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## Modelling and Control of Higher Rating DC Motor with GA-Based Optimized PID Controller

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### Abstract

The usage of the common kind dc motors has its extended past. It has been used in the productions for numerous ages currently. They deliver humble means and exact method of controller. In addition, they have high efficiency and have a high starting torque versus falling speed characteristics which helps high starting torque and helps to prevent sudden load rise. But with such characteristics, the dc motors have some deficiencies that needed to be attended to which gave rise to design of some other alternative types of dc motors. For this purpose, a brief perspective will be considered on how the DC motors could be compensated in terms of control and stability. Therefore, this report would present a theoretical background of DC motors, design of simple model of basic DC motors. The PID controller is applied in various fields of engineering, and it is also a very important tool in telecommunication system. If there is a system and stability is desired, then PID could be very useful. The results analysis and discussion is presented.

**Keywords:** BLDC motor, genetic algorithm, machine modelling, PID controller, ZN method

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## INTRODUCTION

### DC Motors

A brief illustration and mathematical representation of DC motors will be discussed in this section based on the general concepts of electromagnetic induction. The most important part of these components that needs detail attention is the main field and the rotating windings (the stator and the rotor, respectively).

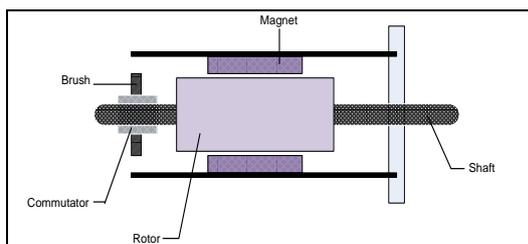


Fig. 1. Sectional Illustration of a DC Motor.<sup>[1]</sup>

As shown in Fig. 1., the stator is formed by the metal carcass with a permanent magnet enclosure which a magnetic field inside the stator windings. At one of the ends is the brush mountings and the brush gear which are used for electrical contacts with the armature (the rotor).<sup>[2]</sup>

### DC MOTOR MODEL

#### Mathematical Model of a Typical DC Motor

A typical dc motor equivalent circuit is illustrated as shown in the circuit shown below in Fig. .

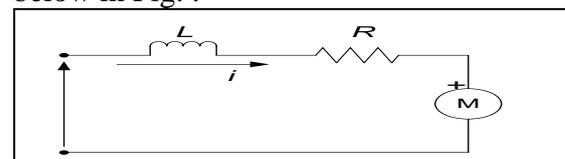
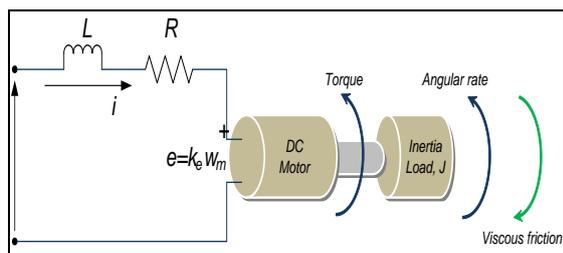


Fig. 2. A Typical DC Motor Equivalent Electrical Circuit.



**Fig. 3.** A Typical DC Motor Electromechanical System Arrangement.<sup>[3]</sup>

The basic component represented are the armature resistance,  $R$  and the armature inductance  $L$ <sup>[4]</sup> in addition, there is the back emf,  $e$ . From the Fig. , the following equations are used to describe the relationship of operation.

Using the Kirchoff's Voltage Law, *KVL*, the following Equation (1) is obtained:

$$V_s = Ri + L \frac{di}{dt} + e \quad (1)$$

At steady state (DC state of zero-frequency,  $V_s = Ri + e$ .

Therefore, for the non-steady state, Equation (1) is rearranged to make provision for the back EMF, as shown in Equation (2) below:

$$e = -Ri - L \frac{di}{dt} + V_s \quad (2)$$

where  $V_s$  is the DC Source voltage and  $i$  is the armature current.

Similarly, considering the mechanical properties of the dc motor, from the Newton's second law of motion, the mechanical properties relative to the torque of the system arrangement in Fig. and Fig. would be the product of the inertia load,  $J$  and the rate of angular velocity,  $\omega_m$  is equal to the sum of all the torques, these follow with Equations (3 and 4) accordingly.

$$J \frac{d\omega_m}{dt} = \sum T_i \quad (3)$$

$$T_e = k_f \omega_m + J \frac{d\omega_m}{dt} + T_L \quad (4)$$

where  $T_e$  is the electrical torque,  $k_f$  the friction constant,

$J$  the rotor inertia,  $\omega_m$  the angular velocity, and  $T_L$  is the supposed mechanical load.

Since, the electrical torque and the back emf could be written as:

$$e = k_e \omega_m \text{ and } T_e = k_t \omega_m \quad (5)$$

where  $k_e$  is the back emf constant and  $k_t$  is the torque constant.

Therefore, re-writing Equations (2 and 3), the Equations (6 and 7) are obtained,

$$\frac{di}{dt} = -i \frac{R}{L} - \frac{k_e}{L} \omega_m + \frac{1}{L} V_s \quad (6)$$

$$\frac{d\omega_m}{dt} = i \frac{k_t}{J} - \frac{k_f}{J} \omega_m + \frac{1}{J} T_L \quad (7)$$

Using Laplace transform to evaluate the two Equations (6 and 7), the following are obtained appropriately (all initial conditions are assumed to be zero):

For Equation (6),<sup>[5-8]</sup>

$$\mathcal{L} \left\{ \frac{di}{dt} = -i \frac{R}{L} - \frac{k_e}{L} \omega_m + \frac{1}{L} V_s \right\} \quad (8)$$

This implies,

$$s i = -i \frac{R}{L} - \frac{k_e}{L} \omega_m + \frac{1}{L} V_s \quad (9)$$

For Equation (7),

$$\mathcal{L} \left\{ \frac{d\omega_m}{dt} = i \frac{k_t}{J} - \frac{k_f}{J} \omega_m + \frac{1}{J} T_L \right\} \quad (10)$$

This implies,

$$s \omega_m = i \frac{k_t}{J} - \frac{k_f}{J} \omega_m + \frac{1}{J} T_L \quad (11)$$

At no load (for  $T_L = 0$ ); Equation (11) becomes:

$$s \omega_m = i \frac{k_t}{J} - \frac{k_f}{J} \omega_m \quad (12)$$

From Equation (12),  $i$  is made the subject for a substitute into Equation (9).

$$i = \frac{s \omega_m + \frac{k_f}{J} \omega_m}{\frac{k_t}{J}} \quad (13)$$

$$\left( \frac{s \omega_m + \frac{k_f}{J} \omega_m}{\frac{k_t}{J}} \right) \left( s + \frac{R}{L} \right) = -\frac{k_e}{L} \omega_m + \frac{1}{L} V_s \quad (14)$$

Equation (14) becomes:

$$\left\{ \left( \frac{s^2 J}{k_t} + \frac{s k_f}{k_t} + \frac{s R J}{k_t L} + \frac{k_f R}{k_t L} \right) + \frac{k_e}{L} \right\} \omega_m = \frac{1}{L} V_s \quad (15)$$

And Equation (15) finally resolved to Equation (16):

$$V_s = \left\{ \frac{s^2 J L + s k_f L + s R J + k_f R + k_e k_t}{k_t} \right\} \omega_m \quad (16)$$

The transfer function is therefore obtained as follows using the ratio of and the angular velocity,  $\omega_m$  to source voltage,  $V_s$ .

That is,

$$G(s) = \frac{\omega_m}{V_s} = \frac{k_t}{s^2 J L + s k_f L + s R J + k_f R + k_e k_t} \quad (17)$$

From these, the transfer function could be derived accordingly as follows:

That is,

$$G(s) = \frac{\omega_m}{V_s} = \frac{k_t}{s^2 J L + (R J + k_f L) s + k_f R + k_e k_t} \quad (18)$$

Considering the following assumptions:

- (1) The friction constant is small, that is,  $k_f$  tends to 0, this implies that
- (2)  $R J \gg k_f L$ , and
- (3)  $k_e k_t \gg R k_f$

And the negligible values zeroed, the transfer function is finally written as;

$$G(s) = \frac{\omega_m}{V_s} = \frac{k_t}{s^2 J L + R J s + k_e k_t} \quad (19)$$

So by re-arrangement and mathematical manipulation on “ $JL$ ”, by multiplying top and bottom of Equation (19) by:

$$\frac{R}{k_e k_t} \times \frac{1}{R}$$

Equation (20) is obtained after the manipulation,

$$G(s) = \frac{\frac{1}{k_e}}{\frac{R J}{k_e k_t} \cdot \frac{L}{R} \cdot s^2 + \frac{R J}{k_e k_t} s + 1} \quad (20)$$

From Equation (13), the following constants are gotten,

The mechanical (time constant),

$$\tau_m = \frac{R J}{k_e k_t} \quad (21)$$

The electrical (time constant),  
Substituting the Equations (21 and 22) into Equation (20), it yields;

$$G(s) = \frac{\frac{1}{k_e}}{\tau_m \cdot \tau_e \cdot s^2 + \tau_m \cdot s + 1}$$

constants.m<sup>[9]</sup>

```
% Start of code
% Sprint Electric flat motor parameters
used in the modeling
%
% Characteristics parameters
R = 120; % Ohms, Terminal
Resistance phase to phase
L = 0.560; % Henrys, Terminal
Inductance phase to phase
Kt = 25.5; % Nm/A, Torque constant
Ks = 374 % rpm/V, Speed constant
tm = 171e-3; % seconds, s, Mechanical
Time constant
J = 92.5e-7; % kg.m^2, Rotor
inertia, given in gcm^2
```

evaluatedconstants.m<sup>[9]</sup>

```
% Evaluated parameters not given
constants
te = L/(R); % seconds, s,
Electrical Time constant
Ke = (R*J)/(tm*Kt); % Back emf
constant
```

topenloop.m<sup>[9]</sup>

```
% Start of code
% includes constant parameters
constants
% includes evaluated constants
evaluatedconstants
% Transfer function
G = tf([1/Ke],[tm*te tm 1]);
% Plots the Step Response diagram
figure;
step(G, 0.5);
title('Open Loop Step Response
diagram');
xlabel('Time, secs')
ylabel('Voltage, volts')
grid on;
```

## HIGH RATING DC MOTOR

The DC motor provided for this thesis is the EC 45 flat Ø45 mm, 145 KW, 190 h.p from sprint electric motors. The order number of the motor is 200142. The parameters used in the modelling are

extracted from the datasheet of this motor with corresponding relevant parameters used. Find below in Table 1 the major extracted parameters used for the modelling task.

**RATINGS OF DC MOTOR**

*Table 1. Motor Parameters Used.*<sup>[8]</sup>

	Sprint electric motor data	Unit	Value
	Values at nominal voltage		
1	Nominal Voltage	V	550
2	No load Speed	rpm	4370
3	No load Current	A	15
4	Nominal Speed	rpm	2550
6	Nominal Current (max. continuous current)	A	335
8	Starting Current	A	65
9	Maximum Efficiency	%	89
10	Terminal Resistance phase to phase	Ω	120
11	Terminal Inductance phase to phase	H	0.560
12	Torque Constant	Nm/A	25.5
13	Speed Constant	rpm/V	374
14	Speed/Torque Gradient	rpm/Nm	176
15	Mechanical time constant	ms	171
16	Rotor Inertia	gcm <sup>2</sup>	92.5

**DC SPRINT ELECTRIC MOTOR MATHEMATICAL MODEL**

The mathematical model of the DC motor is modelled based on the parameters from Table 1 using the Equation (23). This is illustrated below:

$$G(s) = \frac{\frac{1}{K_e}}{\tau_m \cdot \tau_e \cdot s^2 + \tau_m \cdot s + 1} \tag{22}$$

So the values for  $K_e$ ,  $\tau_m$  and  $\tau_e$  need to be calculated to obtain the motor model.

From equation,

$$\tau_e = \frac{L}{R}$$

$$\tau_e = \frac{0.560}{120}$$

$$\tau_e = 4.6667 \times 10^{-3} \tag{23}$$

But  $\tau_m$  is a function of R, J,  $K_e$  and  $K_t$ ,

Where

$$R = 120 \ \Omega; J_{Rotor} = 92.5 \text{ gcm}^2 = 9.25 \times 10^{-6} \text{ Kg m}^2; K_t = 25.5 \text{ Nm/A}; \tau_m = 171 \text{ msec};$$

From Equation (21),  $K_e$  could be obtained: That is,

$$\tau_m = \frac{R \cdot J}{K_e \cdot K_t} = 0.171 \text{ sec}$$

$$K_e = \frac{R \cdot J}{\tau_m \cdot K_t} = \frac{120 \times 9.25 \times 10^{-6}}{0.171 \times 25.5}$$

$$= 2.5455 \times 10^{-3} \frac{\text{V} \cdot \text{secs}}{\text{rad}}$$

Therefore, the G(s) becomes:

$$G(s) = \frac{392.85}{4.667 \times 10^{-3} \times 0.171 \cdot s^2 + 0.171 \cdot s + 1}$$

$$G(s) = \frac{392.85}{0.798 \times 10^{-3} \cdot s^2 + 0.171 \cdot s + 1} \tag{24}$$

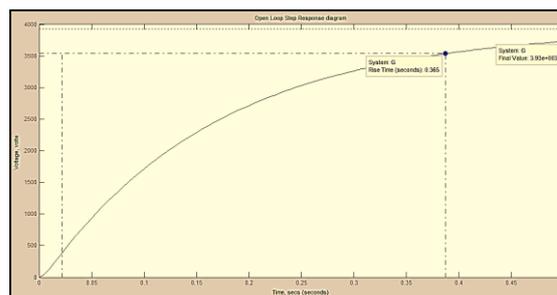
The G(s) derived above in the Equation (24) is the open loop transfer function of the DC Sprint Electric motor using all necessarily sufficient parameters available.

**OPEN LOOP ANALYSIS OF MOTOR MODEL**

The open loop analysis would be done using the MATLAB<sup>®</sup>/SIMULINK<sup>®</sup>. And the corresponding stability analysis is given likewise to see the effect thereafter when there is closed loop system incorporation.

**Open Loop Analysis Using MATLAB m-File**

With the aid of the DC motor parameters provided, the open loop analysis is done by considering the stability factors and making the necessary plots for this analysis. The below plot include the step response. For this, separate m-files were created for the constants, evaluated constants and the main files (Figure 4).



**Fig. 4. Open Loop Step Response.**

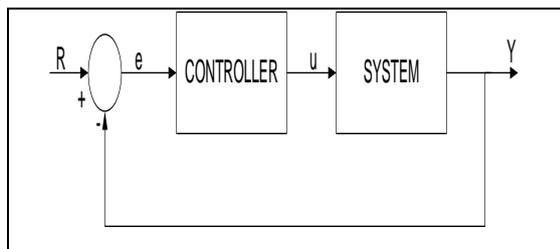
**PID DESIGN CONCEPT**

The proportional-integral-derivative (PID) controller is about the most common and useful algorithm in control systems engineering. In most cases, feedback loops are controlled using the PID algorithm. The main reason why feedback is very important in systems is to be able to attain a set-point irrespective of disturbances or any variation in characteristics of any form ref.<sup>[5]</sup>

The PID controller is always designed to correct error(s) between measured process value(s) and a particular desired set-point in a system.

A simple illustration on how the PID works is given below:

Consider the characteristics parameters – proportional (P), integral (I), and derivative (D) controls, as applied to the diagram below in Fig. 5., the system, S is to be controlled using the controller, C; where controller, C efficiency depends on the P, I and D parameters.



**Fig. 5.** A Typical System With a Controller.

The controller provides the excitation needed by the system and it is designed to control the overall behaviour of the system.

The PID controller has several categories of structural arrangements. The most common of these are the series and parallel structures and in some cases, there are the hybrid form of the series and the parallel structures.

The following shows the typical illustrative diagrams of common PID controller structures. Typically, the function of the form shown in Equation (25) is applicable in this kind of PID controller design.

$$K_P + \frac{K_I}{s} + K_D \cdot s = \frac{K_D s^2 + K_P s + K_I}{s} \quad (25)$$

where  $K_P$  is the Proportional gain,  $K_I$  the Integral gain, and  $K_D$  is the Derivative gain.

$$u = K_P e + K_I \int e dt + K_D \frac{de}{dt} \quad (26)$$

More also, it should be noted that the major goal of the PID parameters is to obtain a fast rise time with minimum overshoot and no (*almost no*) steady-state error.

**PID CONTROLLER TUNING PARAMETERS**

Under this section a critical analysis would be done on the PID tuning criteria and the parameters involved.<sup>[6]</sup> Before a detail analysis is done, a quick look at the tuning methods is considered first and thereafter, specific tuning parameters are computed for the BLDC maxon motor. Some of the generally used tuning methods are the Trial and Error method, the Ziegler-Nichols method (1<sup>st</sup>), Improved Ziegler-Nichols method (2<sup>nd</sup>), Cohen-Coon method, Genetic Algorithms and so on. For this work, the Ziegler-Nichols tuning method would be given a priority (Figure 6).<sup>[7]</sup>

**Trial and Error Tuning Methods**

This method is crude but could help in getting an overview of what the PID parameters could be like and their effects on the whole system model. It is particularly time consuming because of its trial and format. But a computational stability rule was needed to set a mark for the trial and effect. This is done by using

the Routh-Hurwitz stability rule as shown below. Under this, emphasis would be mainly on the PID combination.

**The Routh–Hurwitz stability rule**

From the various designs needed for this trial, a brief stability check is needed to make the trial and error at the first instance. It would be observed that the only design near the perfect (open-loop – which is without compensation or controller) is the PID. To have a more appropriate trial and error value, the following steps would be followed for only the PID structure.

From the PID controller Equation (27),  

$$K_P + \frac{K_I}{s} + K_D \cdot s = K_P \times \left(1 + \frac{1}{T_I \cdot s} + T_D \cdot s\right) \quad (27)$$

Similarly,  

$$K_P + \frac{K_I}{s} + K_D \cdot s = \frac{K_P \cdot s + K_I + K_D \cdot s^2}{s} \quad (28)$$

This is used in the m-file tclosedloopPID\_TrialError4.m and it is convuled with the motor model.

Keeping the  $K_P$  part, with  $T_I$  and  $T_D$  set to infinity and zero respectively. A controller gain,  $K_C$  could be obtained that would sustain the oscillation output. This value serves as the ultimate gain,  $K_{CU}$ . For a proper oscillation,  $K_C$  is set to be less than  $K_{CU}$ .

Therefore, we have:  

$$1 + K_{CU} \cdot G(s) = 0 \quad (29)$$

$$1 + K_{CU} \cdot \frac{392.85}{0.798 \times 10^{-3} \cdot s^2 + 0.171 \cdot s + 1} = 0 \quad (30)$$

Equation (30) becomes,  

$$0.798 \times 10^{-3} \cdot s^2 + 0.171 \cdot s + 1 + 392.85 \cdot K_{CU} = 0 \quad (31)$$

So, for stability purposes,  $K_{CU}$ 's range of values could be obtained by using the Routh-Hurwitz condition of stability. This is computed below:

$s^2$	$0.768 \times 10^{-3}$	$1 + 392.85 \cdot K_{CU}$
$s^1$	$0.171$	$0$
$s^0$	$1 + 392.85 \cdot K_{CU}$	$-$

According to Routh–Hurwitz condition, the obtained characteristics Equation (31) should be spread into column as shown above and the  $s^0$  is evaluated as follows (because it has the assumed unknown  $K_{CU}$  which would be evaluated):

$$s^0(1st\ row) = - \frac{\begin{vmatrix} 0.768 \times 10^{-3} & 1 + 392.85 \cdot K_{CU} \\ 0.171 & 0 \end{vmatrix}}{0.171}$$

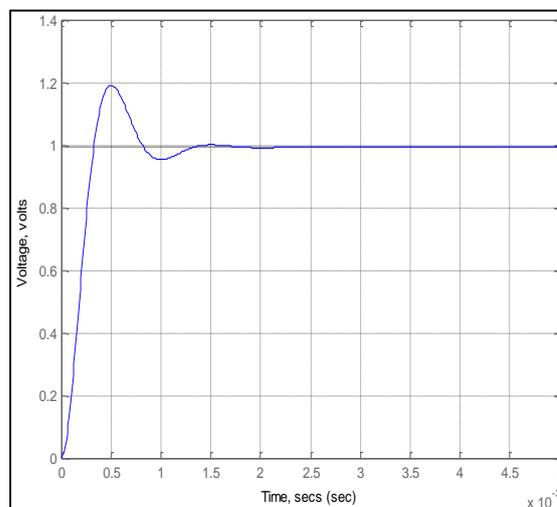
$$s^0(1st\ row) = - \frac{(0.768 \times 10^{-3} \times 0) - (1 + 392.85 \cdot K_{CU})(0.171)}{0.171}$$

$$= 1 + 392.85 \cdot K_{CU}$$

For stability sake, the 1st column after the s-column must not have any sign change (that is, no change from + to – or – to +). Therefore,  $s^0(1st\ row)$ , must be greater than zero.

This implied that,  
 $1 + 392.85 \cdot K_{CU} > 0$

Then,  
 $392.85 \cdot K_{CU} > -1$   
 $K_{CU} > \frac{-1}{392.85} = -0.002545$   
 $K_U = \text{abs}(K_{CU}) = 0.002545$



**Fig. 6.** Closed Loop Response of the System with PID Controller

## OVERVIEW OF GENETIC ALGORITHM

Genetic algorithms try to imitate the properties of natural selection and natural genetics. It begins with initial population that consists of coded strings of binary numbers, called chromosomes that hold possible solutions of the problem. Their performance is evaluated by a fitness function. Strings with higher fitness value are likely to survive to the next generation as parents. It uses three operators namely selection, crossover and mutation to produce next generations. This procedure is repeated until it finds the optimal solution to the problem.<sup>[10]</sup>

## TUNING OF PID CONTROLLER USING GA APPROACH

The values of the PID controller parameters  $K_p$ ,  $T_i$  and  $T_d$  are optimized using GA to ensure optimal control performance. The steps involved in evaluating best possible controller gains are listed below.

### Initialization of Parameters

To initialize GA, we have to define certain initial parameters. The performance of the designed controller depends heavily on how well we assigned the values to these parameters. Parameters with their assigned values are listed below:

INITIALIZATION PARAMETERS

Generations	100
Population size	30
Bit length of the considered chromosome	6
Maximum number of generations	0.8
Selection method	Arithmetic Crossover
Crossover probability	0.8
Mutation type	Uniform mutation
Mutation probability	0.05

### Objective Function for the GA

To calculate the fitness of each chromosome, selection of objective function is very critical. The objective functions are mean of the Squared Error (MSE), Integral of Time multiplied by

Absolute Error (ITAE), Integral of Absolute Magnitude of the Error (IAE), and Integral of the Squared Error (ISE).<sup>[4]</sup> An integral Absolute Error (IAE) criterion is used in this paper for calculating performance index of controller as mentioned below.

$$IAE = \int_0^T e(t) dt$$

where  $T = 1$  sec.

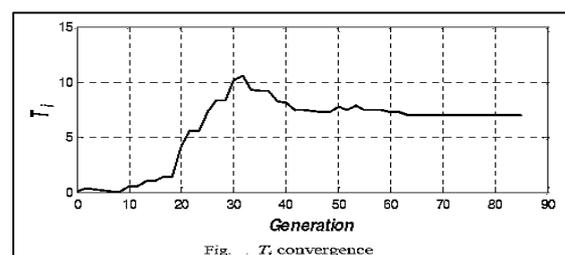
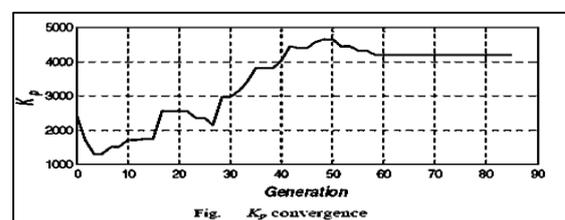
### The Fitness Values

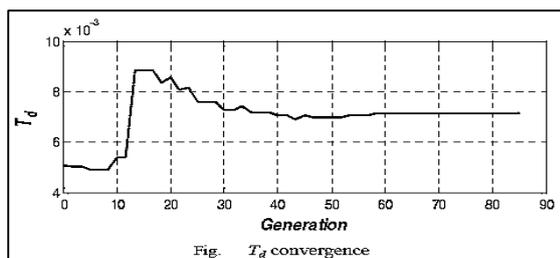
Purpose is to diminish the worth of performance index. The rightest chromosome will be the unique which has minimum value of that performance index,<sup>[5]</sup> so we can describe fitness value as

$$Fitness\ value = \frac{1}{PerformanceIndex}$$

### Termination Criteria

Termination of algorithm takes place when the value of the fitness function for the best point in the current population is less than or equal to fitness limit and the change in the value of fitness function is less than function tolerance. The best member of the population was selected and plotted in the graph. The variation of the PID parameters  $K_p$ ,  $T_i$  and  $T_d$  are shown, respectively.





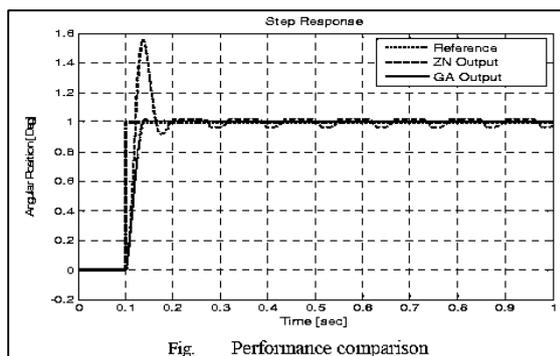
PID parameters acquired after GA optimization are listed below:

TABLE OPTIMIZED GAIN USING GENETIC ALGORITHMS

$K_p$	$T_i$	$T_d$
4200	7	0.0071429

## SIMULATION RESULTS AND COMPARISON

With optimized PID gains using GA, the step response of the controlled system is shown in below figure along with the response obtained from ZN method,



It is apparent from the above step response that the controller performance with GA optimized gains is quite efficient as compared to the response obtained from ZN method. It is clear from that the controller designed with ZN method has high overshoots as was observed in the step response, while the controller with GA optimized gains precisely follows the input signal.

## CONCLUSION

In this paper, modelling of DC motor and its optimized PID position control design using GA is presented. Comparative study is carried out in which the response of GA-

based controller is compared with the controller designed using ZN method. It is obvious from simulation results that the controller performance with GA optimized gains is much more efficient than the ZN method in terms of rise time, settling time, overshoot and set point tracking. However ZN method is good for providing the starting values of PID gains for GA optimization. Hence it can be concluded that GA provides an efficient controller tuning methodology for obtaining improved and accurate performance of a system.

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