UNIT - II

* FILTERS *

Filter:- A filter is a N/W which allows required wanted frequencies & blocks the undesired frequencies.

- The Range of frequencies that can be allowed is called "Transmission Band" (or) "pass band".

- The Range of frequencies that can be blocked are called "stop Band" (or) "Attenuation band".

- Filters consists atleast 'one' pass band & atleast 'one' stop band.

Classification of Filters:-

① Depending on Structures (or) Shapes:

i) π-section ii) T-section iii) L-section

② Depending on components used:

i) Active Filters ii) passive Filters.

③ Depending on frequencies allowed:

i) LPF (low pass Filters) ii) HPF (High pass F) iii) BPF (Band Pass F) iv) BRF | BEF (Band Rejected Filter, Band Elim...)

* Based on the Relation b/w series Arm Impedence (Z_1) & shunt Arm Impedence (Z_2):

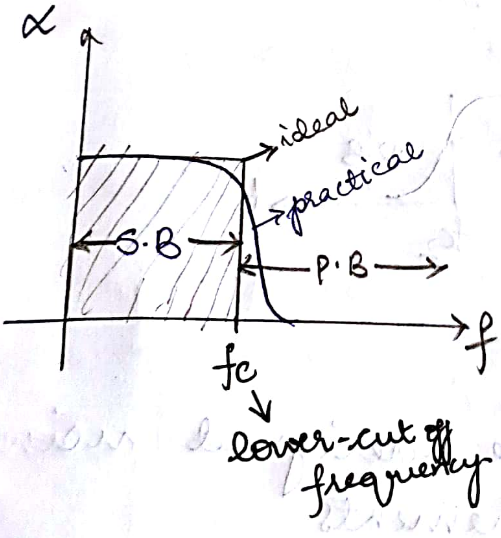
(i) Constant - K Filters / prototype filters

(ii) m- derived Filters

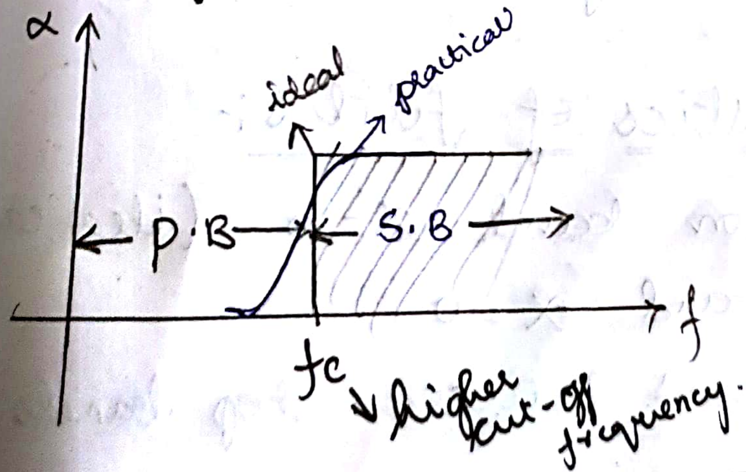
*→ The frequencies which can isolate the stop band & pass bands are called "cut off Frequencies".

Characteristics:-

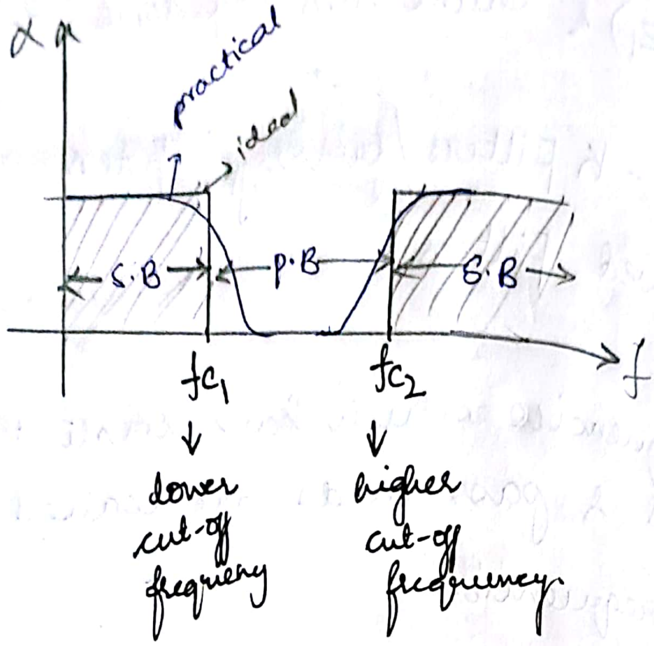
1) ~~High~~ pass Filter (HPF):-



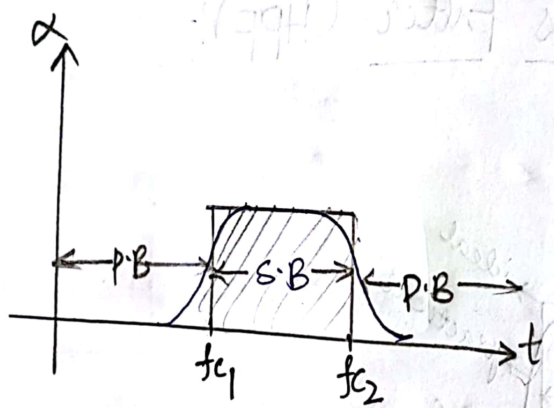
2) Low pass Filter (LPF):-



3.) Band pass Filter (BPF):-



4.) Band Reject Filter (BRF):



Note:-

passive Filters are designed using purely reactive elements
ex:- Inductors, capacitors.

Characteristics of Filters:-

i) In attenuation band $\alpha = \infty$ (ideal case)
→ In pass band $\alpha = 0$
→ Sharp transitions between stop bands

& pass bandw

Condition for cut-off frequencies of a Filter :-

If $Z_1 \rightarrow$ series arm Impedence
$Z_2 \rightarrow$ shunt " "

w.k.T

$$\cosh(p) = 1 + \frac{Z_1}{2Z_2}$$

w.k.T $p = \alpha + j\beta$

α → Attenuation const

β → phase shift

$$\cosh(\alpha + j\beta) = 1 + \frac{Z_1}{2Z_2}$$

$$\cosh\alpha \cdot \cosh(j\beta) + \sinh\alpha \cdot \sinh(j\beta) = 1 + \frac{Z_1}{2Z_2}$$

$$\cosh\alpha \cdot \cos\beta + j\sinh\alpha \cdot \sin\beta = 1 + \frac{Z_1}{2Z_2}$$

since $1 + \frac{Z_1}{Z_2}$ is a real value

$$\boxed{\cosh\alpha \cosh\beta = 1 + \frac{Z_1}{2Z_2}} \longrightarrow (1A)$$

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$$\boxed{\sinh\alpha \cdot \sin\beta = 0} \longrightarrow (1B)$$

From eqn (1B)

$\sinh(\alpha) = 0$ (or) $\sin\beta = 0$

i.e $\alpha = 0$ (or) $\beta = 0$ (or) $\pm\pi$

CASE-1: $\alpha = 0$ [pass band]

from eqn (A)

$\cosh(0) = 1$

$$\cos\beta = 1 + \frac{Z_1}{2Z_2}$$

$$\boxed{\beta = \cos^{-1}\left[1 + \frac{Z_1}{2Z_2}\right]} \longrightarrow (2)$$

W.K.T range of $\cos\beta$ is $[-1, 1]$

$$-1 < \cos\beta < 1$$

$$-1 < 1 + \frac{Z_1}{2Z_2} < 1$$

$$-2 < \frac{Z_1}{2Z_2} < 0$$

$$\boxed{-1 < \frac{Z_1}{4Z_2} < 0}$$

case-ii :- $\beta = 0$ (or) $\pm\pi$ [stop bands]

if $\beta = 0 \Rightarrow \cos\beta = 1$

if $\beta = \pm\pi \Rightarrow \cos\beta = -1$

If $\cos\beta = 1$ i.e $\beta = 0$:-

From eqn (1A)

$$\cosh\alpha[1] = 1 + \frac{Z_1}{2Z_2}$$

$$\boxed{\alpha = \cosh^{-1}\left[1 + \frac{Z_1}{2Z_2}\right]}$$

W.K.T range of $\cosh\alpha$ is $[1, \infty]$

$$1 < \cosh\alpha < \infty$$

$$1 < 1 + \frac{Z_1}{2Z_2} < \infty$$

$$0 < \frac{Z_1}{2Z_2} < \infty$$

$$\boxed{0 < \frac{Z_1}{4Z_2} < \infty}$$

If $\cos\beta = -1$ [i.e $\beta = \pm\pi$] :-

$$\cosh(\alpha)(-1) = 1 + \frac{Z_1}{2Z_2}$$

$$\cosh\alpha = -\left[1 + \frac{Z_1}{2Z_2}\right]$$

$$\boxed{\alpha = \cosh^{-1}\left(1 + \frac{Z_1}{2Z_2}\right)}$$

$$1 < \cosh\alpha < \infty$$

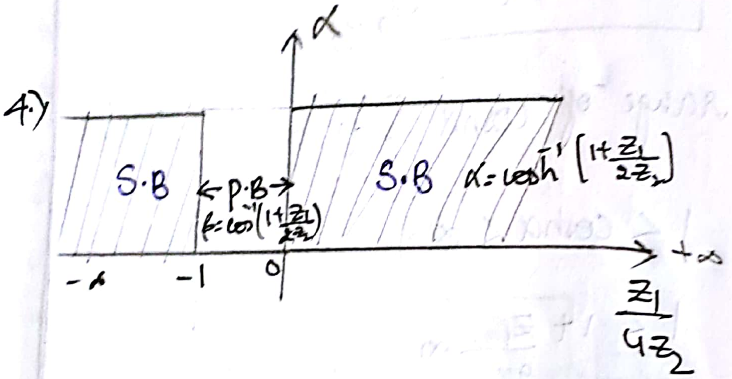
3.) $$1 < -\left[1+\frac{Z_1}{2Z_2}\right] < \infty$$

$$-1 > 1+\frac{Z_1}{2Z_2} > -\infty$$

$$-2 > \frac{Z_1}{2Z_2} > -\infty$$

$$-1 > \frac{Z_1}{4Z_2} > -\infty$$

4.)



condition for deriving cut-off frequency.

$$\boxed{\frac{Z_1}{4Z_2} = -1}$$

Constant K-Filters:-

If Z_1 - series arm Impedences &
Z_2 - shunt " "

then

In Constant K-filters, Z_1 & Z_2 have relation

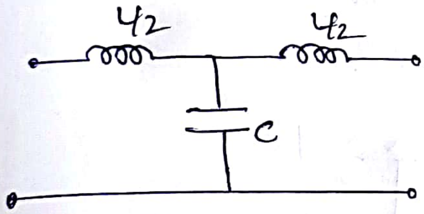
$$\boxed{Z_1 \cdot Z_2 = R_K^2}$$

where R_K - is a constant called : "Design impedence" / "Nominal Impedence"

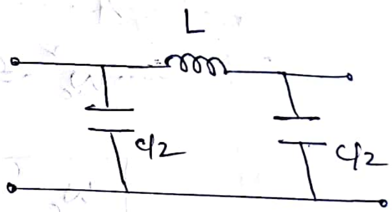
→ constant K-filters are also called "prototype filters" bcoz these are the fundamental blocks used for m-derived filters..

Constant-K LPF :-

standard constant-K LPF :-



T- type



π - type

$$\frac{Z_1}{2} = \frac{j\omega L}{2}.$$

$$\boxed{Z_1 = j\omega L}$$

$$\boxed{Z_2 = \frac{1}{j\omega C}}$$

w.k.T const K- filter

$$Z_1 Z_2 = R_K^2$$

$$\frac{j\omega L}{j\omega C} = R_K^2$$

$$R_k^2 = \frac{L}{C}$$

$$\boxed{R_k = \sqrt{\frac{L}{C}}} \quad \text{—— (4)}$$

<u>Higher</u> <u>cut-off</u> <u>Frequency</u> (f_c):-

$$\frac{Z_1}{4Z_2} = -1 \quad \text{for } f = f_c$$

$$\frac{j\omega_c L}{4\left[\frac{1}{j\omega_c C}\right]} = -1$$

$$\frac{\cancel{j^2}\,\omega_c^2 LC}{4} = \cancel{-1}$$

$$\omega_c^2 = \frac{4}{LC}$$

$$\boxed{\omega_c = \frac{2}{\sqrt{LC}}}$$

$$f_c = \frac{\omega}{2\pi} = \frac{1}{\cancel{2}\pi}\left[\frac{\cancel{2}}{\sqrt{LC}}\right]$$

$$\boxed{\therefore f_c = \frac{1}{\pi\sqrt{LC}}} \quad \text{—— (5)}$$

Characteristic Impedence :- (Z_{OT}) of T-N/w :-

we know that

$$Z_{OT} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$= \sqrt{\frac{(j\omega L)^2}{4} + \left[\frac{j\omega L}{j\omega C}\right]}$$

$$Z_{OT} = \sqrt{\frac{L}{C} - \frac{\omega^2 L^2}{4}}$$

$$Z_{OT} = \sqrt{\frac{L}{C}\left[1 - \frac{\omega^2 LC}{4}\right]} \qquad \left[\frac{4}{LC} = \omega_c^2\right]$$

$$= \sqrt{\frac{L}{C}\left[1 - \frac{\omega^2}{\omega_c^2}\right]}$$

$$= \sqrt{\frac{L}{C}} \sqrt{1 - \frac{\omega^2}{\omega_c^2}} \qquad f_c = \frac{\omega_c}{2\pi}$$

$$\omega_c = 2\pi f_c$$

$$= R_K \sqrt{1 - \left(\frac{2\pi f}{2\pi f_c}\right)^2}$$

$$\boxed{Z_{OT} = R_K \sqrt{1 - \left(\frac{f}{f_c}\right)^2}} \rightarrow (6a)$$

Characteristic Impedence of π-N/W :-

we know that

$$Z_{OT} \cdot Z_{O\pi} = Z_1 Z_2$$

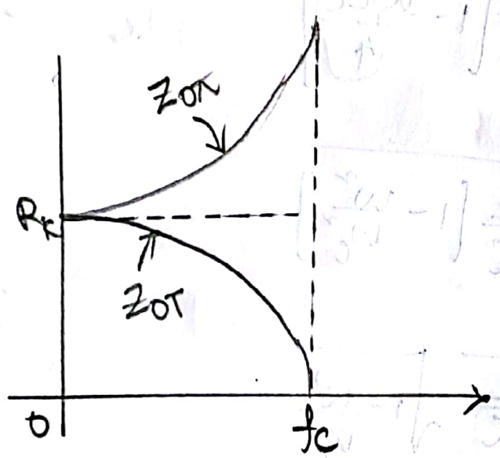
$$Z_{O\pi} = \frac{Z_1 Z_2}{Z_{OT}}$$

$$Z_{O\pi} = \frac{R_K^2}{R_K\sqrt{1-\left(\frac{f}{f_c}\right)^2}}$$

$$\boxed{Z_{O\pi} = \frac{R_K}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}}} \rightarrow (6b)$$

If $f=0 \Rightarrow Z_{OT} = R_K$ & also $Z_{O\pi} = R_K$

If $f=f_c \Rightarrow Z_{OT} = 0$ & $Z_{O\pi} = \infty$



Design of Low pass Filter when R_K & f_c are known:-

we know that

$$f_c = \frac{1}{\pi\sqrt{LC}} \rightarrow (5) \quad ; \quad R_k = \sqrt{\frac{L}{C}} \rightarrow (4)$$

$$(4)\times(5) \Rightarrow f_c \cdot R_k = \frac{1}{\pi\sqrt{LC}} \times \sqrt{\frac{L}{C}}$$

$$= \frac{1}{\pi}\sqrt{\frac{L}{LC\times C}}$$

$$f_c \cdot R_K = \frac{1}{\pi}\sqrt{\frac{1}{c^2}}$$

$$f_c \cdot R_K = \frac{1}{\pi C}$$

$$\boxed{C = \frac{1}{\pi f_c \cdot R_K}}$$

consider $\frac{R_K}{f_c} = \sqrt{\frac{L}{C}}\left[\pi \cdot \sqrt{LC}\right]$

$$= \sqrt{\frac{LL\cancel{C}}{\cancel{C}}} \cdot \pi$$

$$\frac{R_K}{f_c} = \pi L$$

$$\boxed{L = \frac{R_K}{\pi f_c}}$$

<u>Attenuation const</u> (α) & <u>phase shift const</u> (β)
<u>of const-K LPF</u> :-

We know that

$$\cosh(P) = 1 + \frac{Z_1}{2Z_2}$$

$$\cosh(\alpha + j\beta) = 1 + \frac{Z_1}{2Z_2} = 1 + \frac{j\omega L}{2\left(\frac{1}{j\omega C}\right)}$$

$$\cosh\alpha \cosh j\beta + \sinh\alpha \sinh j\beta = 1 - \frac{\omega^2 LC}{2}$$

$$\cosh\alpha \cos\beta + \sinh\alpha \cdot (j\sin\beta) = 1 - \frac{\omega^2 LC}{2}$$

$$\cosh\alpha\cos\beta + j\sinh\alpha\sin\beta = 1 - \frac{\omega^2 LC}{2}$$

$$\cosh\alpha\cos\beta + j\sinh\alpha\sin\beta = 1 - \omega^2\frac{\cancel{4}\,2/\omega_c^2}{\cancel{2}}$$

$$\cosh\alpha\cos\beta + j\sinh\alpha\sin\beta = 1 - \frac{2\omega^2}{\omega_c^2}$$

equating Real & Imaginary parts

$$\cosh\alpha\cos\beta = 1 - \frac{2\omega^2}{\omega_c^2} \longrightarrow (6c)$$

$$\sinh\alpha\sin\beta = 0 \longrightarrow (6d)$$

$$\boxed{\text{If } \alpha = 0 \text{ (or) } \beta = 0 \text{ or } \pm\pi}$$

case-(i):- If $\alpha = 0$

from eqn (6c) $\cosh(0) = 1$

$$(1)\cos\beta = 1 - \frac{2\omega^2}{\omega_c^2}$$

$$\cos\beta = 1 - \frac{2\omega^2}{\omega_c^2}$$

$$\cos^2\beta/2 - \sin^2\beta/2 = 1 - \frac{2\omega^2}{\omega_c^2}$$

$$\cancel{1} + 2\sin^2\beta/2 = \cancel{1} + \frac{2\omega^2}{\omega_c^2}$$

$\sin^2\beta/2 = \frac{\omega^2}{\omega_c^2}$

$(\sin\beta/2)^{\cancel{2}} =$

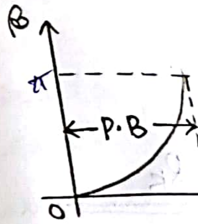
$\beta/2 =$

$\beta =$

β

at $f = 0 \Rightarrow$

$f = f_c \Rightarrow$



Note:- If $f <$

$f >$

case-(ii):-

$\cos\pi =$

Sub

(−

$$\sin^2 \beta/2 = \frac{\omega^2}{\omega_c^2}$$

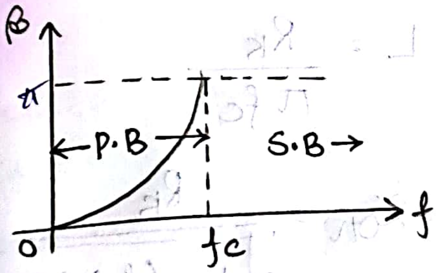
$$\left(\sin \beta/2\right)^{\cancel{2}} = \left(\frac{\omega}{\omega_c}\right)^{\cancel{2}}$$

$$\beta/2 = \sin^{-1}\left(\frac{\omega}{\omega_c}\right)$$

$$\boxed{\beta = \sin^{-1}\left(\frac{\omega}{\omega_c}\right) \text{ (or) } \beta = \sin^{-1}\left(\frac{f}{f_c}\right)}$$

at $f = 0 \Rightarrow \beta = 0$

$f = f_c \Rightarrow \beta = \pi$



Note :- If $f < f_c \rightarrow \beta = 2\sin^{-1}\left(\frac{f}{f_c}\right)$

$f > f_c \rightarrow \beta = \pi$ (maintained to be constant) assumption

case-ii :- $\beta = \pi$ [Neglecting 0 & $-\pi$ ∴ since after $f \geq f_c$ the phase shift β is maintained to be const i.e 'π' only]

$\cos\pi = -1$

Ans. 6c

$$(-1)\cosh\alpha = 1 - \frac{2\omega^2}{\omega_c^2}$$

$$\cosh\alpha = \frac{2\omega^2}{\omega_c^2} - 1$$

$$\boxed{\begin{aligned} \alpha &= \cosh^{-1}\left[\frac{2\omega^2}{\omega_c^2} - 1\right] \\ \alpha &= \cosh^{-1}\left[\frac{2f^2}{f_c^2} - 1\right] \end{aligned}}$$

Summary of Constant-K LPF:-

→ $Z_1 Z_2 = R_K^2$

$$R_K = \sqrt{\frac{L}{C}} \quad ; \quad f_c = \frac{1}{\pi\sqrt{LC}}$$

$$C = \frac{1}{\pi R_K f_c} \quad ; \quad L = \frac{R_K}{\pi f_c}$$

$$Z_{0T} = R_K\sqrt{1-\left(\frac{f}{f_c}\right)^2} \quad ; \quad Z_{0\pi} = \frac{R_K}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}}$$

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problems:-

Q1.) Design a constant-K LPF (π & T) to operate a load of 600Ω & have a cut-off frequency 3KHz

Sol: Given:- $f_c = 3KHz$; $R_K = 600\Omega$

W.K.T; $C = \dfrac{1}{\pi R_K f_c} = \dfrac{1}{\pi \times 600 \times 3\times10^3}$

$C = 0.176\mu F$

$L = \dfrac{R_K}{\pi f_c} = \dfrac{6\ldots}{\pi \ldots}$

$L/2 = 31.83mH$;

T-type:-

π-type:-

Q2.) Design ... a cut-o... impedan...

Sol:- $f_c = \ldots$

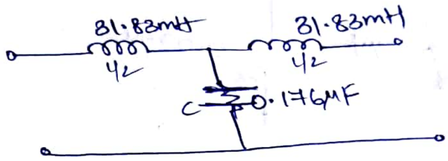
$C/2 = 0\ldots$

$\boxed{C = 0.176\mu F}$

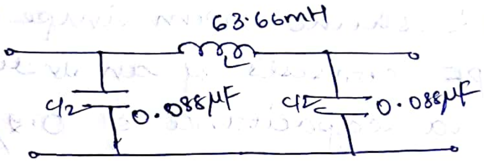
$$L = \frac{R_K}{\pi f_c} = \frac{600}{\pi \times 3 \times 10^3} = \boxed{63.66\,mH}$$

$L/2 = 31.83\,mH$; $C/2 = 0.088\,\mu F$.

T-type :-



π-type :-



Q2.) Design the elements of prototype LPF have a cut-off frequency of 8KHz & Design impedence of 600Ω.

Sol :- $f_c = 8KHz$; $R_K = 600\Omega$

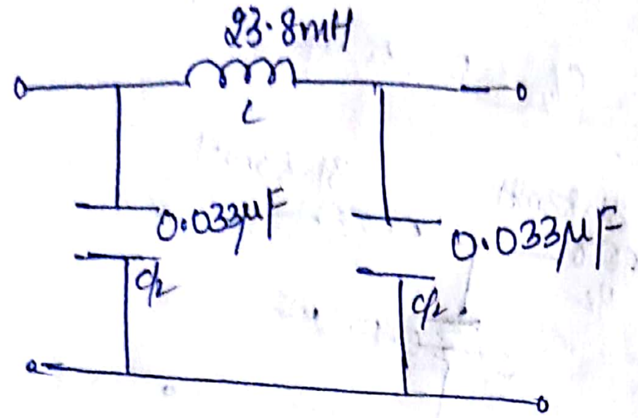
$$C = \frac{1}{\pi R_K f_c} \quad ; \quad L = \frac{R_K}{\pi f_c}$$

$$C = \frac{1}{\pi \times 600 \times 8 \times 10^3} \quad ; \quad L = \frac{600}{\pi \times 8 \times 10^3}$$

$\boxed{C = 0.066\mu F}$; $L = 0.0238$

$\boxed{L = 23.8\,mH}$

$C/2 = 0.033\mu F$; $L/2 = 11.9\,mH$



Q3) Series & shunt arm impedences of a T-type LPF consists of an inductance of 60mH & a capacitance of 0·2μF respectively. Calculate:

i) Cut-off Frequency
ii) Design Impedence
iii) Attenuation const & phase shift const at 5KHz

Sol:- $f_c = ?$, $R_k = ?$, $\alpha = ?$, $\beta = ?$

i) $f_c = ?$

Given;- $L = 60mH$, $C = 0{\cdot}2\mu F$

$$f_c = \frac{1}{\pi\sqrt{LC}} \Rightarrow \frac{1}{\pi \times \sqrt{60\times10^{-3} \times 0{\cdot}2\times10^{-6}}}$$

$$f_c = 2905{\cdot}75\ Hz$$

$$\boxed{f_c = 2{\cdot}91 KHz}$$

ii) $R_K = ?$

$$R_K = \sqrt{\frac{L}{C}}$$

$$= \sqrt{\frac{60\times10^{-3}}{0.2\times10^{-6}}}$$

$$\boxed{R_K = 547.72\,\Omega}$$

iii) $\alpha = ?$; $\beta = ?$

$$\alpha = \cosh^{-1}\left[\frac{2f^2}{f_c^2} - 1\right] \quad ; \quad \beta = \sin^{-1}\left[\frac{f}{f_c}\right]$$

$$f = 5\,KHz > f_c$$

$$\alpha = \cosh^{-1}\left[2\left[\frac{5K}{2.91K}\right)^2 - 1\right]$$

$$\boxed{\alpha = 2.27\,Np}$$

Now phase shift const (β):-

$f = 5KHz$, $\therefore f > f_c$ then $\boxed{\beta = \pi}$

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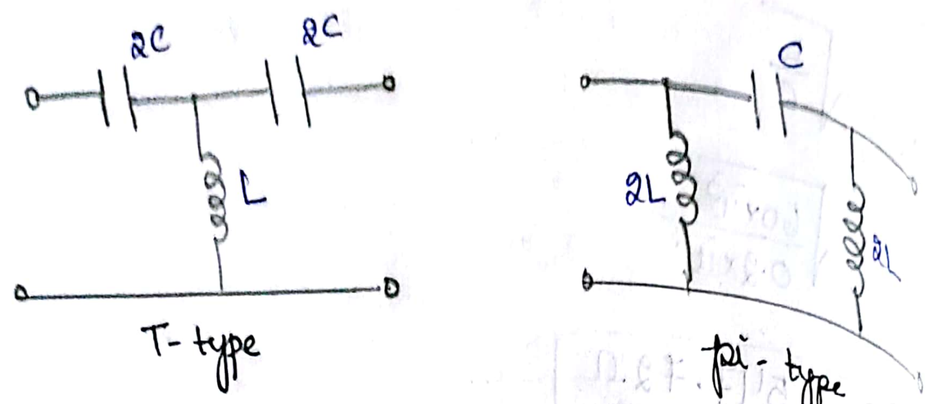
...espectively

...5 at ~~5KHz~~ 5KHz

Constant-K HPF :-



$Z_1 = \frac{1}{j\omega C}$; $Z_2 = j\omega L$

Design impedence (R_K) :-

W.K.T for constant K- filter

$$Z_1 Z_2 = R_K^2$$

$$R_K^2 = \frac{1}{(j\omega C)} (j\omega L)$$

$$\boxed{R_K = \sqrt{\frac{L}{C}}}$$

Cut-off frequency :-

W.K.T $\frac{Z_1}{4Z_2} = -1$ at $f = f_c$

$$\frac{1}{j\omega_c C} = -4(j\omega_c L)$$

$$1 = -4j^2\omega_c^2 LC$$

$$\boxed{\omega_c^2 = \frac{1}{4LC}}$$

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$$(2\pi f_c)^2 = \frac{1}{4LC}$$

$$f_c^2 = \frac{1}{4LC \times 4\pi^2}$$

$$\boxed{f_c = \frac{1}{4\pi\sqrt{LC}}}$$

Characteristic Impedence :-

W.K.T for T- N/W

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$= \sqrt{\frac{1}{j^2\omega^2C^2 \times 4} + \frac{L}{C}}$$

$$= \sqrt{\frac{L}{C} - \frac{1}{4\omega^2C^2}}$$

$$= \sqrt{\frac{L}{C}\left[1 - \frac{1}{4\omega^2 LC}\right]}$$

$$= \sqrt{\frac{L}{C}}\sqrt{1 - \frac{\omega_c^2}{\omega^2}}$$

$$\boxed{Z_{0T} = R_K\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

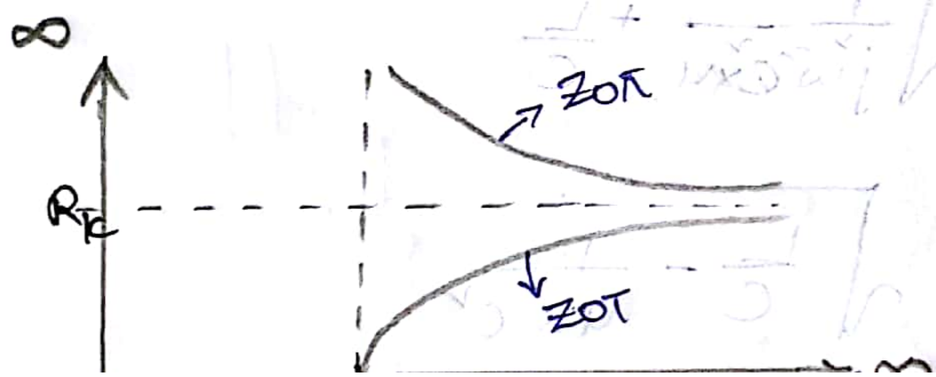
W.K.T

$$Z_{0\pi} \cdot Z_{0T} = Z_1 Z_2$$

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}}$$

$$= \frac{R_k^{\cancel{2}}}{\cancel{R_k}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\boxed{Z_{0\pi} = \frac{R_k}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}}$$



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Design of constant-K HPF if f_c & R_K are given :-

W.K.T for constant-K HPF,

$$f_c = \frac{1}{4\pi\sqrt{LC}} \quad \text{and}$$

$$R_K = \sqrt{\frac{L}{C}}$$

$$f_c \times R_K = \sqrt{\frac{L}{C}} \times \frac{1}{4\pi\sqrt{LC}}$$

$$= \frac{1}{4\pi}\sqrt{\frac{L}{C^2 L}}$$

$$\boxed{C = \frac{1}{4\pi f_c R_K}}$$

$$\frac{R_K}{f_c} = \sqrt{\frac{L}{C}} \cdot 4\pi\sqrt{LC}$$

$$\frac{R_K}{f_c} = 4\pi L$$

$$\boxed{L = \frac{R_K}{4\pi f_c}}$$

Attenuation & phase-shift constants:-

W.K.T $\cosh p = 1 + \frac{Z_1}{2Z_2}$

$$\cosh(\alpha + j\beta) = 1 + \frac{\left(\frac{1}{j\omega C}\right)}{2(j\omega L)}$$

$$\Rightarrow \cosh\alpha \cdot \cos\beta + j\sinh\alpha \sin\beta = 1 + \frac{1}{2j^2\omega^2 LC}$$

$$\cosh\alpha \cos\beta + j\sinh\alpha \sin\beta = 1 - \frac{1}{2\omega^2 LC}$$

$$\cosh\alpha \cos\beta + j\sinh\alpha \sin\beta = 1 - \frac{2\omega_c^2}{\omega^2}$$

comparing both sides

$$\cosh\alpha \cos\beta = 1 - 2\left(\frac{f_c}{f}\right)^2$$

$$\sinh\alpha \cdot \sin\beta = 0$$

$\therefore$ $\sinh\alpha \sin\beta$ becomes zero if either $\boxed{\alpha = 0}$ (or) $\boxed{\beta = 0 \text{ or } \pm\pi}$

Case (i):- $\boxed{\alpha = 0}$

$$\cosh\alpha = \cosh(0) = 1$$

$$\cos\beta = 1 - 2\left(\frac{f_c}{f}\right)^2$$

$$\boxed{\beta = \cos^{-1}\left[1 - 2\left(\frac{f_c}{f}\right)^2\right]}$$

(or)

$$\cos^2\beta/2 - \sin^2\beta/2 = 1 - 2\left(\frac{f_c}{f}\right)^2$$

$$\cancel{1} - \cancel{2}\sin^2\beta/2 = \cancel{1} - \cancel{2}\left(\frac{f_c}{f}\right)^2$$

$$\sin\beta/2 = \left(\frac{f_c}{f}\right)$$

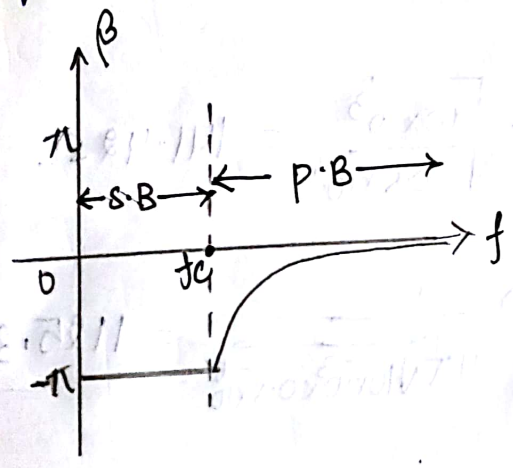
$$\boxed{\beta = 2\sin^{-1}\left[\frac{f_c}{f}\right]}$$

At $f = f_c \Rightarrow \beta = \pi$

$f = \infty \Rightarrow \beta = 0$

NOTE:-

But practically it has been observed that phase shift $\boxed{\beta = -\pi}$ for $f = f_c$. This is due to the properties of HPF or exactly opposite to LPF.



case (ii): consider $\boxed{\beta = -\pi}$

$\cos(\beta) \Rightarrow \cos(-\pi) = -1$

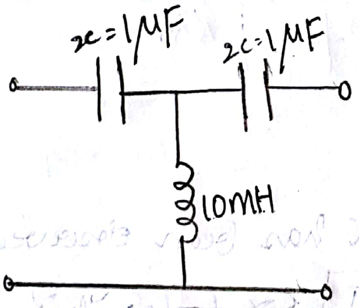
$\cosh\alpha(-1) = 1 - 2\left(\frac{f_c}{f}\right)^2$

$-\cosh\alpha = 1 - 2\left(\frac{f_c}{f}\right)^2$

$$\boxed{\alpha = \cosh^{-1}\left[2\left(\frac{f_c}{f}\right)^2 - 1\right]}$$

problems:

Q1:- For a const k HPF given below. Calc f_c & R_K. consider operating freq = 10 KHz



Sol: $C = 0.5\mu F$

$L = 10mH$

$R_K = \sqrt{\frac{L}{C}} \Rightarrow \sqrt{\frac{10\times10^{-3}}{0.5\times10^{-6}}} = 141.42\,\Omega$

$f_c = \frac{1}{4\pi\sqrt{LC}} \Rightarrow \frac{1}{4\pi\sqrt{10\times10^{-3}\times0.5\times10^{-6}}} = 1125.39\,Hz$

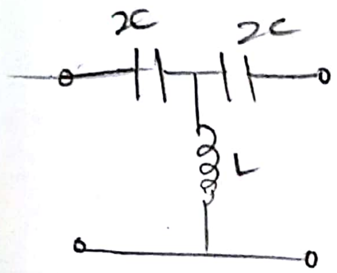
Q.> Design a constant-K HPF with cut-off Frequency 5KHz & Design Impedance 600Ω & also calculate

1> Attenuation & phase-shift constants at $f = 2KHz$

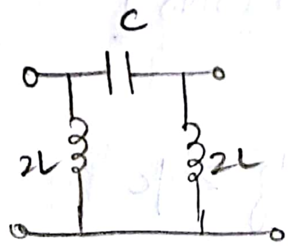
2> At $f = 10KHz$

3> Z_{OT} at $f = 7KHz$ = 419.91

sol:-



T-type



pi-type

$$C = \frac{1}{4\pi f_c R_K} = \frac{1}{4\pi \times 5 \times 10^3 \times 600} = 0.02653 \mu F$$

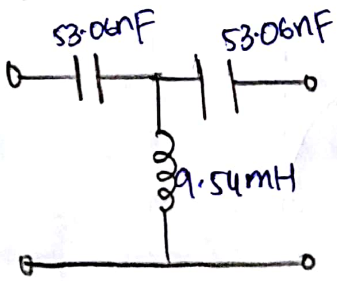
(or)

26.53nF

$2C \Rightarrow 53.06nF$

$$L = \frac{R_K}{4\pi f_c} \Rightarrow \frac{600}{4\pi \times 5 \times 10^3} = 9.54mH$$

$2L \Rightarrow 19.08mH$



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ii) $f = 2\,kHz < f_c$

$\therefore \boxed{\beta = \pi}$

$$\alpha = \cosh^{-1}\left[2\left(\frac{f_c}{f}\right)^2 - 1\right]$$

$$\alpha = \cosh^{-1}\left[2\left(\frac{5}{2}\right)^2 - 1\right]$$

$$\boxed{\alpha = 3.12\,Np}$$

iii) $f = 10\,kHz > f_c$

$$\therefore \beta = 2\sin^{-1}\left[\frac{f_c}{f}\right]$$

$$\beta = 60$$

$$\boxed{\beta = \frac{\pi}{3}}$$

$$\alpha = \cosh^{-1}\left[2\left(\frac{f_c}{f}\right)^2 - 1\right]$$

$\boxed{f > f_c}$

(*) For frequencies $> f_c$, we cannot calculate attenuation const 'α'.

iii) $Z_{0T} = R_K\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

Q3) Design a const K-LPF with design Impedence 500Ω & cut-off frequency $f_c = 3KHz$. Calculate Attenuation & phase-shift constants at 1KHz & 5KHz respectively.

Sol:- $R_k = 500\Omega$

$f_c = 3KHz$.

LPF:

$$C = \frac{1}{\pi f_c R_k} = \frac{1}{\pi \times 3\times10^3 \times 500} = 0.212\mu F$$

$C/2 = 0.106\mu F$

$$L = \frac{R_k}{\pi f_c} = \frac{500}{\pi \times 3\times10^3} = 0.053H \Rightarrow 53.05mH$$

$L/2 = 26.52mH$

given $f = 1KHz$

$\therefore$ Here $f_c > f$

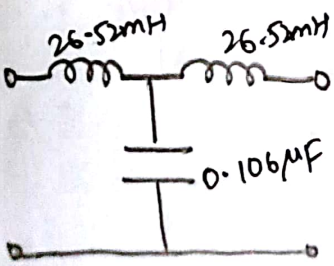
then $\beta = 2\sin^{-1}\left[\frac{f}{f_c}\right]$

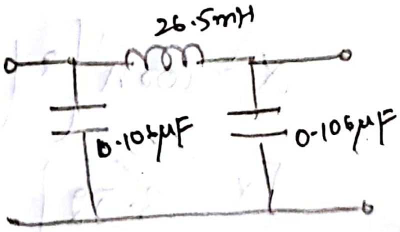
$= 2\sin^{-1}\left[\frac{1}{3}\right]$

$\beta = 38.94$

$\boxed{\beta \simeq 39}$ or $\boxed{\beta = 0.67 rad}$

$\therefore \alpha$ is not defined. coz $(f < f_c)$





At $f = 5 KHz$.

Here $f > f_c$

$\boxed{\beta = \pi}$

$\therefore \alpha = \cosh^{-1}\left[2\left(\frac{f}{f_c}\right)^2 - 1\right]$

$= \cosh^{-1}\left[2\left(\frac{5}{3}\right)^2 - 1\right]$

$\boxed{\alpha = 2.197 \, Np}$

HPF :-

$C = \frac{1}{4\pi f_c \cdot R_K} = \frac{1}{4\pi \times 3 \times 10^3 \times 500} \Rightarrow 53.05 nF$

$L = \frac{R_K}{4\pi f_c} \Rightarrow 0.013 H$

or

$13.26 mH$

At $f = 1 KHz$

$\therefore f_c > f$

$\boxed{\beta = -\pi}$

$\alpha = \cosh^{-1}\left[2\left(\frac{f_c}{f}\right)^2 - 1\right]$

$\boxed{\alpha = 3.52}$

At $f = 5 KHz$

$\therefore f_c < f$

$\beta = \sin^{-1}$ [

$\boxed{\beta = -36.}$

'α' not

→ At $f = 5KHz$

$\therefore f_c < f$

$$\beta = \sin^{-1}\left[\frac{f_c}{f}\right]$$

$$\boxed{\beta = -36.86}$$

'α' not defined.

13/8/18

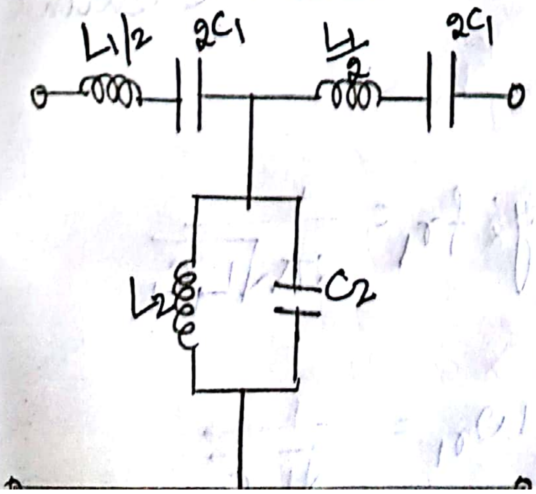
Constant-K Band pass Filter (BPF):-

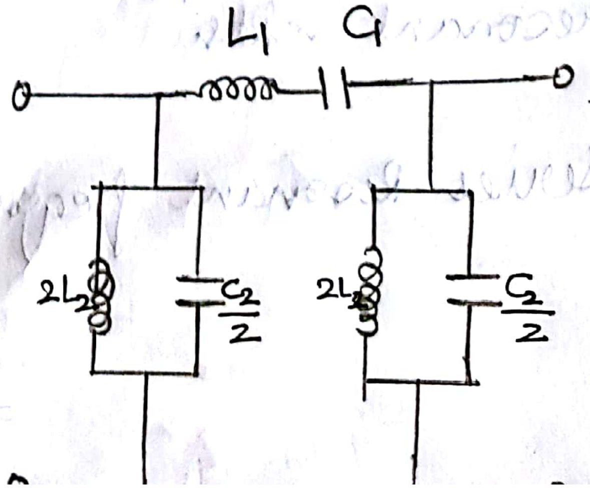
A Band pass Filter is a N/W which allows signals b/w a range of frequency f_{c1} to f_{c2}

f_{c1} is lower cut-off Frequency & (lcf)

f_{c2} is higher " " " (hcf)

Practically, a BPF can be obtained by properly joining a LPF & HPF in such a way that lcf of HPF is less than the hcf of LPF;





For T- N/W;

$$Z_1 = j\omega L_1 - \frac{j}{\omega C_1}$$

$$\boxed{Z_1 = j\left[\omega L_1 - \frac{1}{\omega C_1}\right]}$$

$$Z_2 = \frac{(j\omega L_2)\left(\frac{1}{j\omega C_2}\right)}{j\omega L_2 + \frac{1}{j\omega C_2}}$$

$$Z_2 = \frac{\frac{j\omega L_2}{\cancel{j\omega C_2}}}{\frac{j^2\omega^2 L_2 C_2 + 1}{\cancel{j\omega C_2}}} = \frac{j\omega L_2}{1-\omega^2 L_2 C_2}$$

$$\boxed{Z_2 = \frac{j\omega L_2}{1-\omega^2 L_2 C_2}}$$

→ Constant-K BPF consists series & shunt resonant ckts.

⊛ Series Resonant frequency; $f_{r_1} = \frac{1}{2\pi\sqrt{L_1 C_1}}$

$$\Rightarrow \omega_{r_1} = \frac{1}{\sqrt{L_1 C_1}}$$

Shunt Resonant Frequency; $f_{r_2} = \frac{1}{2\pi\sqrt{L_2 C_2}}$

$$\Rightarrow \omega_{r_2} = \frac{1}{\sqrt{L_2 C_2}}$$

→ In constant-K BPF design, L_1, C_1 & L_2, C_2 are selected in such a way that

$\omega_{r_1} = \omega_{r_2} = \omega_0$ [common Resonant Freq]

$$\omega_0 = \frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{L_2 C_2}}$$

$$\omega_0^2 = \frac{1}{L_1 C_1} = \frac{1}{L_2 C_2}$$

$$\boxed{L_1 C_1 = L_2 C_2 = \frac{1}{\omega_0^2}}$$

Design Impedence:- $[R_K]$

W.K.T for const-K BPF

$$Z_1 Z_2 = R_K^2$$

$$j\left(\omega L_1 - \frac{1}{\omega C_1}\right)\left[\frac{j\omega L_2}{1-\omega^2 L_2 C_2}\right] = R_K^2$$

$$j^2\left[\frac{\omega^2 L_1 C_1 - 1}{\cancel{\omega} C_1}\right]\left[\frac{\cancel{\omega} L_2}{1-\omega^2 L_2 C_2}\right] = R_K^2$$

$$R_K^2 = \frac{L_2}{C_1}\left[\frac{1-\omega^2 L_1 C_1}{1-\omega^2 L_2 C_2}\right]$$

At $\omega = \omega_0$ " $L_1 C_1 = L_2 C_2$ "

$$\boxed{R_K = \sqrt{\frac{L_2}{C_1}}}$$ (or) $$\boxed{\therefore L_1C_1 = L_2C_2}$$

$$\boxed{R_K = \sqrt{\frac{L_1}{C_2}}}$$

Cutt-off frequency :-

W.K.T

from condition of cut-off freq.

$$\frac{Z_1}{4Z_2} = -1$$

$$\Rightarrow Z_1 = -4Z_2$$

$$j\left[\omega L_1 - \frac{1}{\omega C_1}\right] = -4\left[\frac{j\omega L_2}{1-\omega^2 L_2C_2}\right]$$

$$\frac{\omega^2 L_1C_1 - 1}{\omega C_1} = \frac{-4\omega L_2}{1-\omega^2 L_2C_2}$$

b.s multiply with 'Z_1'

$$Z_1 Z_1 = -4Z_1Z_2$$

$$Z_1^2 = -4Z_1Z_2$$

$$Z_1^2 = -4R_K^2$$

$$\boxed{Z_1 = \pm 2jR_K}$$

i.e $Z_1 = -j2R_K$

$Z_1 = j2R_K$

$$[Z_1]_{f=f_{c_1}}$$

$$j\left[\omega_{c_1}L_1 - \frac{1}{\omega_{c_1}\ldots}\right.$$

$$\frac{\omega_{c_1}^2 L_1 C\ldots}{\omega_{c_1}\ldots}$$

$$\omega_{c_1}^2 \ldots$$

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i.e $Z_1 = -j2R_K$ at $f = f_{c_1}$

$Z_1 = j2R_K$ at $f = f_{c_2}$

$$[Z_1]_{f=f_{c_1}} = [-Z_1]_{f=f_{c_2}}$$

$$j\left[\omega_{c_1}L_1 - \frac{1}{\omega_{c_1}C_1}\right] = -j\left[\omega_{c_2}L_1 - \frac{1}{\omega_{c_2}C_1}\right]$$

$$\frac{\omega_{c_1}^2 L_1 C_1 - 1}{\omega_{c_1}\cancel{C_1}} = -\left[\frac{\omega_{c_2}^2 L_1 C_1 - 1}{\omega_{c_2}\cancel{C_1}}\right]$$

$$\frac{\omega_{c_1}^2 L_1 C_1 - 1}{\omega_{c_1}} = \frac{1 - \omega_{c_2}^2 L_1 C_1}{\omega_{c_2}}$$

$$\frac{\omega_{c_1}}{\omega_{c_2}} = \frac{\omega_{c_1}^2 L_1 C_1 - 1}{1 - \omega_{c_2}^2 L_1 C_1}$$

$$\frac{\omega_{c_1}}{\omega_{c_2}} = \frac{\frac{\omega_{c_1}^2}{\omega_0^2} - 1}{1 - \frac{\omega_{c_2}^2}{\omega_0^2}}$$

$$\frac{\omega_{c_1}}{\omega_{c_2}} = \frac{\omega_{c_1}^2 - \omega_0^2}{\omega_0^2 - \omega_{c_2}^2}$$

$$\omega_{c_1}[\omega_0^2 - \omega_{c_2}^2] = (\omega_{c_1}^2 - \omega_0^2)\,\omega_{c_2}$$

$$\omega_{c_1}\omega_0^2 - \omega_{c_2}^2\omega_{c_1} = \omega_{c_1}^2\omega_{c_2} - \omega_0^2\omega_{c_2}$$

$$\omega_0^2(\omega_{c_1}+\omega_{c_2}) = \omega_{c_1}\omega_{c_2}(\cancel{\omega_{c_1}+\omega_{c_2}})$$

$$\omega_0^2 = \omega_{c_1}\omega_{c_2}$$

$$\boxed{\omega_0 = \sqrt{\omega_{c_1}\omega_{c_2}}}$$

(or)

$$\boxed{f_0 = \sqrt{f_{c_1} f_{c_2}}}$$

Design of Constant-K BPF :-

W.K.T for const-K BPF,

$$Z_1 = -j2R_K \quad \text{at } f = f_{c_1}$$

$$\cancel{j}\left[\omega_{c_1}L_1 - \frac{1}{\omega_{c_1}C_1}\right] = -\cancel{j}2R_K$$

$$\frac{\omega_{c_1}^2 L_1 C_1 - 1}{\omega_{c_1}C_1} = -2R_K$$

$$1-\omega_{c_1}^2 L_1 C_1 = 2R_K\,\omega_{c_1}C_1$$

$$1-\frac{\omega_{c_1}^2}{\omega_0^2} = 2R_K\,\omega_{c_1}C_1 \qquad \left[\because \omega_0^2 = \frac{1}{L_1C_1}\right]$$

$$C_1 = \frac{\left[1-\frac{\omega_{c_1}^2}{\omega_0^2}\right]}{2R_K\,\omega_{c_1}}$$

$$C_1 = \frac{\omega_0^2 - \omega_{c_1}^2}{2R_K \omega_{c_1} \omega_0^2}$$

$$C_1 = \frac{4\pi^2 \left[f_0^2 - f_{c_1}^2\right]}{2R_K \, 2\pi f_{c_1} \cdot 4\pi^2 f_0^2}$$

$$C_1 = \frac{f_{c_1} f_{c_2} - f_{c_1}^2}{4\pi R_K f_{c_1} \, f_{c_1} f_{c_2}} \qquad \left[\because f_0 = \sqrt{f_{c_1} f_{c_2}}\right]$$

$$C_1 = \frac{\cancel{f_{c_1}} \left[f_{c_2} - f_{c_1}\right]}{4\pi R_K \cancel{f_{c_1}} \, f_{c_1} f_{c_2}}$$

$$\boxed{C_1 = \frac{f_{c_2} - f_{c_1}}{4\pi R_K f_{c_1} f_{c_2}}}$$

W.K.T

$$\omega_0^2 = \frac{1}{L_1 C_1}$$

$$L_1 = \frac{1}{\omega_0^2 C_1}$$

$$= \frac{\cancel{4\pi} R_K \cancel{f_{c_1} f_{c_2}}}{\cancel{4\pi^2} \cancel{f_0^2} (f_{c_2} - f_{c_1})}$$

$$\boxed{L_1 = \frac{R_K}{\pi (f_{c_2} - f_{c_1})}}$$

W.K.T. $R_K = \sqrt{\frac{L_2}{C_1}}$ or $\sqrt{\frac{L_1}{C_2}}$

let $R_K^2 = \frac{L_2}{C_1}$

$$L_2 = R_K^2 C_1$$

$$= \frac{R_K^{\cancel{2}} (f_{c_2} - f_{c_1})}{4\pi \cancel{R_K} f_{c_1} f_{c_2}}$$

$$\boxed{L_2 = \frac{R_K (f_{c_2} - f_{c_1})}{4\pi f_{c_1} f_{c_2}}}$$

Also $R_K^2 = \frac{L_1}{C_2}$

$$C_2 = \frac{L_1}{R_K^2}$$

$$C_2 = \frac{\cancel{R_K}}{\pi (f_{c_2} - f_{c_1}) R_K^{\cancel{2}}}$$

$$\boxed{C_2 = \frac{1}{\pi R_K (f_{c_2} - f_{c_1})}}$$

$$\boxed{L_1 = \frac{R_K}{\pi (f_{c_2} - f_{c_1})} \qquad L_2 = \frac{R_K (f_{c_2} - f \ldots}{4\pi f_{c_1} \ldots}}$$

Summary

1) $L_1 C_1 = L_2 C \ldots$

2) $R_K = \sqrt{\frac{L_2}{C_1} \ldots}$

3) $Z_1 = -j2R \ldots$

$Z_2 = j2 \ldots$

4) $f_0 = \sqrt{\ldots}$

5) $L_1 = \frac{\ldots}{\pi \ldots}$

6) $L_2 = \ldots$

$$\therefore L_1 = \frac{R_K}{\pi(f_{c_2} - f_{c_1})} \quad ; \quad C_1 = \frac{f_{c_2} - f_{c_1}}{4\pi R_K f_{c_1} f_{c_2}}$$

$$L_2 = \frac{R_K(f_{c_2} - f_{c_1})}{4\pi f_{c_1} f_{c_2}} \quad ; \quad C_2 = \frac{1}{\pi R_K (f_{c_2} - f_{c_1})}$$

Summary of const-K BPF:-

1.) $L_1 C_1 = L_2 C_2 = \frac{1}{\omega_0^2}$

2.) $R_K = \sqrt{\frac{L_2}{C_1}}$ or $\sqrt{\frac{L_1}{C_2}}$

3.) $Z_1 = -j2R_K$ at $f = f_{c_1}$ [L.C.F]

$Z_1 = j2R_K$ at $f = f_{c_2}$ [H.C.F]

4.) $f_0 = \sqrt{f_{c_1} f_{c_2}}$ or $\omega_0 = \sqrt{\omega_{c_1} \omega_{c_2}}$

5.) $L_1 = \frac{R_K}{\pi(f_{c_2} - f_{c_1})}$; $C_1 = \frac{f_{c_2} - f_{c_1}}{4\pi R_K f_{c_1} f_{c_2}}$

6.) $L_2 = \frac{R_K(f_{c_2} - f_{c_1})}{4\pi f_{c_1} f_{c_2}}$; $C_2 = \frac{1}{\pi R_K (f_{c_2} - f_{c_1})}$

Problems:-

Q1.> Design a prototype (const-K) BPF with cut-off frequencies 2KHz & 5KHz respectively & design impedence of 600Ω. Also calculate Resonant frequency.

Sol:- Given:-

$R_k = 600\Omega$

$f_{c_1} = 2KHz = 2000Hz$

$f_{c_2} = 5KHz = 5000Hz$

$f_{c_2} - f_{c_1} = 3000Hz$

Resonant freq $f_0 = \sqrt{f_{c_1} f_{c_2}}$

$= \sqrt{(2000)(5000)}$

$f_0 = \underline{3.16 KHz}$

$L_1 = \frac{R_k}{\pi(f_{c_2} - f_{c_1})} \Rightarrow \frac{6\cancel{00}}{\pi(30\cancel{00})} = \underline{63.66mH}$

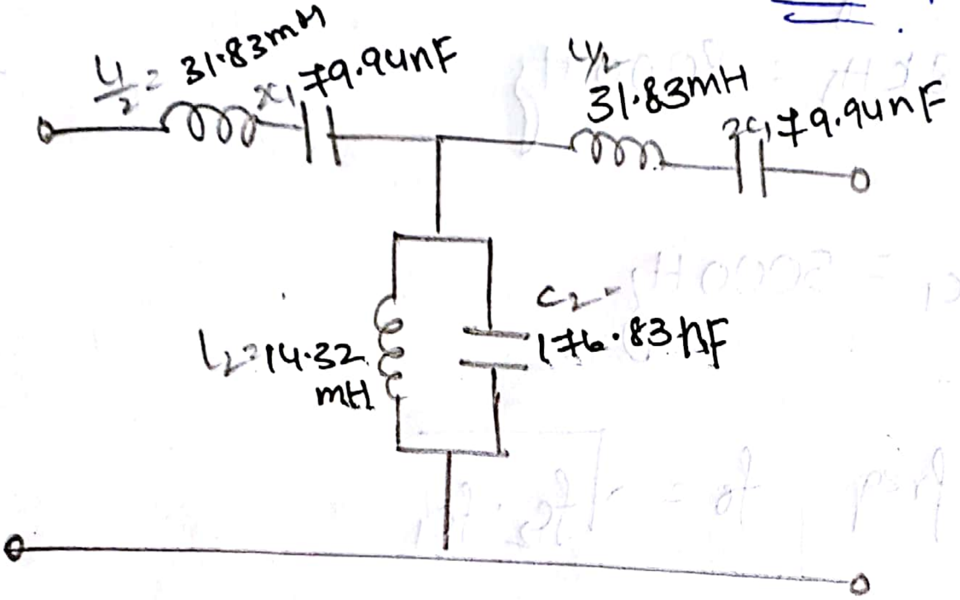
$C_1 = \frac{f_{c_2} - f_{c_1}}{4\pi R_k f_{c_1} f_{c_2}} \Rightarrow \frac{3000}{4\pi \times 600 \times 2000 \times 5000} \Rightarrow \underline{39.79nF}$

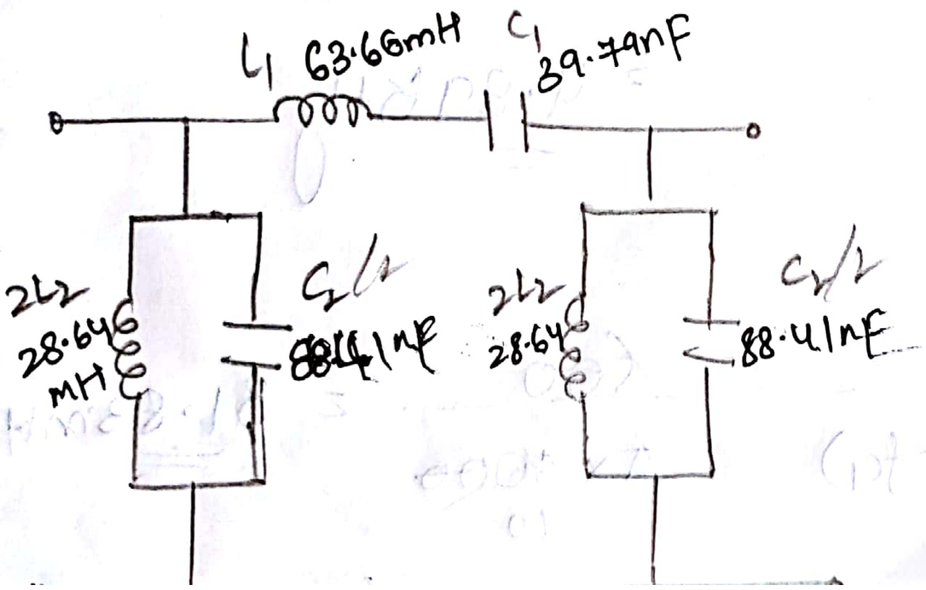
$$L_2 = \frac{R_k(f_{c_2} - f_{c_1})}{4\pi f_{c_1} f_2} \Rightarrow \frac{600(3000)}{4\pi \times 2000 \times 5000}$$

$$= 14.32\,\text{mH}$$

$$C_2 = \frac{1}{\pi R_k(f_{c_2} - f_{c_1})} = \frac{1}{\pi \times 600 \times 3000}$$

$$= 176.83\,\text{nF}$$





Q2.> Design a const-K BPF with cut-off frequencies 3KHz & 8KHz respectively & design Impedence 500Ω. Also calculate Resonant Freq.

Sol:- Given:-

$R_k = 500\,\Omega$

$f_{c_1} = 3\,KHz = 3000\,Hz$

$f_{c_2} = 8\,KHz = 8000\,Hz$

$f_{c_2} - f_{c_1} = 5000\,Hz$

Resonant freq $f_0 = \sqrt{f_{c_2} \cdot f_{c_1}}$

$= \sqrt{(8000)(3000)}$

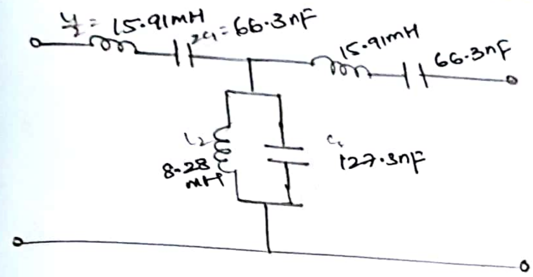
$= 4.89\,KHz$

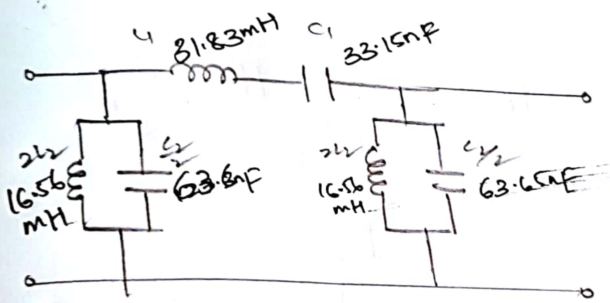
$L_1 = \dfrac{R_k}{\pi(f_{c_2} - f_{c_1})} = \dfrac{\cancel{500}}{\pi \times \cancel{5000}_{10}} = 31.83\,mH$

$C_1 = \dfrac{f_{c_2} - f_{c_1}}{4\pi R_k f_{c_1} f_{c_2}} \Rightarrow \dfrac{5000}{4\pi \times 500 \times 3000 \times 8000} = 33.15\,nF$

$L_2 = \dfrac{R_k(f_{c_2} - f_{c_1})}{4\pi f_{c_1} f_{c_2}} \Rightarrow \dfrac{500 \times 5000}{4\pi \times 3000 \times 8000} = 8.28\,mH$

$$C_2 = \frac{1}{\pi R_k (f_{c_1} - f_{c_2})} \Rightarrow \frac{1}{\pi \times 500 \times 5000} \Rightarrow \underline{127.3\text{nF}}$$





NOTE:-

For constant-k BPF;

$$Z_{0T} = R_k \sqrt{1 - \left[\frac{x^2 - 1}{2nx}\right]^2}$$

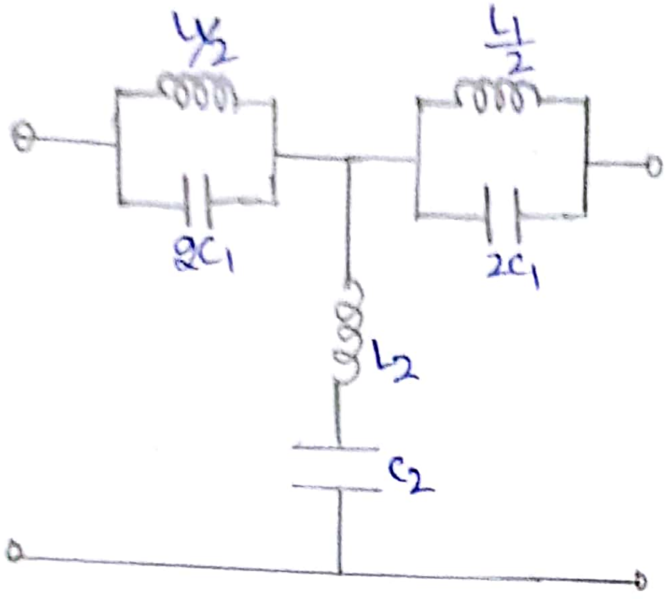
$$\alpha = \cosh^{-1}\left[\frac{(1-x^2)^2}{2n^2x^2} - 1\right]$$

$$\beta = \cos^{-1}\left[1 - \frac{(1-x^2)^2}{2n^2x^2}\right]$$

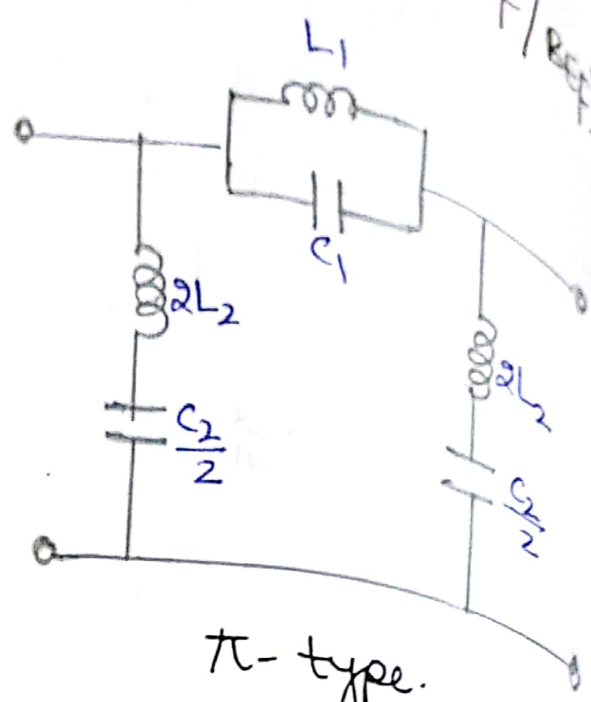
where

$$x = \left(\frac{f}{f_0}\right) \quad ; \quad n = \frac{f_{c_2} - f_{c_1}}{2f_0}$$

(BSF/BEF)



T-type



π-type.

$$Z_1 = \frac{(j\omega L_1)\left(\frac{1}{j\omega C_1}\right)}{j\omega L_1 + \frac{1}{j\omega C_1}}$$

$$Z_1 = \frac{\frac{j\omega L_1}{\cancel{j\omega C_1}}}{\frac{j^2\omega^2 L_1 C_1 + 1}{\cancel{j\omega C_1}}}$$

$$\boxed{Z_1 = \frac{j\omega L_1}{1-\omega^2 L_1 C_1}}$$ → Series arm impedence

$$Z_2 = j\omega L_2 + \frac{1}{j\omega C_2}$$

...ies arm
...onant frequency $f_{r_1} = \frac{1}{2\pi\sqrt{L_1C_1}}$

$$f_{r_2} = \frac{1}{2\pi\sqrt{L_2C_2}}$$

...series & shunt arm elements L_1C_1 & L_2C_2
...re selected in such a way that:

$$\boxed{f_{r_1} = f_{r_2} = f_0}$$

$$\frac{1}{2\pi\sqrt{L_1C_1}} = \frac{1}{2\pi\sqrt{L_2C_2}} = f_0$$

$$\omega_0^2 = \frac{1}{L_1C_1} = \frac{1}{L_2C_2}$$

$$\boxed{L_1C_1 = L_2C_2 = \frac{1}{\omega_0^2}}$$

Design Impedence (R_K):-

W.K.T

$$Z_1 Z_2 = R_K^2$$

$$\left[\frac{\cancel{j\omega} L_1}{1-\omega^2 L_1C_1}\right]\left[\frac{1-\omega^2 L_2C_2}{\cancel{j\omega} C_2}\right] = R_K^2$$

$$\frac{L_1}{C_2}\left[\frac{1-\omega^2 L_2C_2}{1-\omega^2 L_1C_1}\right] = R_K^2$$

$\therefore$ since $\boxed{L_1C_1 = L_2C_2}$

$$\therefore R_k^2 = \frac{L_1}{C_2} = \frac{L_2}{C_1}$$

$$\boxed{R_k = \sqrt{\frac{L_1}{C_2}} = \sqrt{\frac{L_2}{C_1}}}$$

Cut-off Frequency:-

condition for cut-off Freq. is:

$$\frac{Z_1}{4Z_2} = -1$$

$$Z_1 = -4Z_2$$

b.s mul. with Z_1

$$Z_1^2 = -4\underbrace{Z_1 Z_2} \qquad \boxed{Z_1 Z_2 = R_k^2}$$

$$\boxed{Z_1 = \pm j2R_k}$$

As already we know, properties of BSF/BEF is exactly opposite to BPF. So, we consider

$$\boxed{\begin{array}{l} Z_1 = j2R_k \text{ at } f = f_{c_1} \text{ (LCF)} \\ Z_1 = -j2R_k \text{ at } f = f_{c_2} \text{ (HCF)} \end{array}}$$

$$[Z_1]_{f=f_{c_1}} = [-Z_1]_{f=f_{c_2}}$$

$$\frac{\cancel{j}\omega_{c_1}\cancel{L_1}}{1-\omega_{c_1}^2 L_1 C_1} = \frac{-\cancel{j}\omega_{c_2}\cancel{L_1}}{1-\omega_{c_2}^2 L_1 C_1}$$

$$\frac{\omega_{c_1}}{1-\omega_{c_1}^2 L_1 C_1} = \frac{-\omega_{c_2}}{1-\omega_{c_2}^2 L_1 C_1}$$

$$\frac{\omega_{c_1}}{\omega_{c_2}} = \frac{\omega_{c_1}^2 L_1 C_1 - 1}{1-\omega_{c_2}^2 L_1 C_1}$$

$$\frac{\omega_{c_1}}{\omega_{c_2}} = \frac{\frac{\omega_{c_1}^2}{\omega_0^2} - 1}{1-\frac{\omega_{c_2}^2}{\omega_0^2}}$$

$$\frac{\omega_{c_1}}{\omega_{c_2}} = \frac{\omega_{c_1}^2 - \omega_0^2}{\omega_0^2 - \omega_{c_2}^2}$$

$$\omega_{c_1}\left[\omega_0^2 - \omega_{c_2}^2\right] = \omega_{c_2}\left[\omega_{c_1}^2 - \omega_0^2\right]$$

$$\omega_0^2 \omega_{c_1} - \omega_{c_1}\omega_{c_2}^2 = \omega_{c_1}^2 \omega_{c_2} - \omega_0^2 \omega_{c_2}$$

$$\omega_0^2 \cancel{(\omega_{c_1} + \omega_{c_2})} = \omega_{c_1}\omega_{c_2}\cancel{(\omega_{c_1} + \omega_{c_2})}$$

$$\omega_0^2 = \omega_{c_1}\omega_{c_2}$$

$$\boxed{\omega_0 = \sqrt{\omega_{c_1}\omega_{c_2}}}$$

or

$$\boxed{f_0 = \sqrt{f_{c_1} f_{c_2}}}$$

Design of BSF :-

consider

$$Z_1 = j2R_k \quad \text{at } f = f_{c_1}$$

$$\frac{\cancel{j}\omega_{c_1} L_1}{1-\omega_{c_1}^2 L_1 C_1} = \cancel{j}2R_k$$

$$\omega_{c_1} L_1 = 2R_k\left[1 - \frac{\omega_{c_1}^2}{\omega_0^2}\right]$$

$$\cancel{2}\pi f_{c_1} L_1 = \cancel{2}R_k\left[\frac{f_0^2 - f_{c_1}^2}{f_0^2}\right]$$

$$\pi f_{c_1} L_1 = R_k\left[\frac{f_{c_1}f_{c_2} - f_{c_1}^2}{f_{c_1}f_{c_2}}\right]$$

$$L_1 = \frac{R_k \cancel{f_{c_1}}(f_{c_2}-f_{c_1})}{\pi \cancel{f_{c_1}} f_{c_1} f_{c_2}}$$

$$\boxed{L_1 = \frac{R_k(f_{c_2}-f_{c_1})}{\pi f_{c_1} \cdot f_{c_2}}}$$

W.K.T

$$R_k^2 = \frac{L_1}{C_2} \Rightarrow C_2 = \frac{L_1}{R_k^2}$$

$$C_2 = \frac{\cancel{R_k}(f_{c_2}-f_{c_1})}{\pi f_{c_1} f_{c_2} R_k^{\cancel{2}}}$$

$$\boxed{C_2 = \frac{(f_{c_2} - f_{c_1})}{\pi f_{c_1} f_{c_2} R_K}}$$

$$L_2 C_2 = \frac{1}{\omega_0^2}$$

$$L_2 = \frac{1}{C_2 4\pi^2 f_0^2}$$

$$L_2 = \frac{\cancel{\pi} \cancel{f_{c_1}} \cancel{f_{c_2}} R_K}{(f_{c_2} - f_{c_1}) 4\pi^{\cancel{2}} f_0^2}$$

$$\cancel{f_{c_1}} \cancel{f_{c_2}}$$

$$\boxed{L_2 = \frac{R_K}{4\pi (f_{c_2} - f_{c_1})}}$$

consider;

$$R_K^2 = \frac{L_2}{C_1} \Rightarrow C_1 = \frac{L_2}{R_K^2}$$

$$C_1 = \frac{\cancel{R_K}}{4\pi (f_{c_2} - f_{c_1}) R_K^{\cancel{2}}}$$

$$\boxed{C_1 = \frac{1}{4\pi R_K (f_{c_2} - f_{c_1})}}$$

problems:-

Q1.) Design a BSF which stops the frequen[illegible] in the range 1000Hz to 4000Hz & the design impedence is given as 500Ω.

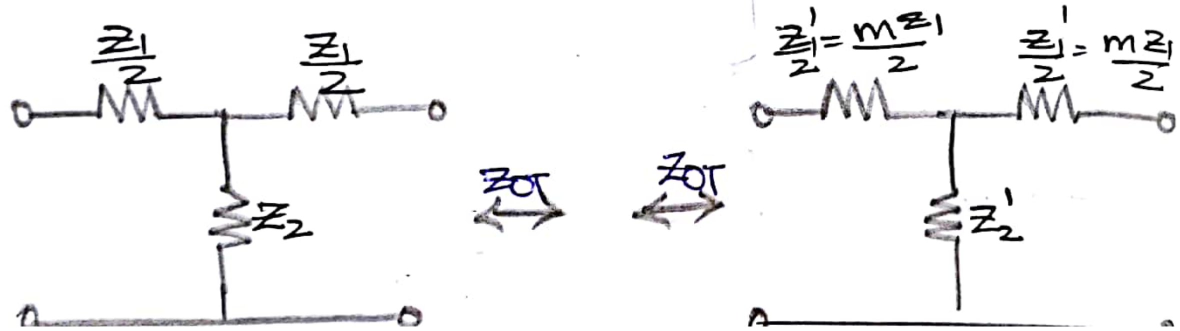
Sol:-

M-derived Filters:-

Def: m-derived filters are modified constant filter with a parameter 'm' such a way that, characteristics impedence of prototype filter is same as that of m-derived.

- Filters can be designed either in T-type or π-type. So, 1st we derive the expression for m-derived T-type & π-type N/ws.

m-derived "T-type":-



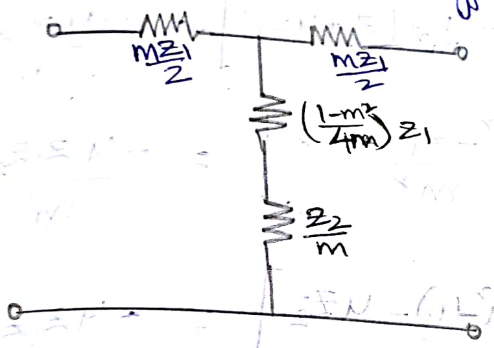
$$\frac{Z_1^2}{4} + Z_1 Z_2 - \frac{m^2 Z_1^2}{4} = m Z_1 Z_2'$$

$$\frac{Z_1^2}{4}(1-m^2) + Z_1 Z_2 = m Z_1 Z_2'$$

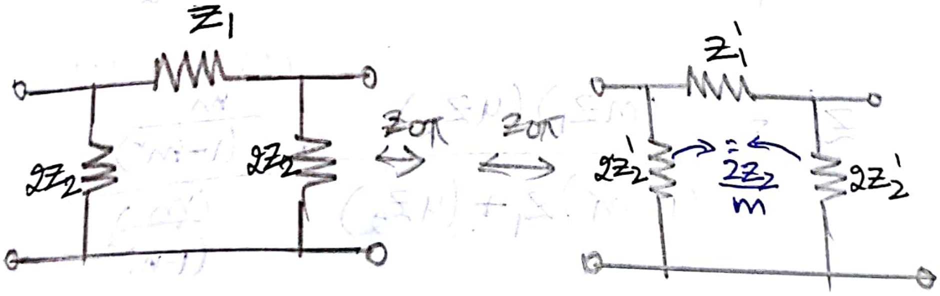
$$\frac{Z_1}{4}[1-m^2] + Z_2 = m Z_2'$$

$$\boxed{Z_2' = \left[\frac{1-m^2}{4m}\right] Z_1 + \frac{Z_2}{m}}$$

Sub^n Z_2' in m-derived T-type N/w



m-derived "π-type":-



prototype π-type

m-derived π-type
(shunt arm imp div with 'm')

$$2Z_2' = \frac{2Z_2}{m}$$

$$\boxed{Z_2' = \frac{Z_2}{m}}$$

$$Z_{0\pi} = Z'_{0\pi}$$

$$\frac{4Z_1Z_2^2}{Z_1+4Z_2} = \frac{4Z_1'Z_2'^2}{Z_1'+4Z_2'}$$

$$\cancel{4}Z_1\cancel{Z_2^2}\left[Z_1' + 4\frac{Z_2}{m}\right] = \frac{\cancel{4}Z_1'\cancel{Z_2^2}}{m^2}\left[Z_1+4Z_2\right]$$

$$Z_1Z_1' + \frac{4Z_1Z_2}{m} - \frac{Z_1'}{m^2}\left[Z_1+4Z_2\right] = 0$$

$$Z_1'\left[Z_1 - \frac{1}{m^2}(Z_1+4Z_2)\right] = -\frac{4Z_1Z_2}{m}$$

$$Z_1'\left[\frac{Z_1m^2 - Z_1 - 4Z_2}{m^{\cancel{2}}}\right] = -\frac{4Z_1Z_2}{\cancel{m}}$$

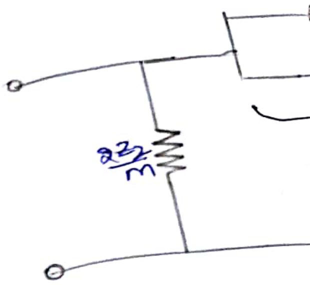
$$Z_1'\left[Z_1(m^2-1) - 4Z_2\right] = -4Z_1Z_2m$$

$$Z_1'\left[4Z_2 + (1-m^2)Z_1\right] = 4Z_1Z_2m$$

$$Z_1' = \frac{(mZ_1)(4Z_2)}{(1-m^2)Z_1 + (4Z_2)} \times \frac{\frac{m}{(1-m^2)}}{\left(\frac{m}{(1-m^2)}\right)}$$

$$Z_1' = \frac{(mZ_1)\left(\frac{4m}{1-m^2}\right)Z_2}{mZ_1 + \left(\frac{4m}{1-m^2}\right)Z_2}$$

$\boxed{Z_1' = (mZ_1)}$



Summary:-

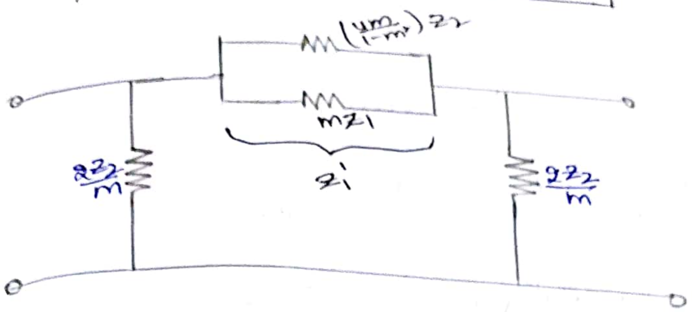
NOTE

1. For m-derive… by 'm' i.e … get $\boxed{Z_2' = }$

2. In m-de… -d by 'm' … get $\boxed{Z_1' =}$

M-Deri…

$$\boxed{Z_1' = (mZ_1) \parallel \left(\frac{4m}{1-m^2}\right) Z_2}$$



Summary:-

NOTE

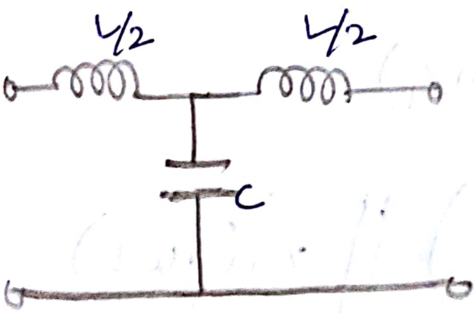
1. For m-derived T-section, series arm is multiplied by 'm' i.e $\boxed{Z_1' = mZ_1}$ so that we can get

$$\boxed{Z_2' = \left(\frac{1-m^2}{4m}\right) Z_1 + \frac{Z_2}{m}}$$

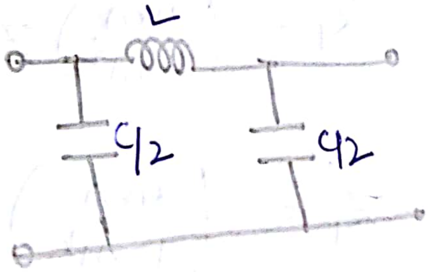
2. In m-derived π-section, shunt arm is divided by 'm' i.e $\boxed{Z_2' = \frac{Z_2}{m}}$ so that we can get

$$\boxed{Z_1' = (mZ_1) \parallel \left(\frac{4m}{1-m^2}\right) Z_2}$$

M-Derived LPF Analysis:-



prototype LPF
T-type



π-type.

For T-type :-

$Z_1 = j\omega L$

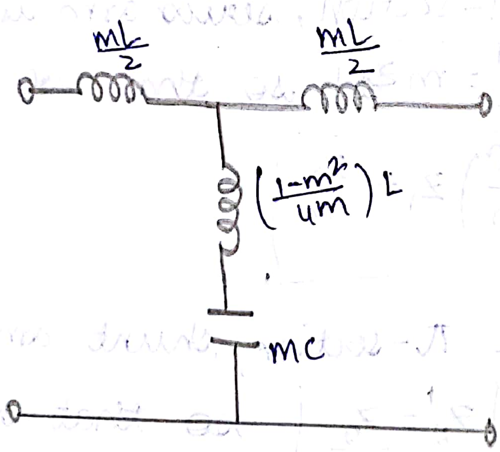
$Z_2 = \frac{1}{j\omega C}$

$Z_1' = m(j\omega L) = j\omega(mL)$

$Z_2' = \left(\frac{1-m^2}{4m}\right)(j\omega L) + \frac{1}{j\omega Cm}$

$Z_2' = j\omega\left(\frac{1-m^2}{4m}\right)L + \frac{1}{j\omega(mC)}$

m-derived LPF (T-type) :-



For π-type :-

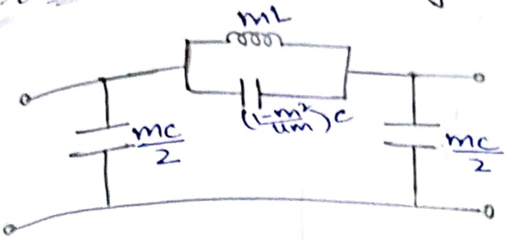
$Z_2' = \frac{Z_2}{m} = \frac{1}{j\omega C \cdot m} = \frac{1}{j\omega(mC)}$

$Z_1' = \left(\frac{4m}{1-m^2}\right) Z_2 \,||\, (mZ_1)$

$Z_1' = \left(\frac{4m}{1-m^2}\right)\left(\frac{1}{j\omega C}\right) \,||\, m(j\omega L)$

$$Z_1' = j\omega(mL) \parallel \left[\frac{1}{j\omega\left(\frac{1-m^2}{4m}\right)C}\right]$$

m-derived LPF (π-type):-



II] Cut-off Frequency:-

consider T-type:-

$$\frac{Z_1'}{4Z_2'} = -1 \quad \text{at } \omega = \omega_c$$

$$Z_1' = -4Z_2'$$

$$j\omega_c(mL) = -4\left[j\omega_c\left(\frac{1-m^2}{4m}\right)L + \frac{1}{j\omega_c(mC)}\right]$$

$$j\omega_c mL = -\cancel{4}\, j\omega_c\left(\frac{1-m^2}{\cancel{4}m}\right)L - \frac{4}{j\omega_c mC}$$

$$\cancel{j}\omega_c L\left[m + \frac{1-m^2}{m}\right] = +\frac{(\cancel{j})4}{\omega_c mC}$$

$$\omega_c L\left[\frac{\cancel{m^2}+1-\cancel{m^2}}{m}\right] = \frac{4}{\omega_c mC}$$

$\omega_c L = \frac{4}{\omega_c C}$

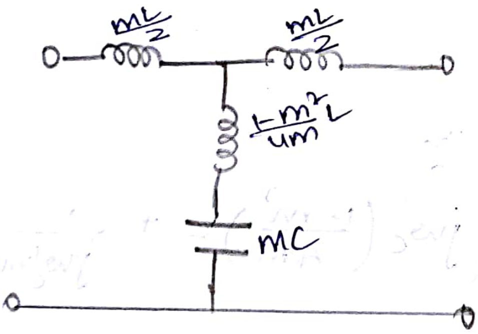
$$\boxed{\omega_c^2 = \frac{4}{LC}}$$

$$4\pi^2 f_c^2 = \frac{4}{LC}$$

$$\boxed{f_c = \frac{1}{\pi\sqrt{LC}}}$$

II] Frequency of Infinite Attenuation [f_∞]:

m-derived T-type LPF :-



$\omega = \omega_0 = \omega_\infty \rightarrow$ (when $\alpha \rightarrow \infty$)

↳ series resonant freq

$$j\omega_\infty \left(\frac{1-m^2}{4m}\right) L + \frac{1}{j\omega_\infty (mC)} = 0$$

$$\frac{j^2\omega_\infty^2 (1-m^2) LC + 4}{j4m\omega_\infty C} = 0$$

$4 - \omega_\infty^2 (1-$

$\omega_\infty^2 (1-$

$(1-$

Characteristics

1> cut-off

prototype

2> Characte

3> Attenua

4> we can

$f = f_\infty$

frequ

ation $[f_\infty]$:

$$4 - \omega_\infty^2 (1-m^2) LC = 0$$

$$\omega_\infty^2 (1-m^2) LC = 4$$

$$(1-m^2) = \frac{4}{\omega_\infty^2 LC}$$

$$1 - m^2 = \frac{\omega_c^2}{\omega_\infty^2} \rightarrow \boxed{\because \omega_c^2 = \frac{4}{LC}}$$

$$m^2 = 1 - \left(\frac{f_c}{f_\infty}\right)^2$$

$$\boxed{m = \sqrt{1 - \left(\frac{f_c}{f_\infty}\right)^2}}$$

Characteristics of m-derived Filter:

1) cut-off frequency remains same as prototype filter

2) Characteristic Impedence also remains same.

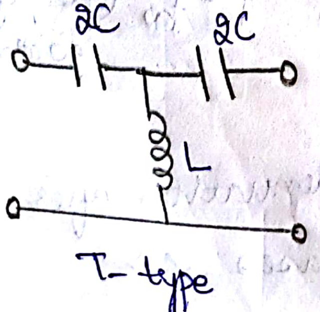
3) Attenuation is infinity at $f = f_\infty$ (freq of ∞ attenuation)

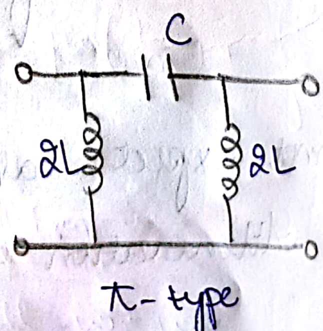
4) we cannot expect same attenuation after $f = f_\infty$. Attenuation decreases as frequency increasing from f_∞ to ∞.

20/8/18,

m-derived HPF Analysis:-

standard prototype HPF:





$Z_1 = \frac{1}{j\omega C}$; $Z_2 = j\omega L$

m-derived T-type:-

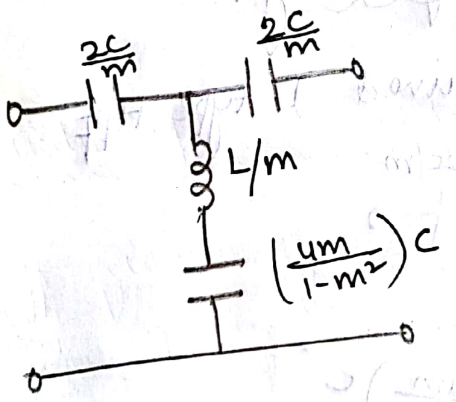
w.k.T for T-type

$$Z_1' = mZ_1 = m\left(\frac{1}{j\omega C}\right) = \frac{1}{j\omega\left(\frac{C}{m}\right)}$$

$$Z_2' = \left(\frac{1-m^2}{4m}\right) Z_1 + \frac{Z_2}{m}$$

$$= \left(\frac{1-m^2}{4m}\right)\left[\frac{1}{j\omega C}\right] + \frac{j\omega L}{m}$$

$$Z_2' = \frac{1}{j\omega\left[\frac{4m}{1-m^2}\right]C} + j\omega\left[\frac{L}{m}\right]$$



→ m-derived T-type

m-derived π-type:-

w.k.T for π-type

$$Z_1' = (mZ_1) \,||\, \left(\frac{4m}{1-m^2}\right) Z_2$$

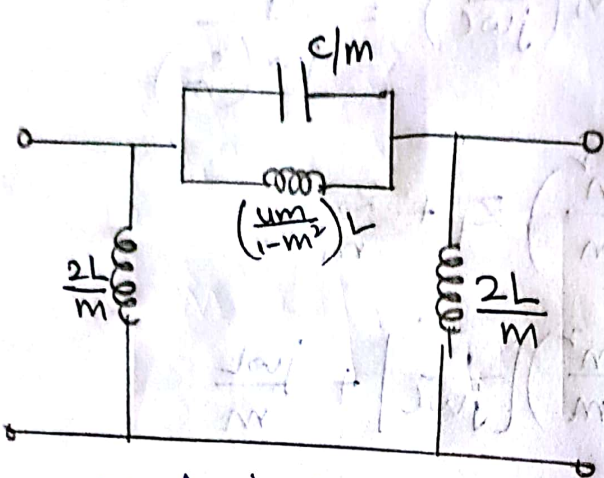
$$Z_1' = m\left(\frac{1}{j\omega C}\right) \,||\, \left(\frac{4m}{1-m^2}\right) j\omega L$$

$$\boxed{Z_1' = \frac{1}{j\omega\left(\frac{C}{m}\right)} \,||\, j\omega\left(\frac{4m}{1-m^2}\right) L}$$

Scanned by CamScanner

$$Z_2' = \frac{Z_2}{m} = \frac{j\omega L}{m}$$

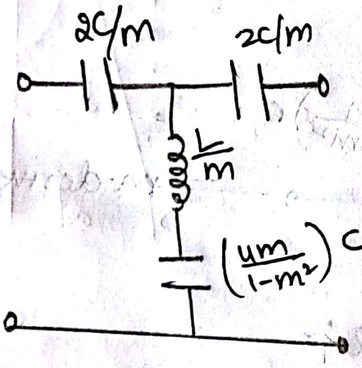
$$Z_2' = j\omega\left(\frac{L}{m}\right)$$



m-derived π-type.

Cut-off Frequency:-

consider m-derived T-type HPF



$$Z_1' = \frac{m}{j\omega C} \quad ; \quad Z_2' = j\omega\left(\frac{L}{m}\right) + \frac{(1-m^2)}{j\omega 4mC}$$

Condition for cut-off Freq is

$$\frac{Z_1'}{4Z_2'} = -1 \quad \text{at } \omega = \omega_c$$

$$\frac{m}{j\omega_c C} = -4\left[j\omega_c\left(\frac{L}{m}\right) + \frac{1-m^2}{j\omega_c 4mC}\right]$$

$$\frac{m}{j\omega_c C} = -4\left[\frac{4j^2\omega_c^2 LC + 1 - m^2}{j\omega_c 4mC}\right]$$

$$-\cancel{m^2} = -4\omega_c^2 LC + 1 - \cancel{m^2}$$

$$1 = 4\omega_c^2 LC$$

$$\boxed{\omega_c^2 = \frac{1}{4LC}}$$

$$4\pi^2 f_c^2 = \frac{1}{4LC}$$

$$f_c^2 = \frac{1}{16\pi^2 LC}$$

$$\boxed{f_c = \frac{1}{4\pi\sqrt{LC}}}$$

Frequency of Infinite Attenuation (f_0):-

At $\omega = \omega_0 \Rightarrow Z_2' = 0$

ω_0 → resonant freq of shunt arm

Let $\omega = \omega_0 = \omega_\infty$

$Z_2' = 0$

$$j\omega\left(\frac{L}{m}\right) + \left(\frac{1-m^2}{j\omega 4mC}\right) = 0$$

$$\frac{j^2\omega_\infty^2 4LC + 1 - m^2}{j\omega_\infty 4mC} = 0$$

$$1-m^2 = 4LC\,\omega^2$$

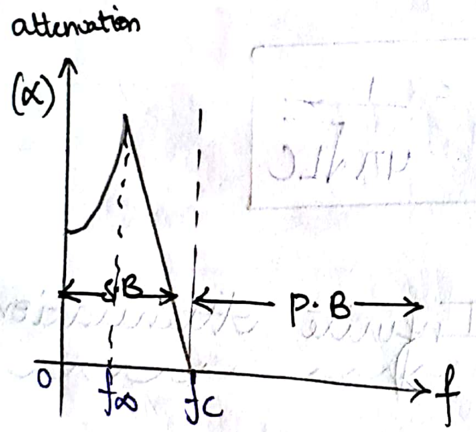
$$1-m^2 = \frac{\omega_\infty^2}{\omega_c^2} \qquad \left[\because \omega_c^2 = \frac{1}{4LC}\right]$$

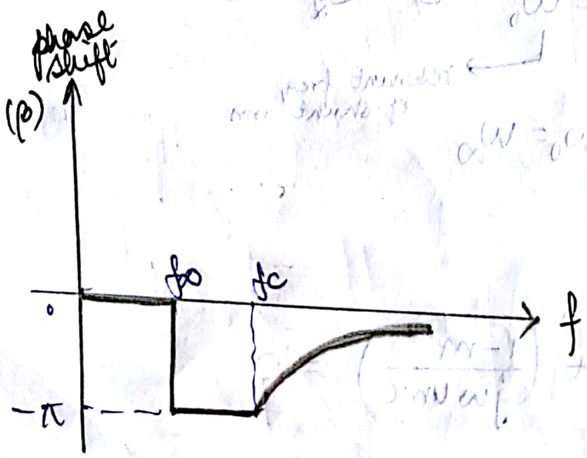
$$1-\frac{f_\infty^2}{f_c^2} = m^2$$

$$\boxed{m = \sqrt{1-\frac{f_\infty^2}{f_c^2}}}$$

Always frequency of attenuation (f_∞) is in Stop Band

VI) Attenuation & phase shift constants :-





$\beta = 0$ 0 to f_∞

$\beta = -\pi$ f_∞ to f_c

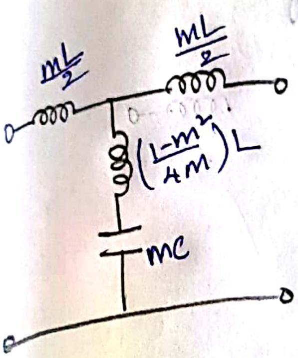
$\beta = -2\sin^{-1}\left[\frac{f_c}{f}\right]$ f_c to ∞

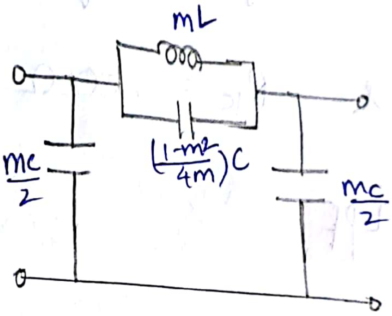
Summary :-

m-derived LPF :-

$$m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2} \quad ; \quad L = \frac{R_K}{\pi f_c}$$

$$C = \frac{1}{\pi R_K f_c}.$$

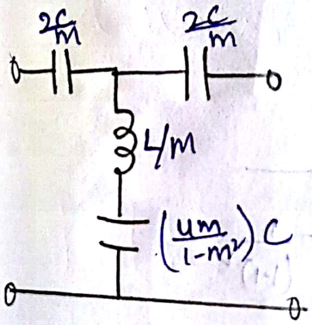


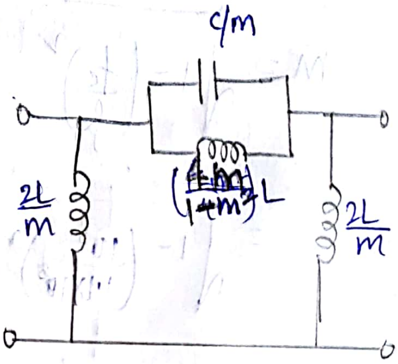


m-derived HPF :-

$$m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2} \quad ; \quad L = \frac{R_K}{4\pi f_c}$$

$$C = \frac{1}{4\pi R_K f_c}$$





problems:-

imp.

Q1.) Design an m-derived LPF filter with cut-off Frequency 1kHz & frequency of attenuation $f_\infty = 1.1$ kHz & design Impedence $R_k = 600\Omega$

sol:- Given $f_c = 1$ kHz, $f_\infty = 1.1$ kHz,

& $R_k = 600\,\Omega$

For LPF:-

$$L = \frac{R_k}{\pi f_c} = \frac{600}{\pi \times 1 \times 10^3} \Rightarrow \underline{190.98\,mH}$$

$$C = \frac{1}{\pi R_k f_c} \Rightarrow \frac{1}{\pi \times 600 \times 10^3} \Rightarrow \underline{530.51\,nF}$$

$$m = \sqrt{1 - \left(\frac{f_c}{f_\infty}\right)^2}$$

$$= \sqrt{1 - \left(\frac{10^3}{1.1 \times 10^3}\right)^2} \Rightarrow \sqrt{1 - \frac{1}{(1.1)^2}}$$

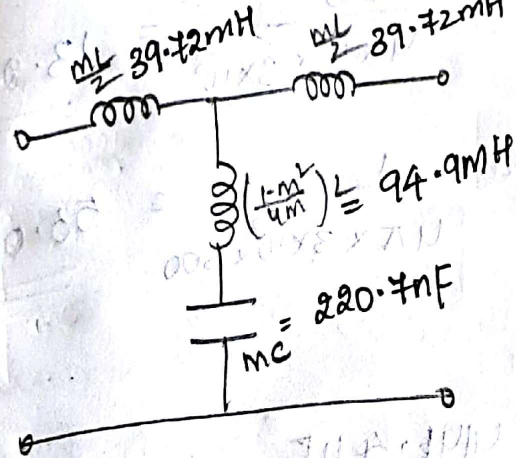
$$m = \underline{0.416}$$

T-type:

$$\frac{mL}{2} = \frac{0.416 \times 190.98}{2} = 39.72\,mH$$

$$\left(\frac{1-m^2}{4m}\right)L = \left(\frac{1-0.416^2}{4\times 0.416}\right)\times 190.98 \Rightarrow 94.9\,mH$$

$$mC \Rightarrow 0.416 \times 530.51 = 220.7\,nF$$

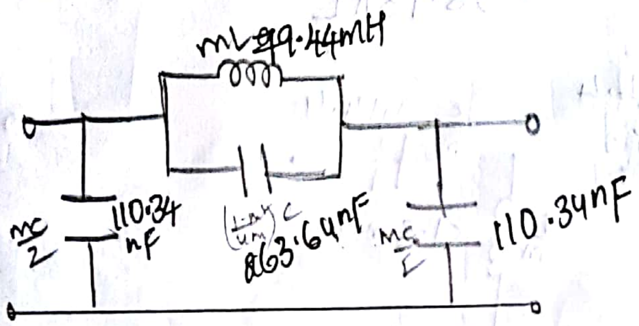


π-type:

$$mL \Rightarrow 0.416 \times 190.98 = 79.44\,mH$$

$$\left(\frac{1-m^2}{4m}\right)C \Rightarrow \frac{1-(0.416)^2}{4\times 0.416}\times 530.51 = 263.64\,nF$$

$$\frac{mC}{2} \Rightarrow \frac{0.416\times 530.51}{2} = 110.34\,nF$$



Q2.) Design an m-derived HPF with cut-off frequency 3kHz & frequency of infinite attenuation $f_\infty = 2.9\,kHz$ & $R_k = 500$.

Sol:- $m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2} = \sqrt{1-\left(\frac{2.9}{3}\right)^2} = \underline{\underline{0.256}}$

$$L = \frac{R_k}{4\pi f_c} = \frac{500}{4\times\pi\times 3\times10^3} = \underline{\underline{13.26\,mH}}$$

$$C = \frac{1}{4\pi f_c R_k} = \frac{1}{4\pi\times 3\times10^3\times 500} = \underline{\underline{53.05\,nF}}$$

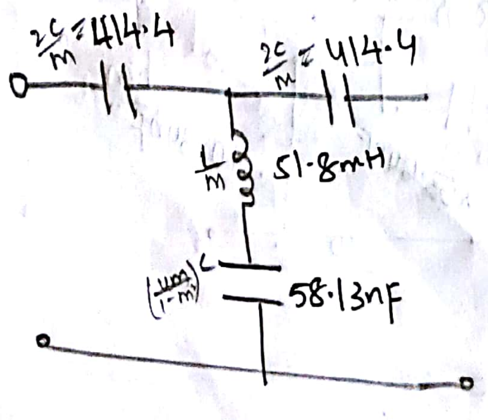
T-type:-

$$\frac{2C}{m} \Rightarrow \frac{2\times53.05}{0.256} = 414.4\,nF$$

$$\frac{L}{m} \Rightarrow \frac{13.26}{0.256} = 51.8\,mH$$

$$\left(\frac{4m}{1-m^2}\right)C \Rightarrow \frac{4\times0.256}{1-(0.256)^2}\times 53.05\times10^{-9}$$

$$\Rightarrow 58.13\,nF$$

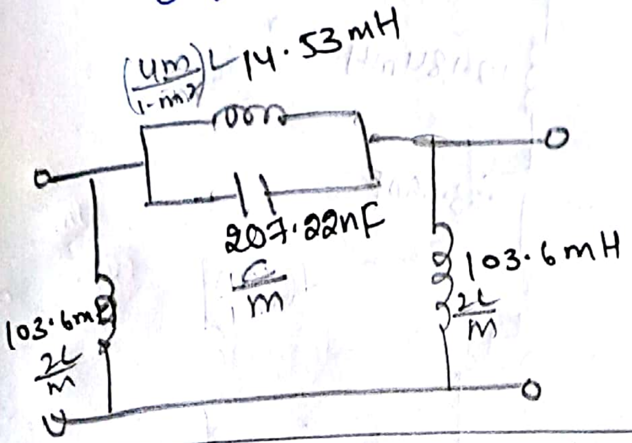


π-type:

$\left(\frac{4m}{1-m^2}\right)L = \frac{4\times 0.256}{1-(0.256)^2} \times 13.26mH \Rightarrow 14.53mH$

$\frac{C}{m} \Rightarrow 207.22nF$

$\frac{2L}{m} \Rightarrow \frac{2\times 13.26}{0.256} = 103.6mH$



Q3) Design an m-derived LPF with cut-off freq $f_c = 5kHz$ & design imp $R_k = 600\Omega$ & $f_\infty = 1.25 f_c$

Sol: $m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2} \Rightarrow \sqrt{1-\left(\frac{5\times10^3}{1.25\times5\times10^3}\right)^2} = \underline{0.6}$

$L = \frac{R_k}{\pi f_c} \Rightarrow \underline{38.19mH}$

$C = \frac{1}{\pi R_k f_c} = \underline{106.1nF}$

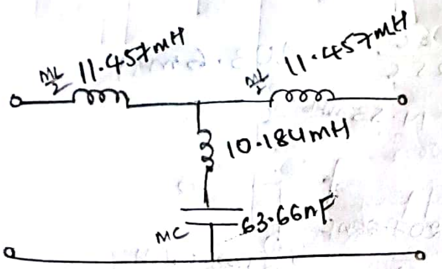
T-type:

$\frac{mL}{2} \Rightarrow \frac{0.6\times 38.19}{2} \Rightarrow 11.457mH$

$mC \Rightarrow 0.6\times106.1nF = 63.66nF$

$$\left(\frac{1-m^2}{4m}\right)L \Rightarrow \left(\frac{1-(0.6)^2}{4(0.6)}\right)\times 38.19$$

$$= 10.184\,mH$$

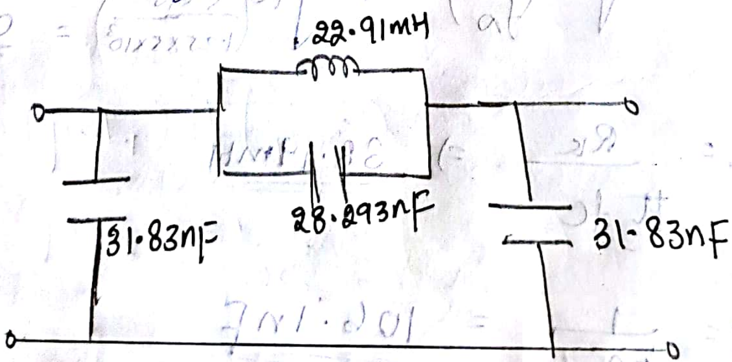


π-type:-

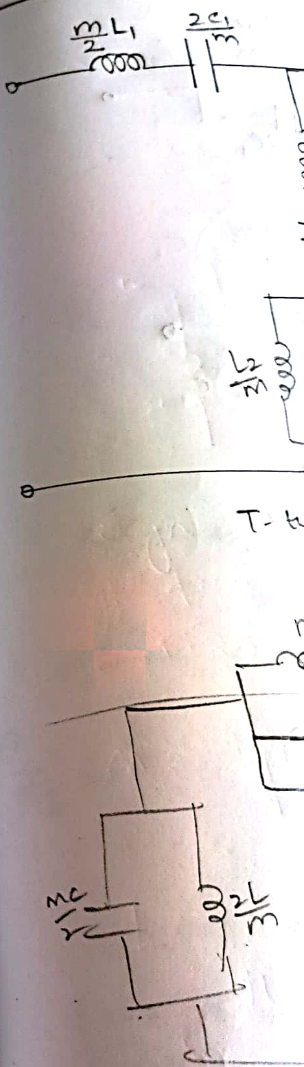
$$mL \Rightarrow 0.6\times 38.19 = 22.91\,mH$$

$$\left(\frac{1-m^2}{4m}\right)C \Rightarrow \frac{1-(0.6)^2}{4\times 0.6}\times 106.1 = 28.293\,nF$$

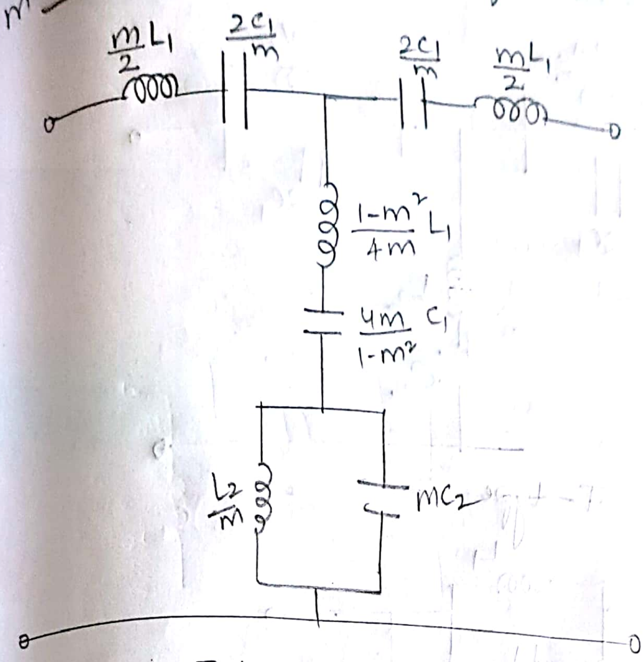
$$\frac{mC}{2} \Rightarrow \frac{0.6\times 106.1}{2} = 31.83\,nF$$



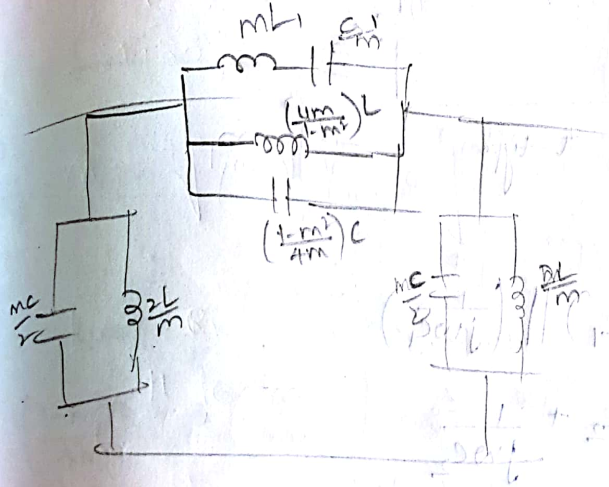
m-derived Band-



m-derived Band-pass filter

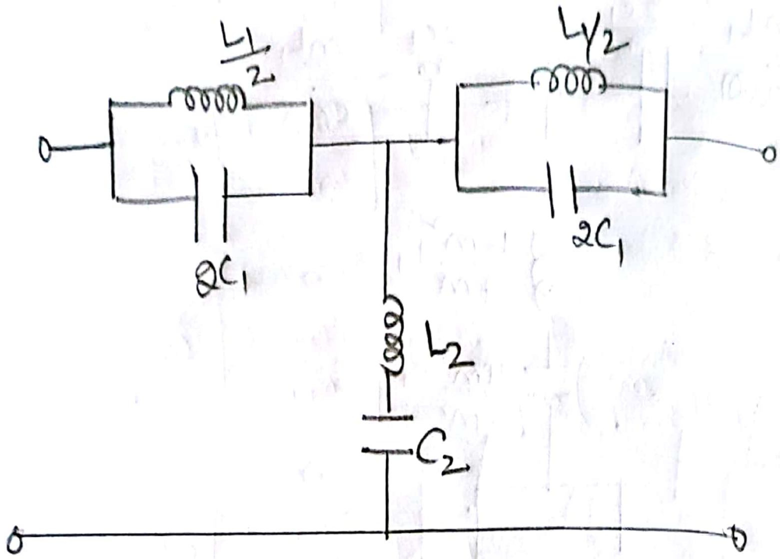


T- type

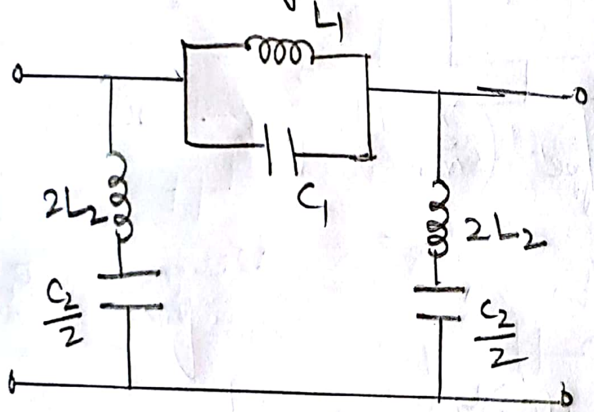


π - type

m-derived "Band Stop" Filter :-



T-type



π-type

W.K.T

$$Z_1 = (j\omega L_1) \,||\, \left(\frac{1}{j\omega C_1}\right)$$

$$Z_2 = j\omega L_2 + \frac{1}{j\omega C_2}$$

For T-type :-

$$Z_1' = mZ_1 = m\left[\frac{(j\omega L_1)\left(\frac{1}{j\omega C_1}\right)}{j\omega L_1 + \frac{1}{j\omega C_1}}\right]$$

$$\cancel{Z_1' = \frac{(j\omega L_1)\left(\frac{1}{j\omega C_1}\right)}{\frac{1}{m}\left[j\omega L_1 + \frac{1}{j\omega C_1}\right]}}$$

$$Z_1' = \frac{(j\omega \cancel{M} L_1)\left(\frac{1}{j\omega C_1}\right)}{\left(j\omega L_1 + \frac{1}{j\omega C_1}\right)} \times \frac{m}{m}$$

$$Z_1' = \frac{j\omega(mL_1) \times \frac{1}{j\omega\left(\frac{C_1}{m}\right)}}{j\omega(mL_1) + \frac{1}{j\omega\left(\frac{C_1}{m}\right)}}$$

$$\boxed{Z_1' = j\omega(mL_1) \,||\, \frac{1}{j\omega\left(\frac{C_1}{m}\right)}}$$

Also

$$Z_2' = \left(\frac{1-m^2}{4m}\right) Z_1 + \frac{Z_2}{m}$$

$\downarrow$

considering

$$\left(\frac{1-m^2}{4m}\right) Z_1 \Rightarrow \left(\frac{1-m^2}{4m}\right)\left[\frac{(j\omega L_1)\left(\frac{1}{j\omega C_1}\right)}{\left[j\omega L_1 + \frac{1}{j\omega C_1}\right]}\right]$$

$$= \frac{j\omega\left(\frac{1-m^2}{4m}\right) L_1 \times \frac{1}{j\omega C_1}}{\frac{\left(\frac{1-m^2}{4m}\right)}{\left(\frac{1-m^2}{4m}\right)}\left[j\omega L_1 + \frac{1}{j\omega C_1}\right]}$$

$\frac{\pi}{k}$

$\frac{*}{\text{Imp step}}$

$$= \frac{j\omega\left[\frac{1-m^2}{4m}\right]L_1\left(\frac{1-m^2}{j\omega 4mC_1}\right)}{j\omega\left[\frac{1-m^2}{4m}\right]L_1 + \frac{1-m^2}{j\omega 4mC_1}}$$

$$= \frac{j\omega\left[\frac{1-m^2}{4m}\right]L_1 \times \frac{1}{j\omega\left(\frac{4m}{1-m^2}\right)C_1}}{j\omega\left(\frac{1-m^2}{4m}\right)L_1 + \frac{1}{j\omega\left(\frac{4m}{1-m^2}\right)C_1}}$$

$$\therefore \left(\frac{1-m^2}{4m}\right)Z_1 \Rightarrow j\omega\left(\frac{1-m^2}{4m}\right)L_1 \,\|\, \frac{1}{j\omega\left(\frac{4m}{1-m^2}\right)C_1}$$

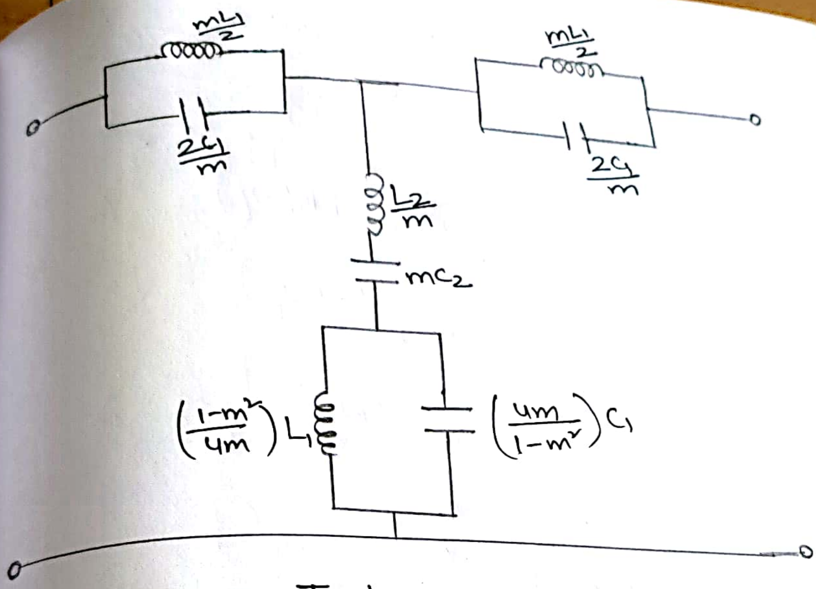
Now considering;

$$\frac{Z_2}{m} = \frac{1}{m}\left[j\omega L_2 + \frac{1}{j\omega C_2}\right]$$

$$= j\omega\left[\frac{L_2}{m}\right] + \frac{1}{j\omega(mC_2)}$$

Now we have $Z_2' = \left(\frac{1-m^2}{4m}\right)Z_1 + \frac{Z_2}{m}$

$$\therefore Z_2' = \left[j\omega\left(\frac{1-m^2}{4m}\right)L_1 \,\|\, \frac{1}{j\omega\left(\frac{4m}{1-m^2}\right)C_1}\right] + \left[j\omega\left(\frac{L_2}{m}\right) + \frac{1}{j\omega(mC_2)}\right]$$



eg; T-type.

NOTE :-

For m-derived Band pass filter :

$$m = \sqrt{1 - \left(\frac{f_{c_2} - f_{c_1}}{f_{\infty_2} - f_{\infty_1}}\right)^2}$$

$f_{c_1} \rightarrow$ lower cut-off Freq

$f_{c_2} \rightarrow$ higher " " "

$\left.\begin{array}{l} f_{\infty_1} \\ f_{\infty_2} \end{array}\right] \rightarrow$ freq of infinite attenuation.

$$\frac{\frac{j\omega L_2}{m}}{1-\omega^2 L_2 C_2} + j\frac{1-m^2}{4m}\left[\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right] \quad 0$$

$$\frac{(1-m^2)L_1}{4m} + \frac{4mC_1}{(1-m^2)}$$ series comb

$$\frac{L_2}{m} \,||\, mC_2$$ — || comb

$$Z_1 = j\omega L_1 - \frac{j}{\omega C_1} \Rightarrow j\omega L_1 + \frac{1}{j\omega C_1}$$

$$Z_2 = j\omega C_2 \,||\, \frac{1}{j\omega C_2}$$

W.K.T

$$Z_1' = mZ_1$$

$$= m\left[j\omega L_1 + \frac{1}{j\omega C_1}\right]$$

$$= j\omega m L_1 + \frac{1}{\frac{j\omega C_1}{m}}$$

$Z_2' =$

$\left(\frac{1-m^2}{4m}\right.$

Now,

$$Z_2' = \left(\frac{1-m^2}{4m}\right) Z_1 + \frac{Z_2}{m}$$

$$\left(\frac{1-m^2}{4m}\right) Z_1 = \left(j\omega L_1 + \frac{1}{j\omega C_1}\right)\left(\frac{1-m^2}{4m}\right)$$

$$= j\omega L_1 \left(\frac{1-m^2}{4m}\right) L_1 + \frac{1}{j\omega\left(\frac{4m}{1-m^2}\right) C_1}$$

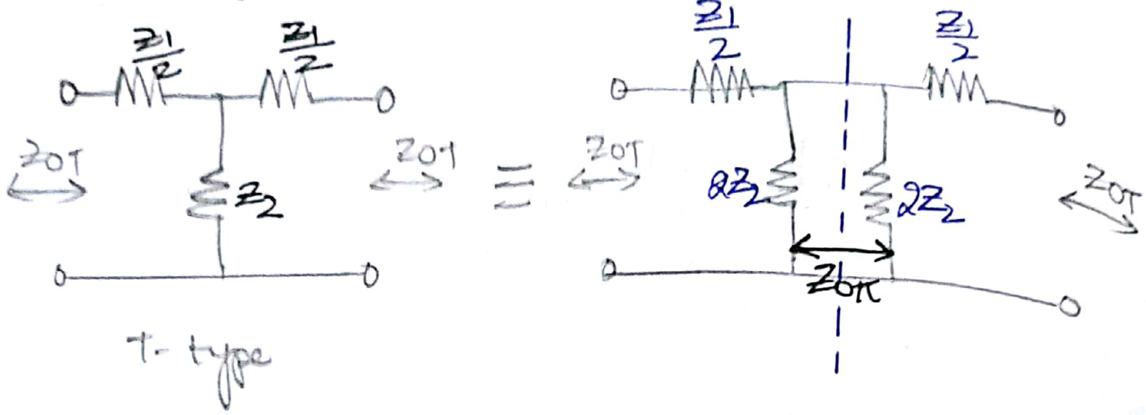
$$\frac{Z_2}{m} = \frac{j\omega L_2 \times \frac{1}{j\omega C_2}}{j\omega L_2 + \frac{1}{j\omega C_2}} \times \frac{1}{m}$$

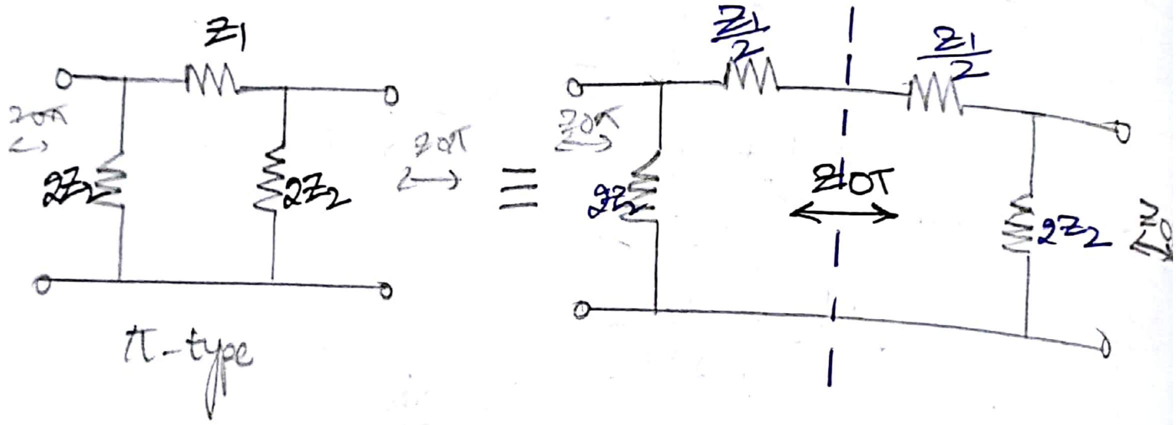
$$= \frac{j\omega L_2 \times \frac{1}{j\omega C_2}}{j\omega m L_2 + \frac{1}{\frac{j\omega C_2}{m}}} \times \frac{m}{m}$$

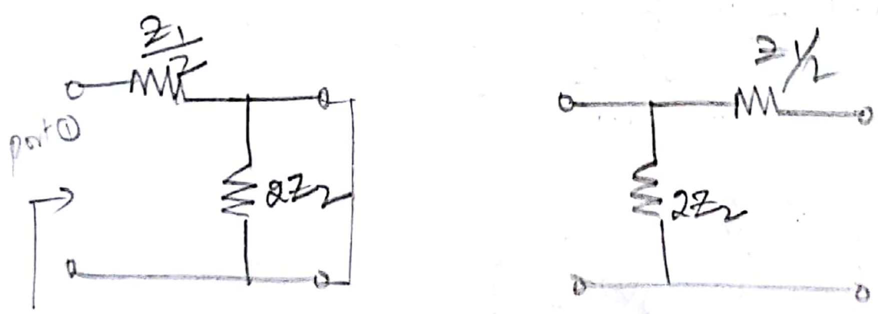
$$= \frac{j\omega L_2 m \times \frac{1}{\frac{j\omega C_2}{m}}}{j\omega m L_2 + \frac{1}{\frac{j\omega C_2}{m}}}$$

$$\frac{Z_2}{m} = j\omega m L_2 \,||\, \frac{1}{\frac{j\omega C_2}{m}}$$

1) prototype half sections:-







$Z_{sc_1} = \frac{Z_1}{2}$; $Z_{sc_2} = \left(\frac{Z_1}{2}\right) \parallel (2Z_2)$

$Z_{oc_1} = \frac{Z_1}{2} + 2Z_2$; $Z_{oc_2} = 2Z_2$

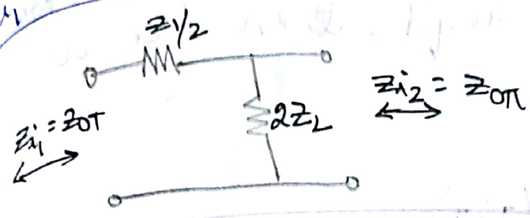
W.K.T

$Z_{i_1} = \sqrt{Z_{sc_1} \cdot Z_{oc_1}}$; $Z_{i_2} = \sqrt{Z_{sc_2} \cdot Z_{oc_2}}$

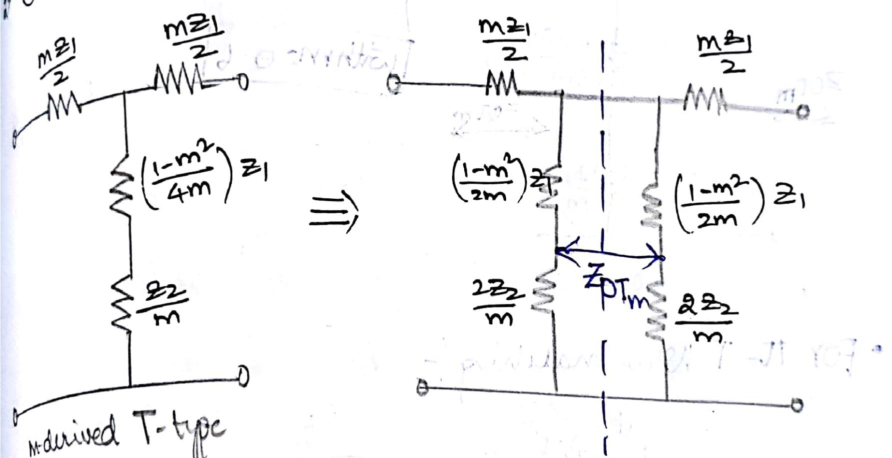
$$Z_{i_1} = \sqrt{\left(\frac{Z_1}{2}\right)\left(\frac{Z_1}{2} + 2Z_2\right)} \quad ; \quad Z_{i_2} = \sqrt{\frac{\left(\frac{Z_1}{2}\right)(2Z_2)}{\left(\frac{Z_1}{2} + 2Z_2\right)}(2Z_2)}$$

$$Z_{i_1} = \sqrt{\frac{Z_1^2}{4} + \frac{2Z_1Z_2}{2}} \qquad = \sqrt{\frac{4Z_1Z_2^2}{Z_1 + 4Z_2}}$$

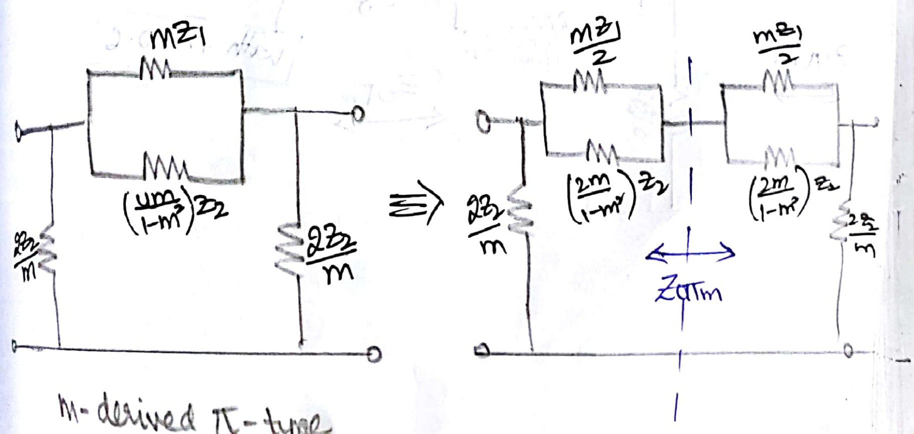
$$\boxed{Z_{i_1} = Z_{0T}} \quad ; \quad \boxed{Z_{i_2} = Z_{0\pi}}$$



m-derived half-sections:-



m-derived T-type

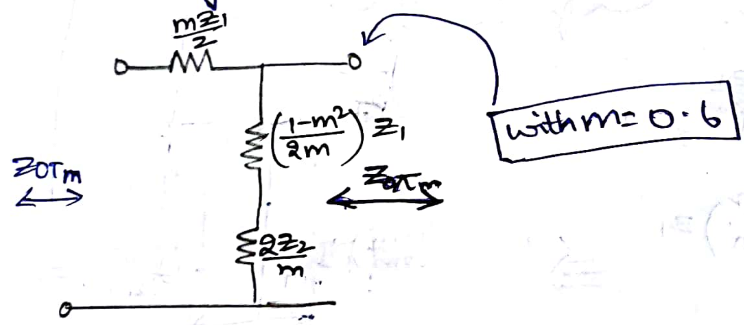


m-derived π-type

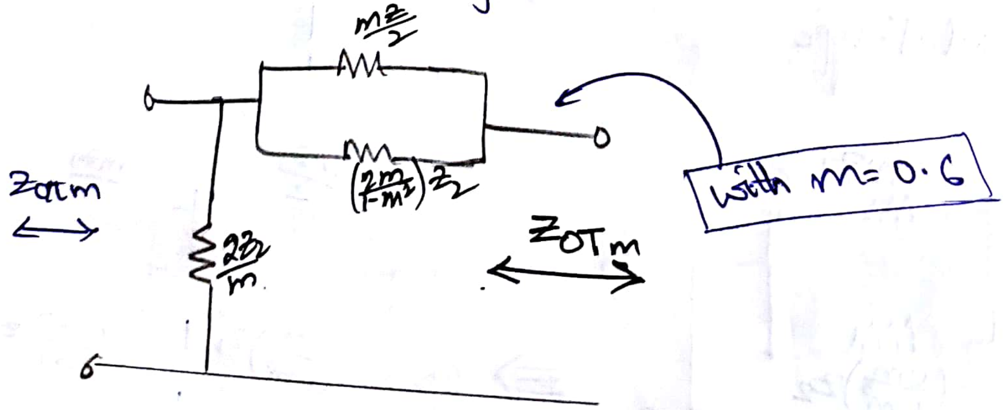
NOTE :-

- If T to π & π to T N/W matching is req'd, then a prototype half section (L-section) is used.
- For m-derived T to π & π to T, N/W matching is req'd, then we select $\boxed{m = 0.6}$

For T-type :- (T-π)



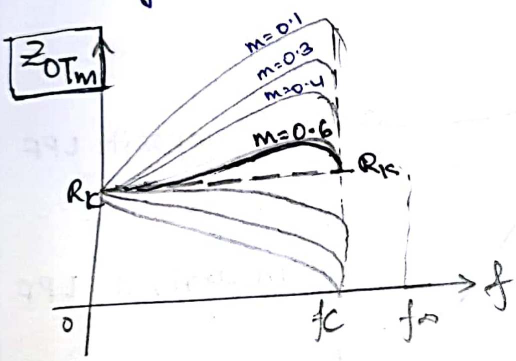
- For π-T N/W matching :-



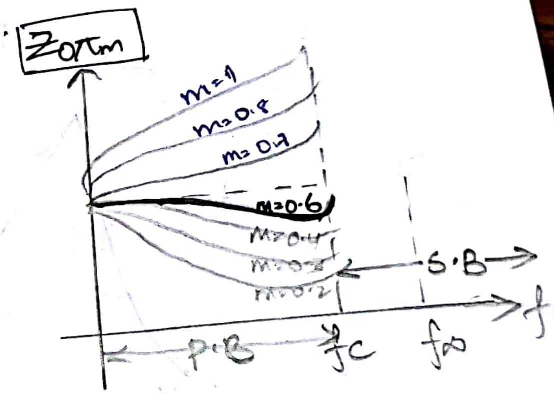
Variation of characteristic impedance with 'm' :-

m-derived T & π-section N/Ws, characteristic impedence changes w.r.t frequency & parameter 'm'.

For m-derived low-pass filter for diff values of 'm' variation of characteristic impedences are given below:



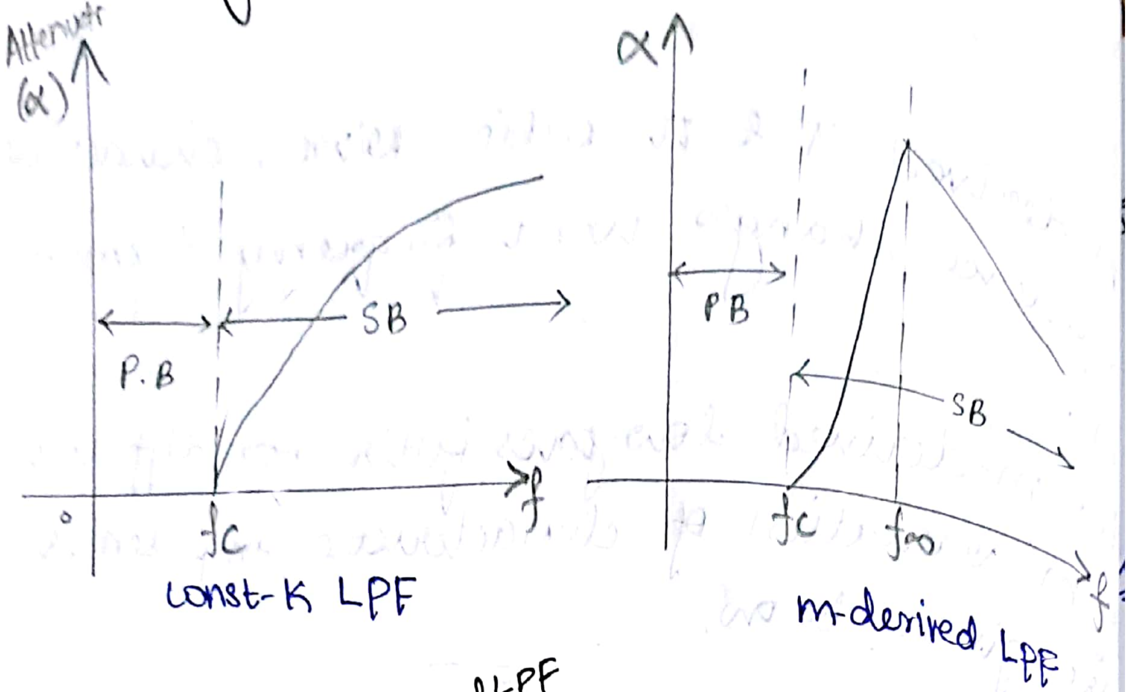
Low-pass filters

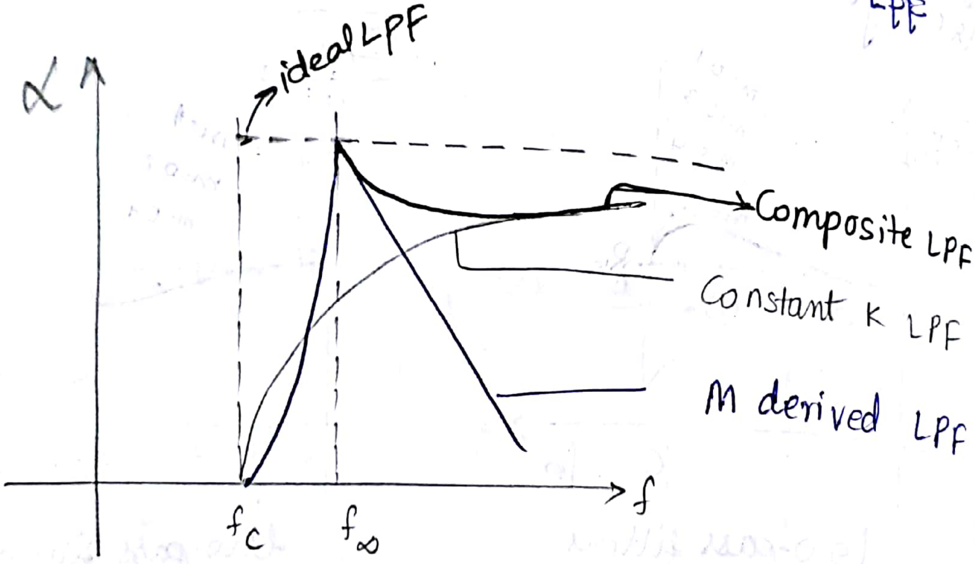


low-pass filters

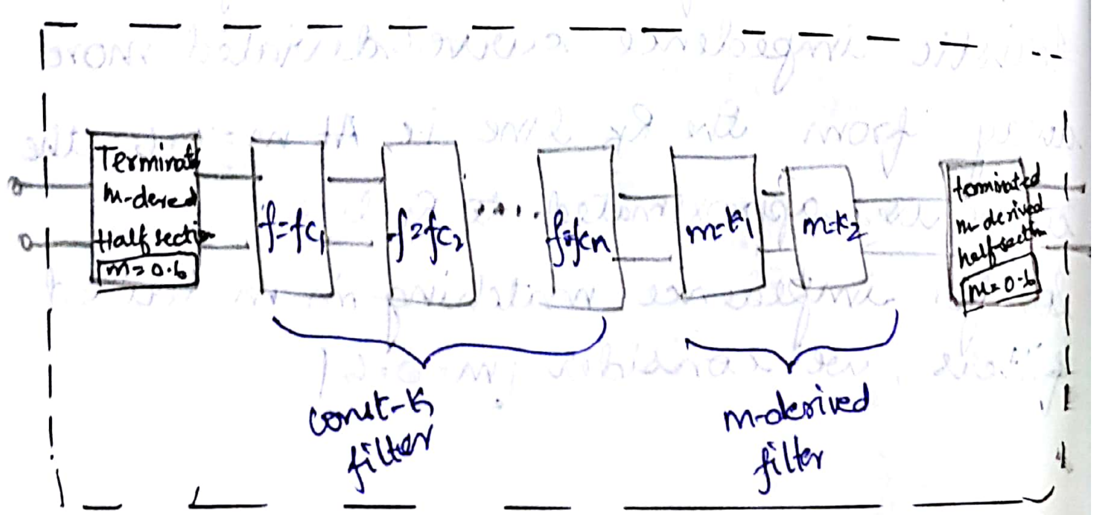
From the graphs, it has been observed that except $\boxed{m=0.6}$ for other values of 'm', characteristic impedence curve is deviated more away from the R_K line i.e At $m=0.6$; the curve is approximated to R_K line.

So, for impedence matching in m-derived filters, we consider $\boxed{m=0.6}$





Block Diagram of composite Filter:-



problems :-

1) Design a composite LPF T-type with cut-off frequency 2 kHz & frequency of ∞ attenuation $f_\infty = 2.05$ kHz & Design impedence $R_k = 500\,\Omega$.

Sol:- $f_c = 2\,kHz$

$f_\infty = 2.05\,kHz$

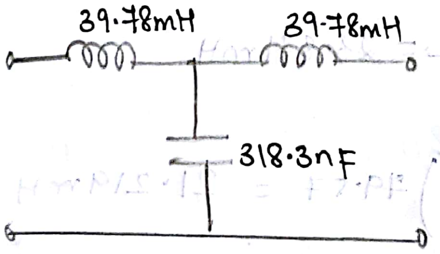
$R_k = 500\,\Omega$

i, const-K LPF :-

$$L = \frac{R_k}{\pi f_c} = \frac{500}{\pi \times 2 \times 10^3} = 79.57\,mH$$

$$\boxed{\frac{L}{2} \Rightarrow 39.78\,mH}$$

$$C = \frac{1}{\pi R_k f_c} = \frac{1}{500 \times 2 \times 10^3 \times \pi} = 318.3\,nF$$



ii, m-derived LPF :-

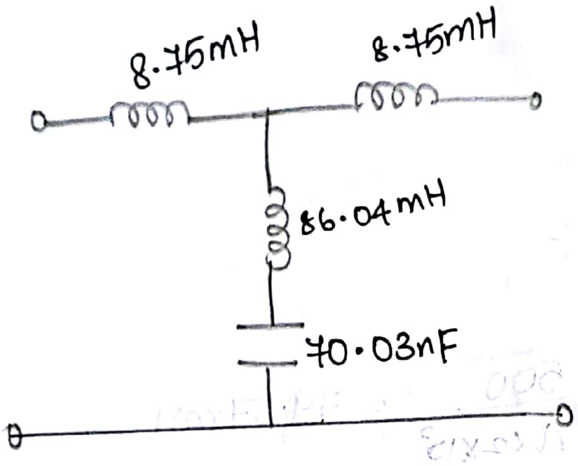
$$m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2} \Rightarrow \sqrt{1-\left(\frac{2}{2.05}\right)^2} \Rightarrow 0.22$$

Scanned by CamScanner

$$\frac{mL}{2} = \frac{0.22 \times 79.57}{2} = 8.75 mH$$

$$\left(\frac{1-m^2}{4m}\right)L \Rightarrow \left(\frac{1-0.22^2}{4 \times 0.22}\right) \times 79.57 = 86.04 mH$$

$$mC \Rightarrow 0.22 \times 318.3 \Rightarrow 70.03 nF$$



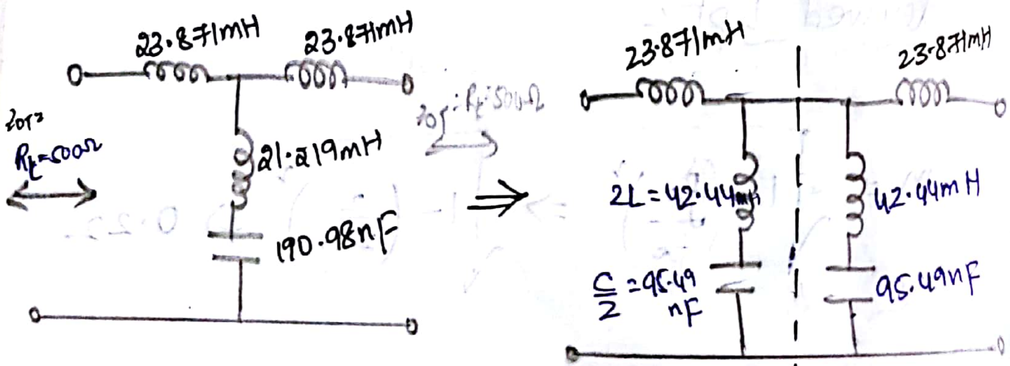
(iii) :- Terminating Half-Sections :-

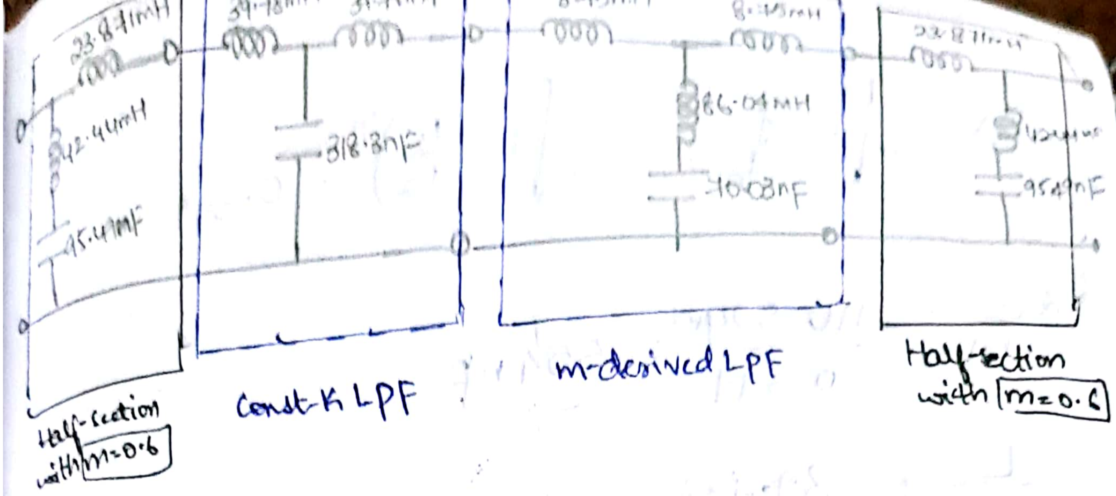
let $\boxed{m = 0.6}$

$$\frac{mL}{2} \Rightarrow \frac{0.6 \times 79.57}{2} = 23.871 mH$$

$$\left(\frac{1-m^2}{4m}\right)L \Rightarrow \left(\frac{1-(0.6)^2}{0.6 \times 4}\right) 79.57 = 21.219 mH$$

$$mC \Rightarrow 0.6 \times 318.3 = 190.98 nF$$





28/8/18

Q) Design a composite HPF T-type to operate at $f_c = 1.2\,KHz$ & $f_\infty = 1.1\,KHz$ & design imp. $R_K = 600\Omega$.

Sol:- $f_c = 1.2\,KHz$

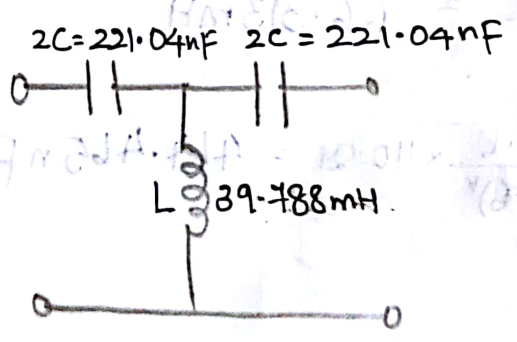
$f_\infty = 1.1\,KHz$

$R_K = 600\,\Omega$

i, const-K HPF :-

$$L = \frac{R_K}{4\pi f_c} = \frac{600}{4\times\pi\times1.2\times10^3} = 39.788\,mH$$

$$C = \frac{1}{4\pi f_c R_K} = \frac{1}{4\times\pi\times1.2\times10^3\times600} = 110.524\,nF$$



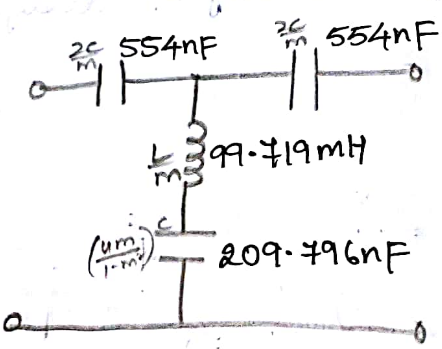
ii, m-derived HPF :-

$$m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2} \Rightarrow \sqrt{1-\left(\frac{1.1}{1.2}\right)^2} = 0.399$$

$$\frac{2C}{m} = \frac{110.524\times 2}{0.399} = 554\,nF$$

$$\frac{L}{m} = \frac{39.788}{0.399} = 99.719\,mH$$

$$\left(\frac{4m}{1-m^2}\right)C \Rightarrow \frac{4\times 0.399}{1-(0.399)^2}\times 110.524 \Rightarrow 209.796\,nF$$



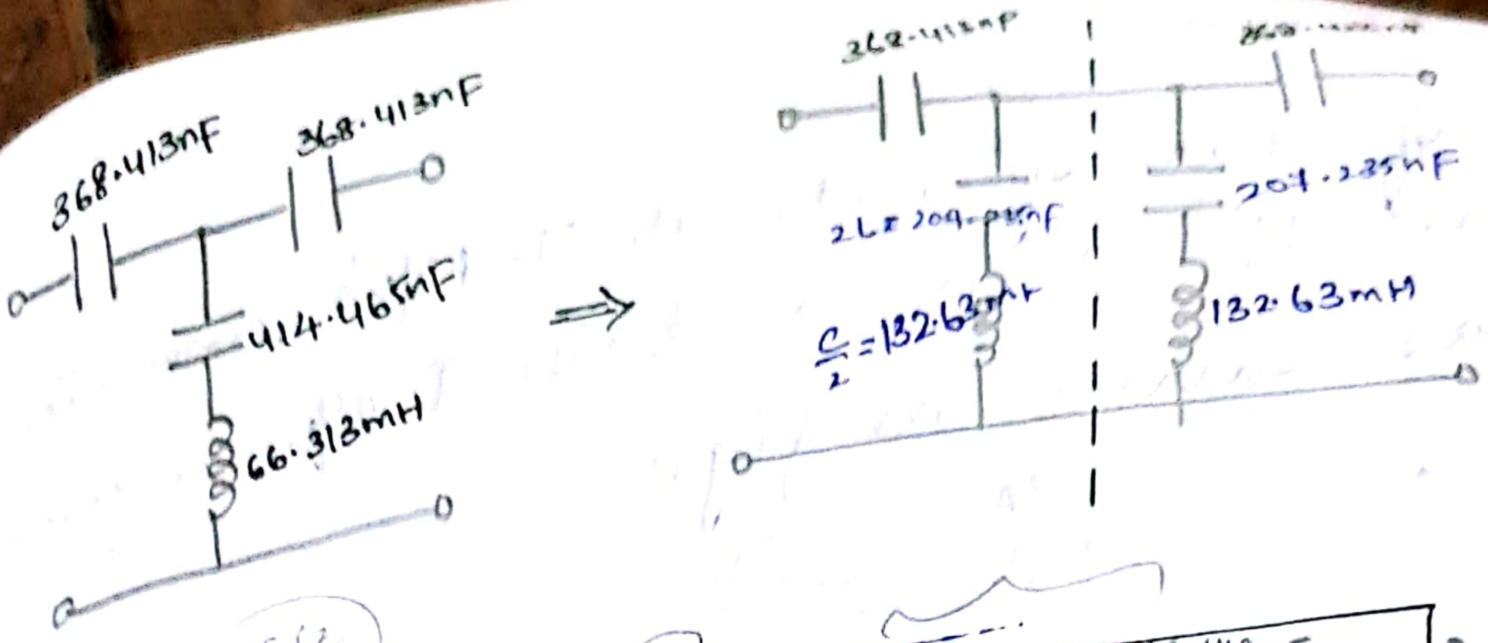
iii, Terminating half-sections;-

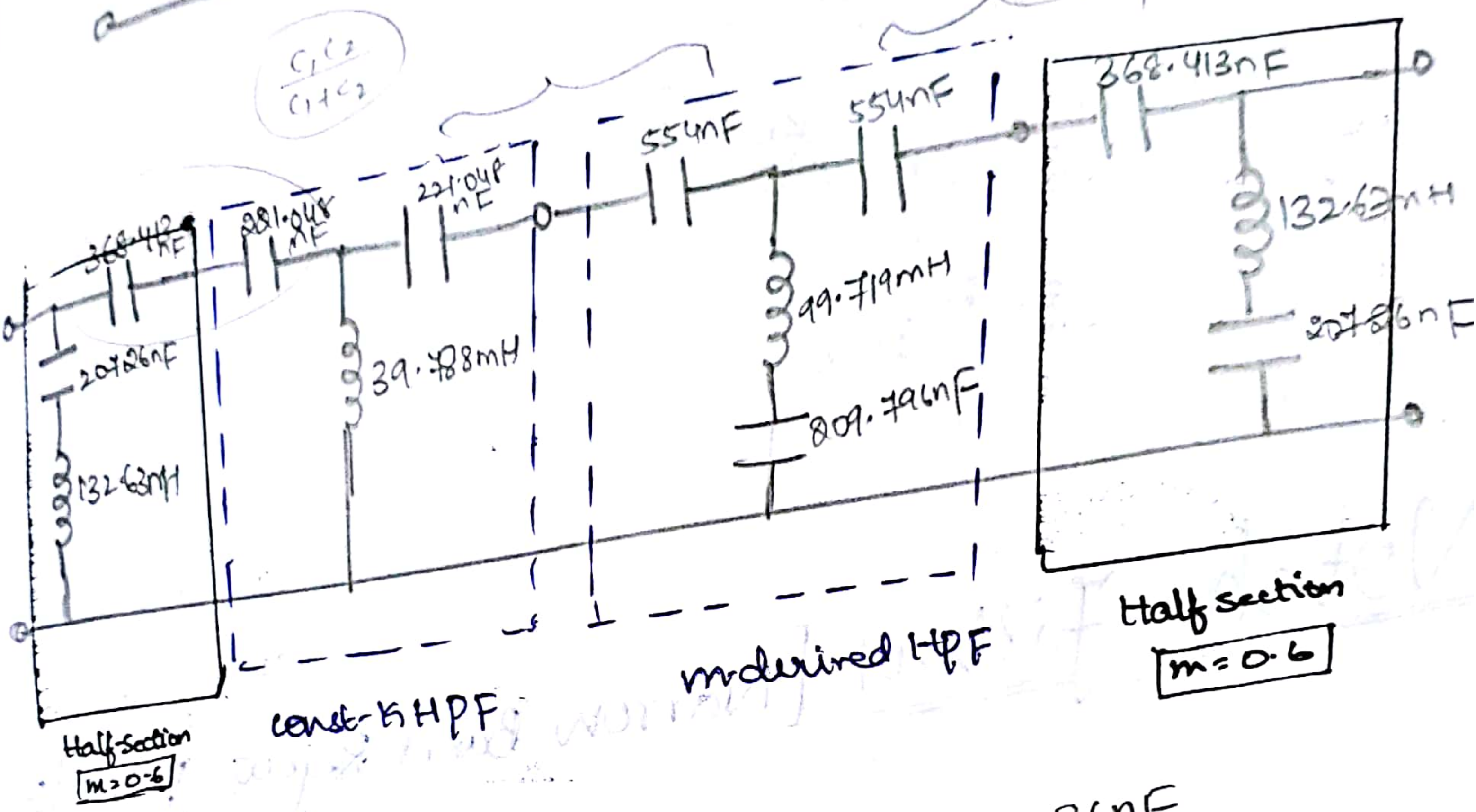
let $\boxed{m = 0.6}$

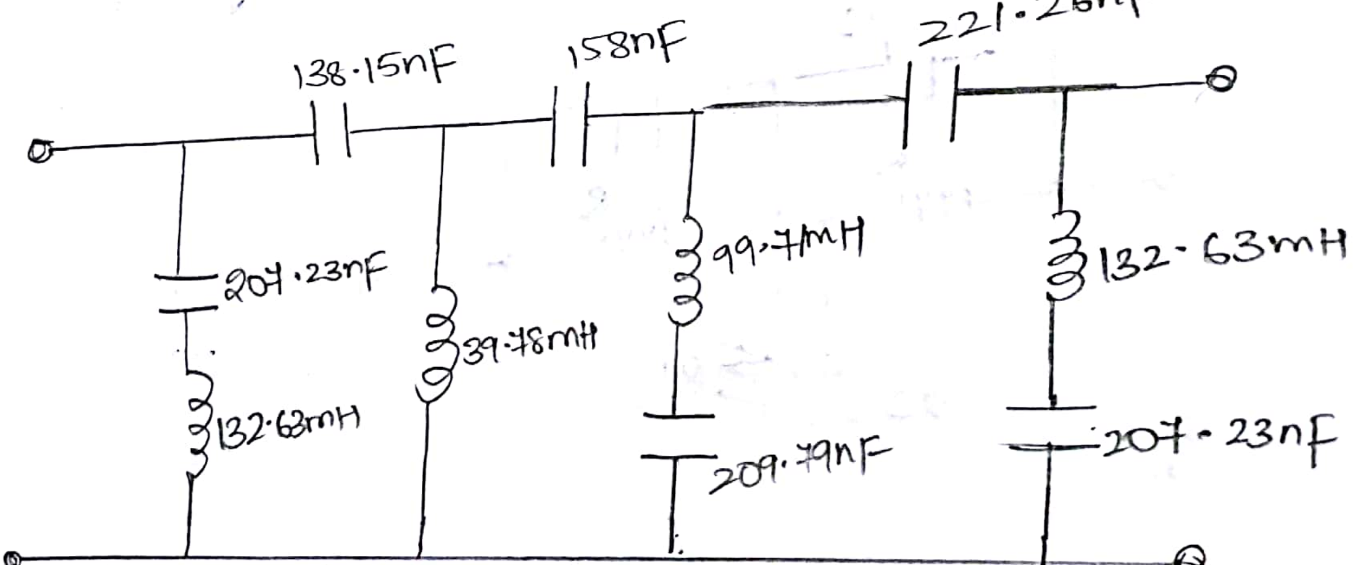
$$\frac{2C}{m} \Rightarrow \frac{2\times 110.524}{0.6} = 368.413\,nF$$

$$\frac{L}{m} \Rightarrow \frac{39.788}{0.6} = 66.313\,mH$$

$$\left(\frac{4m}{1-m^2}\right)C \Rightarrow \frac{4\times 0.6}{1-(0.6)^2}\times 110.524 = 414.465\,nF$$

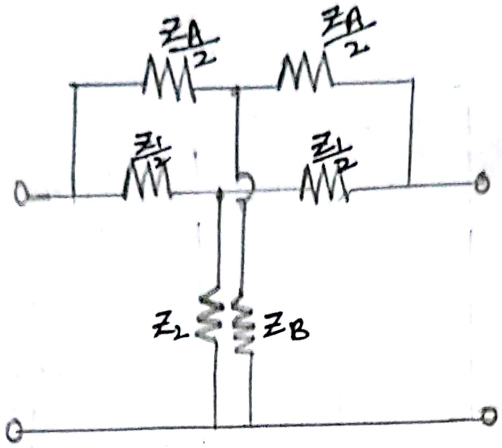

368.413nF
368.413nF
414.465nF
66.313mH
368.413nF
207.235nF
2L = 209.95nF
C/2 = 132.63mH
132.63mH


368.413nF
207.26nF
132.63mH
Half-Section
m=0.6
221.04nF
221.04nF
39.788mH
const-K HPF
554nF
554nF
99.719mH
209.796nF
m-derived HPF
368.413nF
132.63mH
207.86nF
Half section
m=0.6

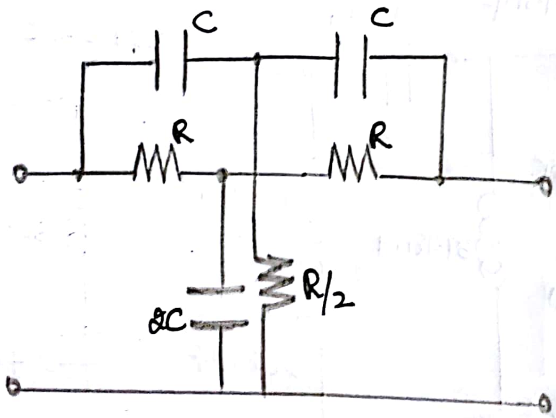

138.15nF
158nF
221.26nF
207.23nF
132.63mH
39.78mH
99.7mH
209.79nF
132.63mH
207.23nF

Twin-T Network :-

If two symmetrical T-N/w connected in ||^el, then a twin-T N/w is formed as shown in fig:-



Notch Filter :- [Narrow Band Reject Filter]



$$\boxed{\text{Reject frequency } (f_0) = \frac{1}{2\pi RC}}$$