)}{|X|} \Rightarrow \frac{-3V_1(s) + 5V_2(s)}{4} \longrightarrow ②$$

$$\therefore \cancel{\alpha_{21} = \frac{I_2(s)}{I_1(s)} \Rightarrow \frac{-3V_1(s) + 5V_2(s)}{5V_1(s) - 3V_2(s)}}$$

form ②

$$4I_2(s) = -3V_1(s) + (-5I_2(s))$$

$$9I_2(s) = -3V_1(s)$$

$$I_2(s) = -\frac{1}{3}V_1(s)$$

$$\Rightarrow V_1(s) = -3I_2(s)$$

Scanned by CamScanner

from ①

$$4I_1(s) = 5\left[-3I_2(s)\right] - 3\left[-I_2(s)\right]$$

$$4I_1(s) = -15I_2(s) + 3I_2(s)$$

$$4I_1(s) = -\cancel{12}^{\,3} I_2(s)$$

$$\boxed{\frac{I_2(s)}{I_1(s)} = -\frac{1}{3}} \Rightarrow \alpha_{21}$$

Poles & Zeroes of Network Fun^c :-

Any N/w function can be denoted as

$$F(s) = \frac{N(s)}{D(s)}$$

where $N(s)$ – Nr polynomial
$D(s)$ – Dr polynomial

So $F(s)$ can be written as:-

$$F(s) = \frac{N(s)}{D(s)} = \frac{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}$$

$$= K \frac{(s-z_1)(s-z_2)\cdots(s-z_n)}{(s-P_1)(s-P_2)\cdots(s-P_m)}$$

where $\boxed{K = \frac{a_n}{b_m}}$ called Scale Factor / Gain Factor

Z_i, $i = 1, 2, 3 \ldots\ldots n \rightarrow$ roots of $N(s) \rightarrow$ zeros

P_j, $j = 1, 2, 3 \ldots\ldots m \rightarrow$ roots of $D(s) \rightarrow$ poles

Z_i & P_j are roots of Nr & Dr polynomials respectively & these may be real or complex conjugate

At $s = Z_i$, $i = 1, 2 \ldots\ldots n$ $\boxed{F(s) = 0}$

i.e N/w func becomes zero ∴ $s = Z_i$ are called "zeros" (or) zero frequencies

At $s = P_j$, $j = 1, 2, 3 \ldots n$ $\boxed{F(s) = \infty}$

i.e N/w func becomes undefined ∴ $s = P_j$ are called "poles" (or) pole frequencies.

Any system can be described by differential eqns.

Q1) Find the poles & zeros of the system described by following D.E.

1) $$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 2\frac{dx}{dt} + x$$

Sol:- Apply L.T

$L\left\{\frac{d^2y}{dt^2}\right\} = s^2 y(s) - s\,y(0) - f'(0)$

$$s^2 Y(s) + 5s\,Y(s) + 6Y(s) = 2s\,X(s) + X(s)$$

$$Y(s)\left[s^2 + 5s + 6\right] = (2s+1)\,X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{2s+1}{s^2+5s+6}$$

$$F(s) = \frac{Y(s)}{X(s)} = \frac{\overset{K}{2}\left(s+\frac{1}{2}\right)}{s+3s+2s+6}$$

$$F(s) = \frac{\left(s+\frac{1}{2}\right)}{(s+3)(s+2)}$$

$\therefore$ $s = -3$ & -2 are "poles" of $F(s)$

& $s = -\frac{1}{2}$ are "zero" of $F(6)$

& $K = 2$ is the gain Factor.

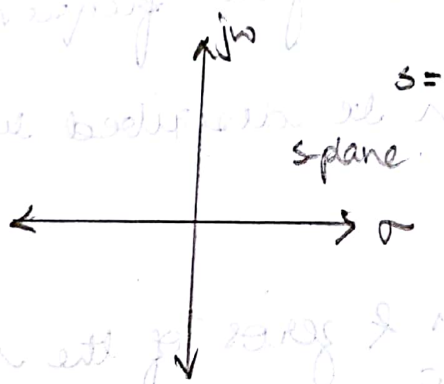
pole-zero plot :-

Zero → 'o'

poles → 'x'

$s = \sigma + j\omega$

s-plane



Significance of pole-zeroes for N/W Fun^{cs} :-

let $F(s) = \frac{N(s)}{D(s)}$

i) For driving pt Impedence fun^{c} :-

$$F(s) = \frac{N(s)}{D(s)} \Rightarrow Z(s) = \frac{V(s)}{I(s)}$$

poles means roots of $I(s)$
zeros " roots of $V(s)$

→ At poles $I(s)=0 \Rightarrow \boxed{Z(s)=\infty}$
i.e N/w func $Z(s)$ becomes O.C at poles.

→ At zeros $V(s)=0 \Rightarrow \boxed{Z(s)=0}$
i.e N/w func $Z(s)$ becomes S.C at zeroes.

ii) For driving pt. Admittance func:-

$$F(s) = Y(s) = \frac{I(s)}{V(s)}$$

poles means roots of $V(s)$
zeros " roots of $I(s)$

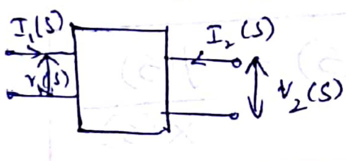
→ At poles $V(s)=0 \rightarrow \boxed{Y(s)=\infty}$
i.e N/w func. $Y(s)$ becomes S.C at poles

→ At zeroes $I(s)=0 \Rightarrow \boxed{Y(s)=0}$
i.e N/w func $Y(s)$ becomes O.C at zeroes.

iii) Transfer Functions:-

consider voltage ratio transfer func

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)}$$



$$V_2(s) = G_{21}(s) \cdot V_1(s)$$

$$V_2(s) = \sum_{i=1}^{n} \frac{K_i}{(s-P_i)} + \sum_{j=1}^{m} \frac{K_j}{(s-P_j)}$$

part-① part-②

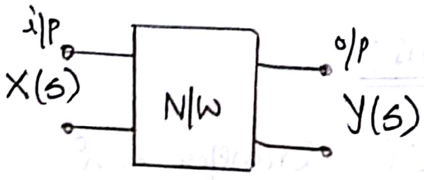
- From part-①; $s = P_i$, $i = 1, 2 \cdots n$ are poles of N/W funᶜ $G_{21}(s)$ & these are called "free oscillation" (or) "Natural oscillations". This oscillation exist without applying i/p also.

- From part-②; $s = P_j$, $j = 1, 2, 3 \cdots n$ are poles of N/W funᶜ $V_1(s)$ & these are called "Forced oscillations".

- K_i & K_j defines the magnitude of free & forced oscillations.

System Function:- [H(s)]

It is defined as ratio of L.T of o/p to L.T of i/p & is denoted by "H(s)".



$$\boxed{H(s) = \frac{Y(s)}{X(s)}}$$

Also

$$y(s) = H(s) \cdot X(s)$$

$$y(s) = \sum_{i=1}^{n} \frac{K_i}{(s-P_i)} + \sum_{j=1}^{m} \frac{K_j}{(s-P_j)} \longrightarrow ①$$

$$y(t) = \sum_{i=1}^{n} K_i e^{P_i t} + \sum_{j=1}^{m} K_j \cancel{K_j} e^{P_j t} \longrightarrow ②$$

if $X(s) = 1$ i.e $x(t) = \delta(t) \rightarrow$ unit impulse

$L[x(t)] = 1$

from eqn ①

$$\boxed{y(s) = H(s) = \sum_{i=1}^{n} \frac{K_i}{(s-P_i)}}$$

$$h(t) = y(t) = \sum_{i=1}^{n} K_i e^{P_i t}$$

→ Natural Response of a system can be explained as a system response to its impulse input.

→ Natural Response is also called "Impulse response" or "Force-free response" or "Transient response".

System Stability & poles locations:-

For a linear system to be stable, the following conditions must be statisfied:

$$\sum_{i=1}^{n} \lim_{t \to \infty} K_i e^{P_i t} = 0$$

't'

i.e P_i must be "$-ve$". Therefore all poles are left half of the s-plane.

Restriction on location of poles & zeroes in the S-plane for Driving pt. Fun^c :-

(or) / Conditions for a driving pt. Fun^c :-

i) The coefficients of $N(s)$ & $D(s)$ must be real & $+ve$

ii) Complex & imaginary parts of poles must be conjugate

iii) The real parts of poles & zeroes must be either zero or $-ve$. ($-ve$ half of s-plane)

iv) If the real part is zero then the pole or zero must be simple. i.e not-repeated. $(j\omega)^2$ ✗, $j\omega$, $-j\omega$

** v) polynomials $N(s)$ & $D(s)$ must not have missing terms from highest order to lowest order unless all even terms (or) odd terms becomes zero. s^3+s^2+1) ✗ (missing s) (or) (s^2+1) ✓ (s^3+1) ✓

vi) The highest degree of $N(s)$ & $D(s)$ may differ by 'zero' or '1' iii, The lowest degree of N(s) & D(s) may differ by 'zero' or '1'

Check whether the following funs are valid driving funcs or not.

i, $F(s) = \dfrac{s^4+2s^3-2s+1}{s^3+s^2-2s+12}$ → "Invalid"
(involves -ve) 1st cond

ii, $F(s) = \dfrac{s^2+s+2}{s^4+5s^3+6s}$ → "Invalid"
(by 6th cond)

iii, $F(s) = \dfrac{s^2+5s+4}{s^2+6s+10}$ → "Valid"

Necessary conditions for Transfer Funcs:-

(or)

Restriction on location of poles & zeroes in s-plane for Transfer Funcs:-

i,

i, to iv, same points

v) The polynomial D(s) may not have any missing terms b/w highest order to lowest order unless all even terms or odd terms are missing

vi) The polynomial N(s) may have missing terms b/w highest order to lowest order.

vii) The degree of N(s) may be as small as "zero" & independent of D(s)

viii) For Voltage Transfer Ratios ('G') & current Transfer Ratios ('α'), the maximum degree of N(s) is the degree of D(s).

ix) For Transfer Impedence Fun^cs & Admittance func^s the maximum degree of N(s) is degree of D(s) + 1 $(n \leq m+1)$