Different Techniques for Boundary Shear Stress Predictions for Open Channel Flow

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Abstract. The fluid motion in a channel is having a direct relation to the boundary shear stress and therefore to define the fluid field and the velocity profile, knowledge of it is essentially required. Prediction and calculation of boundary shear force distributions in open channel flow are crucially required in many engineering problems such as channel design, calculation of energy losses and sedimentation. It is seen that the boundary shear stress distribution in various types of channel varies with the shape, type and patterns of the channel. In the case of the straight channel, boundary shear stress distribution varies with the different width-depth ratio, while the meandering channel boundary shear distribution varies with sinuosity, aspect ratio, and meandering. The compound channel is all the way different and boundary shear distribution is a combination of the floodplain and main channel (straight or meandering). It is thus very significant to study various methodologies adopted, identified, and used for accurate estimation of boundary shear stress distribution in various natural and artificial open channels. In the present work, critical appraisal of different approaches used for boundary shear stress distributions in channels is discussed. It has been found from the review that most common methods used by different researchers globally are Vertical Depth Method (VDM), Normal Depth Method (NDM) Guo and Julien Method (GJM), Ramana Prasad and Russell Manson Method (PMM), Knight et al. Method (KAM), Merged Perpendicular Method (MPM) and Yang and Lim Method (YLM) because these methods are simple, robust and easy to use for computing boundary shear distribution in open channels.

Keywords: open channel flow, Compound channel, boundary shear stress distribution

1 Introduction

The non-uniformity of boundary shear stress distribution over the wetted perimeter of a channel cross-section is widely proven, even for steady flows in straight prismatic channels with a simple cross-sectional geometry. This is mainly due to the anisotropy of the turbulence which produces transverse gradients of Reynolds stresses and secondary circulations (Gessner, 1973). Tominaga *et al*. (1989) and Knight and Demetriou (1983) showed that the boundary shear stress increases where the secondary currents flow towards the wall and decreases when they flow away from the wall. Other governing factors in the distribution of shear stress of a straight open channel are the geometry of the cross-section, longitudinal and lateral boundary roughness distributions and sediment concentration (Chlebek and Knight, 2006; Khodashenas *et al*., 2008). Several direct and indirect measurement techniques for boundary shear stress are reported in the literature (Al-Hamid, 1991). The widely practiced indirect measurement technique is Preston's (1954) method which has been considered for the boundary shear stress measurements of the data sets used in this research. Due to the shortcomings, limitations and demerits of these measuring techniques, determining the actual shear stress distribution along the wetted perimeter is very difficult (Patel, 1965) and hence, various empirical, analytical and computational methods have been developed to predict the boundary Shear Stress (Khodashenas *et al*., 2008).

2. STATE-OF-ART

Several decades ago, Leighly (1932) proposed that conformal mapping to be used to study the boundary shear stress distribution in open-channel flow. In the absence of secondary currents, he pointed out that, the boundary shear stress acting on the bed must be balanced by the downstream component of the weight of water contained within the boundary orthogonally.

Hydraulic radius separation Einstein (1942) method is still extensively used in laboratory studies and engineering practice.

Zheleznyakov (1965) was probably the first investigator, considering the interaction between the main channel and the adjoining floodplain in his studies. He demonstrated the effect of momentum transfer mechanism, which was responsible for decreasing the overall rate of discharge for floodplain depths just above the bank-full level in his laboratory work while its significance is only in very small depths because as floodplain depth increased, the nature of flow does not behave in the same way as that earlier in small depths.

Ghosh and Roy (1970) presented the boundary shear distribution in rough as well as smooth open channels of trapezoidal and rectangular cross sections, by direct measurement of shear drag on an isolated length of the test channel using the technique of three point suspension system suggested by Bagnold.

Both Ghosh and Jena (1973) and Ghosh and Mehata (1974) reported studies on boundary shear distribution in straight two-stage channels for smooth and rough boundaries. They found that the distribution of shear is non-uniform and the location of maximum shear on the bed and side to be some distance from the centerline and free surface respectively.

Myers and Elswy (1975) studied the shear stress distribution in channels of complex sections and the effect of interaction mechanism. In comparison to the values under the isolated condition, the results showed a decrement of 22 percent in channel shear and increment up to 260 percent in floodplain shear. This possibly indicates regions of erosion and scour of the channel and flow distribution in alluvial compound sections.

Rajaratnam and Ahmadi (1979) under the smooth boundary condition studied the flow interaction between the straight main channel and symmetrical floodplain. The results demonstrated the transport of longitudinal momentum from the main channel to the floodplain. Due to flow interaction, a considerable increase in the bed shear in the floodplain near the junction with the main channel is seen whereas in the main channel it decreased. The effect of interaction reduced as the flow depth in the floodplain increased.

Knight (1981) proposed an empirically derived equation that presented the percentage of the shear force carried by the walls as a function of the breadth/depth ratio and the ratio between the Nikuradse equivalent roughness sizes for the bed and the walls.

Wormleaton, Alen, and Hadjipanos (1982) used "divide channel" method for the assessment of discharge while undertook a series of laboratory tests in straight channels with symmetrical floodplains. From the measurement of boundary shear, apparent shear stress at the horizontal, vertical and diagonal interface plains originating from the main channel-floodplain junction could be evaluated. An apparent shear stress ratio was given which was a useful yardstick in selecting the best methodology of dividing the channel for discharge calculation.

Knight and Demetriou (1983) performed experiments in straight, symmetrical compound channels to investigate the discharge characteristics, boundary shear stress and boundary shear force distributions in the section. They came out with equations for calculating the percentage of shear force carried by floodplain and also the proportions of the total flow in various sub-areas of the compound section in terms of two dimensionless channel parameters.

Knight and Hamed (1984) worked in the same direction of that of Knight and Demetriou (1983) but considering rough floodplains. The floodplains were roughened progressively in six steps to study the influence of different roughness between the main channel and floodplain to the process of lateral momentum transfer. They presented, equations for the shear force percentages carried by floodplains and the apparent shear force in vertical, horizontal, diagonal, and bisector interface plains were given using four dimensionless channel parameters.

Knight and Patel (1985) stated some of the laboratory experiments results in relation to the distribution of boundary shear stresses in smooth closed ducts of a rectangular cross section for aspect ratios between 1 and 10. The distributions were shown to be impelled by the number and the shape of the secondary flow cells, which, in turn, rest upon the aspect ratio.

Knight, Yuan, and Fares (1992) gave details of the experimental data of SERC-FCF concerning boundary shear stress distributions in meandering channels all over the path of one complete wavelength. They also reported the experimental data on velocity vectors, surface topography, and turbulence for the two types of meandering channels of sinuosity 1.374 and 2.043 respectively. They inspected the effects of channel sinuosity, secondary currents, and cross section geometry on the value of boundary shear in meandering channels and gave a momentum force balance for the flow.

Knight and Sterling (2000) detected the distribution of boundary shear stress in a partially full circular conduit with and without a smooth, flat bed for a data ranging from 0.375<F<1.96 and 6.5*104<R<3.42*105, using Preston-tube technique. The distribution of boundary shear stress is shown to depend on Froude number and geometry.

Yang and McCorquodale (2004) came up with a method for computing three-dimensional Reynolds shear stresses and boundary shear stress distribution in smooth rectangular channels by considering an order of magnitude analysis to integrate the Reynolds equations. An abbreviated relationship between the lateral and vertical terms was hypothesized with which the Reynolds equations become solvable. This relationship was in the form of a power law with an exponent of n = 1, 2, or infinity

Guo and Julien (2005) proposed a technique to define average bed and sidewall shear stresses in smooth rectangular open-channel flows after resolving the continuity and momentum equations. The analysis revealed that the shear stresses were functions of three components: (1) interfacial shear stress; (2) gravitational; and (3) secondary flows.

Khatua (2008) extended the work of Patra and Kar (2000) to meandering compound channels. Considering five parameters (sinuosity *Sr*, amplitude, relative depth, width ratio and aspect ratio), obtained general equations representing the total shear force percentage carried by floodplain. The proposed equations are simple, quite reliable and gave good results with the observed data for a straight compound channel of Knight and Demetriou (1983) as well as for the meandering compound channel.

Lashkar and Fathi (2010) did experiments to determine the contribution of the wall shear force on total boundary shear force. A nonlinear regression-based technique was conducted to inspect the results and develop equations to determine the percentage of wall and bed shear force on the wetted perimeter of the rectangular channels.

Khatua (2010) stated the distribution of boundary shear force for highly meandering channels having distinctly different sinuosity and geometry. Based on the work, the interrelationship between the boundary shear, sinuosity, and geometry parameters has been revealed. The models are also proven using the well-published data of other investigators.

3. Categorization of Different Methodologies

3.1 Geometrical methods

Geometrical methods count on dividing the channel cross-section into sub-regions. The shear force along each of the segments of the boundary subdivided is found by balancing the forces against the weight of fluid in the corresponding sub-region. Leighly's (1932) method, Vertical depth Method (VDM), Einstein's (1942) method, Normal Area Method (NAM), Vertical Area Method (VAM), Merged Perpendicular Method (MPM) (Khodashenas and Paquier, 1999) and Normal Depth Method (NDM) (Lundgren and Johnson, 1964) are among the stated geometrical methods in literature.

3.2 Empirical methods

Empirical methods are generally simple regression technique developed from curve fitting to measured experimental data. Perhaps the very first model of such kind is Knight's (1981) model. His model was further developed by him and his colleagues (Knight *et al.*, 1984a & b and 1994), and other researchers (Flintham and Carling, 1988).

Pizzuto (1991) and Olivero *et al*. (1999) also suggested similar simple models for the boundary shear stress.

3.3 Analytical methods

Analytical methods are based on the law of continuity, momentum equations and energy transportation. Some of these methods lead to a geometric solution for solving the shear stress in open channels. Some of the analytical methods include the work of Yang and Lim (1997, 2005), Zheng and Jin (1998), Guo and Julien (2005) and Bilgil (2005).

3.4 Computational methods

Perhaps, more accurate way of finding the boundary shear stress distribution is by means of a turbulence closure model to elucidate the governing equations of motion. For instance, Christensen and Fredsoe (1998) used the Reynolds stress turbulence model (RSM) and De Cacqueray *et al*. (2009) used the SSG Reynolds stress turbulence model in computational fluid dynamics (CFD) software to predict the boundary shear stress in open channels for solving the equations of motion.

When examining sediment transport and the evolution of river morphology, it is essential to estimate the boundary shear stress distribution. However, accurate computation of the local shear stress is a challenging task even using sophisticated turbulence models. As an alternative, various empirical, analytical or simplified computational methods were developed. Most of them were focused on the computation of the local, the mean wall, and the mean bed shear stresses in straight and prismatic channels of rectangular, trapezoidal and circular with or without flatbed or compound cross-sections. In total, these methods rely on different assumptions, which may top to an approach dependent shear stress.

The aim of the particular research is to provide a quantitative assessment of various existing methods for the computation of the boundary shear stress in open channel flow. These seven methods were preferred because they provide a method, sufficiently general to compute the boundary shear stress, and because they are simple enough for engineering application.

4. Review of Typical Methods

4.1 Vertical Depth Method (VDM)

This method adopts that the local shear stress τ_i on one wetted perimeter point *i* is proportional to the local water depth h_i as

$$\tau_i = \rho g h_i J \tag{1}$$

where ρ is the water density, g is the gravitational acceleration and J is the energy slope. The arbitrary cross-sectional shape can be considered for the application of the VDM, although the method ignores secondary currents and the transfer of momentum between the main channel and its floodplains. Furthermore, the roughness distribution along the wetted perimeter is presumed to be homogeneous.

4.2 Normal Depth Method (NDM)

Lundgren and Jonsson (1964) acclaimed that the concept of "vertical depth" is not applicable for the calculation of the boundary shear stress distribution for the steep side slope. They instead of the VDM used the Normal Depth Method (NDM) in which $\mathrm{h_i}$ of Eq. (1) is replaced by $\mathrm{h_{Ni}}$, where $\mathrm{h_{Ni}}$ is the flow depth designated along the line normal to the wetted perimeter and the equation is

$$\tau_i = \rho g h_{Ni} J \tag{2}$$

The Fig.(1) (S. R. KHODASHENAS et. al., 2008) shows the Schematic illustrations of the VDM and NDM

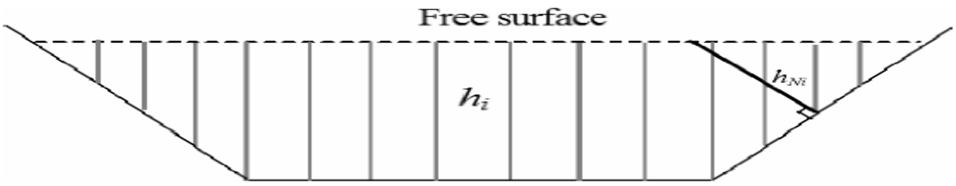

Fig. (1) Schematic illustrations of the VDM and NDM (S. R. KHODASHENAS et. al., 2008)

4.3 Merged perpendicular method (MPM)

Khodashenas and Paquier (1999) established a geometrical method to estimate the local shear stress in an irregular cross-section. This Merged Perpendicular Method (MPM) was derived from the Normal Area Method that depends on Einstein's (1942) hydraulic radius separation concept, which is "a cross-sectional region bounded by walls dividing into three sub-areas, corresponding to sidewalls and bed, respectively". The wetted area is divided into small sub-areas using the lines normal to the wetted perimeter according to the following procedure (Fig. 2) (El kadi Abderrezzak, 2006)

i. The wetted perimeter P is divided into small segments i of length P_i.

ii. Two perpendiculars L_{i-1} and L_i are drawn from the limits of each segment i. Lines L_{i-1} and L_i are considered of the order 1.

iii. When two adjacent perpendiculars cross at a common point, one single line of order 2 elongates them. This line is the bisector of the two perpendiculars. For example, the angle between the horizontal plane and $L_{i,\,i-1}$ which results from the intersection of L_{i-1} and L_i is

$\hat{L}_{i,i-1} = 1/2(\hat{L}_i + \hat{L}_{i-1})$ with "^" as the angle between the horizontal plane and the line L_i.

iv. When two lines of order j and k respectively intersect at a mutual point, they are elongated by one single line of the order $j + k$. The angle between this line and the horizontal plane is achieved through the weighted mean of the angles between the previous lines and the horizontal plane. For example, the order of $L_{i,i-1,i+1}$ following from the intersection of $L_{i,i-1}$ (order 2) and L_{i+1} (order 1) is 3. The angle between $L_{i,i-1,i+1}$ and the horizontal plane is (Fig. 2)

$\hat{L}_{i,i-1,i+1} = 1/3(\widehat{2L}_{i,i-1} + \hat{L}_{i+1})$

v. For each section *i* the local hydraulic radius $R_{hi} = S_i/P_i$ between the final lines is figured with S_i as the flow area. The local shear stress τ_i is then

$$\tau_i = \rho g R_{hi} J \quad (3)$$

The mean boundary stresses $\bar{\tau}_{(b)}$ and $\bar{\tau}_{(w)}$ representing respectively on the bed and the sidewall is defined by numerical integration of the local values.

The MPM delivers results of more practical and credible than the given by the Vertical Depth Method, the Normal Area Method and the Normal Depth Method, predominantly because the local shear stresses achieved in convex corners are higher than in concave corners, i.e. zones where the flow velocity is usually low (Khodashenas and Paquier, 1999). But this method disregards the transfer of momentum between the main channel and its floodplains and the secondary flow structures. Furthermore, the roughness distribution besides the wetted perimeter is not considered when the wetted area is divided into sub-areas.

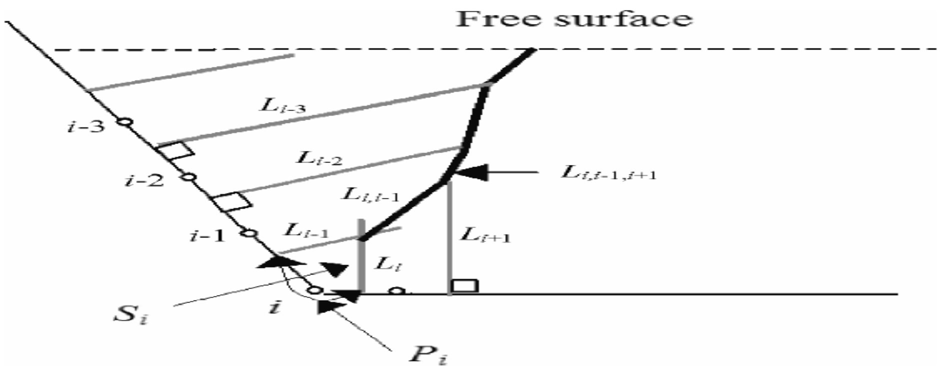

Fig.(2) Schematic illustrations of the areas determined by MPM (El kadi Abderrezzak, 2006)

4.4 Yang and Lim method (YLM)

4.4.1 General description of the method

Yang and Lim (1997, 2002, 2005) obtained an analytical method, which is akin to the MPM, to compute the distribution of shear stress in prismatic channels with a non-uniform boundary roughness. Their method is established on the concept of "surplus energy transport through a minimum relative distance toward the nearest boundary" in steady, uniform and fully developed turbulent flow. Yang and Lim described the relative distance as the ratio of the shortest geometric distance to the energy dissipation capability of the boundary. For a smooth boundary, the characteristic length representing the energy dissipation capacity of the boundary is scaled using the viscous length scale v/u_*, with v as the kinematic fluid viscosity and u_* as the shear velocity. For a rough boundary, the characteristic length is scaled using the boundary roughness height. From this concept, Yang and Lim divided the flow area into sub-areas according to the cross-sectional shape and roughness composition of its wetted perimeter. Meanwhile, the secondary currents are not taken into account.

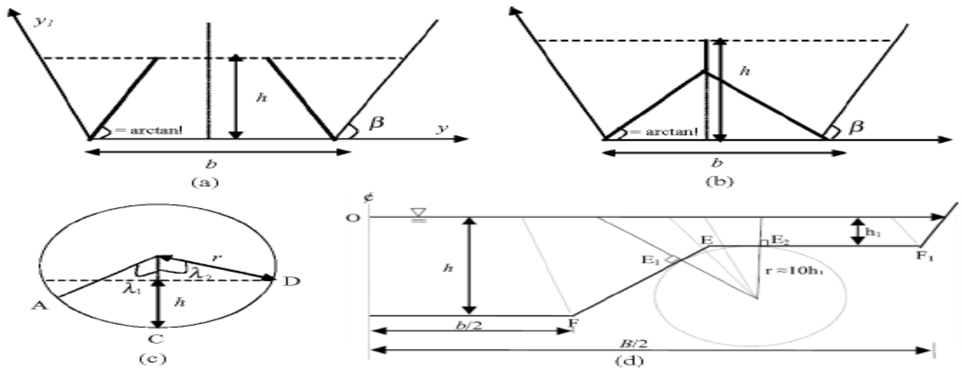

Fig.(3) Schematic cross-sections to which the YLM is applied (a)Trapezoidal shallow wide channel,(b) Trapezoidal deep narrow channel,(c) Circular conduit,(d) Compound channel (S. R. Khodashenas et. al., 2008)

4.4.2 Smooth trapezoidal channel

For wide channels, the intersection of division lines is located above the water surface (Fig. 3a) (S.R. Khodashenas et. al., 2008) if

$$\frac{b}{h} \geq 2\,\frac{1-\cos\beta}{\sin\beta} \tag{6}$$

where h is the flow depth, b is the channel bed width, β is the angle between the sidewall and water surface. The shear stress distributions $\tau_{(b)}$ and $\tau_{(w)}$ respectively, alongside the bed and the sidewall can be represented by (El kadi Abderrezzak, 2006)

$$\begin{aligned} &\tau_{(b)}y = \rho g \xi y J && 0 \leq y \leq \frac{h}{\xi} \\ &\tau_{(b)}y = \rho g h J && \frac{h}{\xi} \leq y \leq b - \frac{h}{\xi} \\ &\tau_{(b)}y = \rho g \xi (b-y) J && b - \frac{h}{\xi} \leq y \leq b \end{aligned} \tag{7}$$

$$\begin{aligned} &\tau_{(w)}y_1 = \rho g y_1 \frac{\psi \sin\beta}{1-\psi\cos\beta} J && 0 \leq y_1 \leq \frac{\psi\cos\beta}{1-\sin\beta} h \\ &\tau_{(w)}y_1 = \rho g \left(\frac{h}{\sin\beta} - y_1\right) \tan\beta\, J && \frac{1-\psi\cos\beta}{\sin\beta} h \leq y_1 \leq \frac{h}{\sin\beta} \end{aligned} \tag{8}$$

where y is the crosswise span-wise distance calculated from channel sidewall, y_1 is the distance alongside the sidewall measured from the channel corner. ξ is the slope of the dividing line and $A = u^*_{(b)}/u^*_{(w)}$ is the ratio of the sidewall energy dissipation capacity to the bottom energy dissipation capacity. These parameters are defined as

$$\begin{aligned} &\xi = \frac{\sin\beta}{\psi - \cos\beta} \\ &\psi^3 + \frac{2h}{b\sin\beta}\psi - 2\left(1 + \frac{h}{b}\frac{1}{\tan\beta}\right) = 0 \end{aligned} \tag{9}$$

The mean boundary stresses $\bar{\tau}_{(b)}$ and $\bar{\tau}_{(w)}$ are given by (El kadi Abderrezzak, 2006)

$$\begin{aligned} &\frac{\bar{\tau}_{(b)}}{\rho g h J} = 1 + \frac{h}{b}\frac{1}{\tan\beta} - \psi\frac{h}{b}\frac{1}{\sin\beta} \\ &\frac{\bar{\tau}_{(w)}}{\rho g h J} = \frac{1}{2}\psi \end{aligned} \tag{10}$$

It is interesting to emphasize that the approach of Yang and Lim (Eq. (8)) is not valid if

$$1 - \psi\cos\beta \leq 0 \tag{11}$$

Then, the authors assumed that the boundary shear stress is uniform and given by $\rho g R_h J$. For narrow channels, the meeting point of division lines is located under the water surface (Fig. 3b) (S. R. Khodashenas et. al., 2008) if

$$\frac{b}{h} \le 2\,\frac{1-\cos\beta}{\sin\beta} \tag{12}$$

The shear stress distributions $\tau_{(b)}$ and $\tau_{(w)}$, correspondingly along the bed and the sidewall are given by (El kadi Abderrezzak,2006)

$$\begin{aligned}
&\tau_{(b)}(y) = \rho g \xi y J \qquad 0 \le y \le \frac{b}{2}\\
&\tau_{(b)}(y) = \rho g \xi (b-y) J \qquad \frac{b}{2} \le y \le b
\end{aligned} \tag{13}$$

$$\begin{aligned}
&\tau_{(w)} y_1 = \rho g y_1 \frac{\psi \sin\beta}{1-\psi\cos\beta} J \qquad 0 \le y_1 \le \frac{1-\psi\cos\beta}{\psi-\cos\beta}\frac{b}{2}\\
&\tau_{(w)}(y_1) = \rho g \left(\frac{b}{2\sin\beta} + \frac{y_1}{\tan\beta}\right) J \qquad \frac{1-\psi\cos\beta}{\psi-\cos\beta}\frac{b}{2} \le y_1 \le h\sin\beta - \frac{b}{2}\cos\beta\\
&\tau_{(w)} y_1 = \rho g \left(\frac{h}{\sin\beta} - y_1\right) \tan\beta\, J \qquad h\sin\beta - \frac{b}{2}\cos\beta \le y_1 \le \frac{h}{\sin\beta}
\end{aligned} \tag{14}$$

The parameters ξ and ψ are defined as

$$\xi = \frac{\sin\beta}{\psi - \cos\beta}$$

$$\left(\frac{1}{\psi}\right)^3 + \frac{b\sin\beta}{2h}\left[\frac{4h}{b\tan\beta}\left(1+\frac{h}{b}\frac{1}{\tan\beta}\right)+1\right]\left(\frac{1}{\psi}\right) - 2\left(1+\frac{h}{b}\frac{1}{\tan\beta}\right) = 0 \tag{15}$$

The mean boundary stresses $\bar{\tau}_{(b)}$ and $\bar{\tau}_{(w)}$ are given by (El kadi Abderrezzak, 2006)

$$\begin{aligned}
&\frac{\bar{\tau}_{(b)}}{\rho g h J} = c\xi\\
&\frac{\bar{\tau}_{(w)}}{\rho g h J} = \frac{b}{4h\psi^2}\,\xi
\end{aligned} \tag{16}$$

As for the case of shallow-deep channels, the YLM is not valid if

$$(1 - \psi \cos \beta) \leq 0 \qquad \text{or} \qquad \frac{2h}{b} \leq \frac{1}{\tan \beta} \tag{17}$$

and the authors presumed again that the boundary shear stress is uniform and equal to $\rho g R_h J$

4.4.3 Rough trapezoidal channel

For channels of homogeneous and rough trapezoidal cross section, the mean, local bed and mean sidewall stresses are obtained using the Eqs (6)–(17) with $\psi = 1$, for the reason that the partition lines are the bisectors of the internal base angles of the trapezoidal channel.

4.4.4 Circular channel with homogeneous boundary roughness

The boundary shear stress distribution in circular conduits of homogeneous boundary roughness and flowing partially full (h/r < 1) is

$$\tau(A) = \left(\frac{2\lambda_2}{2\lambda_2 - \sin 2\lambda_2}\right)\left[1 - \left(1 - \frac{h}{2}\right)^2 \left(\frac{1}{cos^2 \lambda_1}\right)\right] \rho g R_h J \tag{18}$$

where $\tau(A)$ is the local boundary shear stress at an angle λ_1 from the normal line OC (Fig. 3c) (S. R. Khodashenas et. al., 2008), λ_1 is the angle between the radius OA and the normal line OC, λ_2 is the angle between the radius drawn to the water surface OD and the normal line OC, r is the conduit radius.

4.4.5 Compound channel

On the base of YLM, Yang *et al*. (2004) projected a method to calculate the local boundary shear stress at the edge E of a floodplain profile shown in Fig. 3(d) (S. R. Khodashenas et. al., 2008). A circle with an experimental radius of $10h_1$ and tangential to (FE) and (F1E) is drawn, with $h1$ as flow depth in the floodplain. The energy in the element bounded by two relative lines (OE1) and (OE2) and the free surface is expected to be dissipated at point E. The local boundary shear stress at E was defined using the wetted perimeter E1EE2 and area of this element. The floodplain and main channel are considered separately as a trapezoidal channel, and equations (6)–(17) can be used to calculate the boundary shear stress distribution. This technique gives a realistic incessant distribution of boundary shear stress from point E to F and from point F1 to E. Though the transformation of momentum between the main channel and its floodplain is ignored.

4.5 Guo and Julien method (GJM)

The mean bed and sidewall shear stresses in the smooth rectangular open channel were defined by Guo and Julien (2005) by solving the momentum and continuity equations. As a first approximation, they determined the average bed and sidewall shear stresses by utilizing conformal mapping, after ignoring secondary currents and by supposing a constant eddy viscosity (Eq. (19)). In a second approximation, they added two lumped empirical correction factors for the effects of secondary currents, variable eddy viscosity and other possible effects (Eq. (20)). The mean bed shear stress is given by

Without Secondary Currents

$$\frac{\bar{\tau}_{(b)}}{\rho g h J} = \frac{4}{\pi^2}\frac{b}{h}\sum_{n=1}^{\infty}(-1)^n \frac{t^{2n-1}-1}{(2n-1)^2} \text{ with } t = e^{-\frac{\pi h}{b}} \quad (19)$$

With Corrections Factors

$$\frac{\bar{\tau}_{(b)}}{\rho g h J} = \frac{4}{\pi} Arctg[\exp\left(\frac{-\pi h}{b}\right)] + \frac{\pi}{4}\frac{h}{b}\left(\frac{-h}{b}\right) \quad (20)$$

The mean sidewall shear stress can be calculated by

$$\frac{\bar{\tau}_{(w)}}{\rho g h J} = \frac{b}{2h}\left(1 - \frac{\bar{\tau}_{(b)}}{\rho g h J}\right) \quad (21)$$

4.6 Ramana Prasad and Russell Manson method (PMM)

Ramana Prasad and Russell Manson (2002) suggested an analytical expression for calculating the percentage shear force $\%SF_w$ taken by the sidewall in prismatic channels of the trapezoidal cross section with homogeneous boundary roughness. The influence of secondary currents was neglected. The percentage shear force $\%SF_w$ is given in the expression of width-depth ratio b/h by

$$\%SF_w = \frac{100\bar{\tau}_{(w)}}{\bar{\tau}_{(w)}+\bar{\tau}_{(b)}\left(\frac{P_{(b)}}{P_{(w)}}\right)} = \begin{Bmatrix} 25\left(4-\frac{b}{h}\right) & \frac{b}{h} \le 2 \\ \frac{100}{\frac{b}{h}} & \frac{b}{h} \ge 2 \end{Bmatrix} \tag{22}$$

where *P(b)* and *P(w)* are bed and sidewalls wetted perimeter, respectively. Knowing $\%SF_w$, it is possible to obtain $\bar{\tau}_{(b)}$ and $\bar{\tau}_{(w)}$ via the equations

$$\frac{\bar{\tau}_{(b)}}{\rho g h J} = (1 - 0.01\%SF_w)\left(1+\frac{P_{(w)})}{P_{(b)}}\right) \tag{23}$$

$$\frac{\bar{\tau}_{(w)}}{\rho g h J} = (1 - 0.01\%SF_w)\left(1+\frac{P_{(b)}}{P_{(w)}}\right) \tag{24}$$

4.7 Knight et al. method (KAM)

Knight *et al*. (1994) offered an empirical equation for calculating the percentage shear force $\%SF_w$ taken by the sidewall in prismatic channels of the trapezoidal or rectangular cross-section with homogeneous boundary roughness. It was improved on the basis of a huge range of experimental data involving both subcritical ($\mathsf{F} < 1$) and supercritical ($\mathsf{F} > 1$) flows in straight channels of rectangular and trapezoidal cross-section, in which $\mathsf{F} = u/(gS/b)^{1/2}$ is the Froude number, *u* the flow velocity, *b* the surface width and *S* the cross-sectional area. Later, Knight and Sterling (2000) explored experimentally the distribution of the boundary shear stress in smooth circular conduits, with or without a flatbed, flowing partially full and established that the percentage shear force carried by the walls is predominantly well reproduced for *P(b)/P(w)* > 1 (Knight and Sterling, 2000) by

$$\%SF_w = C_{cf} exp\left[-3.23\log_{10}\left(\frac{P_{(b)}}{P_{(w)}C_2}+1\right)+4.6052\right] \tag{25}$$

For $\mathsf{F} < 1$: $C_2 = 1.50$, $C_{cf} = 1$ for *P(b)/P(w)* < 6.546, C_{cf} =0.5875*(P(b)/P(w))*0.28471 for *P(b)/P(w)* ≥ 6.546, and

For F > 1: C_2 = 1.38, C_{cf} = 1 for $P(b)/P(w)$ < 4.374, C_{cf} =0.6603$(P(b)/P(w))$0.28125 for $P(b)/P(w) \geq$ 4.374.

5. CONCLUSIONS

This analysis report provides a comparison of different existing methods for computing the boundary shear stress distribution in prismatic channels of simple cross-sectional shape, including the rectangular, circular, and trapezoidal with and without flatbed and compound sections, and uniform boundary roughness. Six technique were picked and tested: Vertical Depth Method (VDM), Merged Perpendicular Method (MPM), Yang and Lim Method (YLM), Guo and Julien Method (GJM), Ramana Prasad and Russell Manson Method (PMM), and the Knight *et al*. Method (KAM). The methods are defined and results are contrasted with laboratory data in terms of the mean bed and mean sidewall shear stresses (or the percentage of the total shear force acting on the sidewalls), and the difference of local shear stress versus perimetric distance.

The experimental database suggests that the local boundary shear stresses and the mean bed and sidewall shear stresses are considerably impelled by the boundary roughness, the cross-sectional shape as well as the presence of secondary flows. The extensively used VDM does not offer reliable results in terms of the local shear stress distribution. The GJM with variable eddy viscosity, Correction factors for secondary currents effects and other likely effects is obtained to give the best prediction of the mean bed and the mean wall shear stresses in smooth rectangular channels. This result verifies the compulsion to take into consideration the consequence of secondary currents. The PMM and KAM could be a good predictor of the wall and the mean bed shear stresses, respectively, in rough rectangular cross-section, and in circular cross-section with a flatbed. MPM and YLM provide the complete predictions of the local shear stress for the trapezoidal, rectangular and circular cross-sections. These two methods yield comparable results, but the MPM has the advantage to be also acclimated to irregular cross-sectional shape. Even for compound cross-sections, MPM and YLM afford a suitable estimation of the local shear stress, except near corners, at the edge of the cross-section and around the main channel-floodplain interface region. These local inconsistencies are directly related to the fact that MPM and YLM do not include the lateral flow exchange between the floodplain and the main channel that initiates secondary cells in the floodplain as well as in the main channel.

In terms of application, the technique presented are helpful engineering tools and easy to implement in numerical models. There are enormous practical problems even considering flows in flumes of the trapezoidal or compound cross-section for which it is essential to recognize the boundary shear stress distribution. The methods may be straightaway extended to arbitrary cross-sectional shape with a non-uniform roughness distribution.

An overall validation of the technique presented earlier would involve further measurements of the boundary shear stress distribution in smooth, intermediate and rough channels of varying cross-sectional shapes. A wide experimental measurement is accessible at www.flowdata.bham.ac.uk, which could be utilized to complete the comparison of the selected methods. The database is from the University of Birmingham and summarizes over 600 complete sets of boundary shear stress and velocity data for different boundary roughness distribution, flow conditions and geometry.

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7. NOTATIONS

b=Channel width of bed

B=Channel width of floodplain bed

Ccf =Constant of Eq. (25)

C2 =Constant of Eq. (25)

F=Froude number

g=Gravitational acceleration

h=Flow depth

h_N =Flow depth calculated along the line normal to the wetted perimeter

$h1$ =Flow depth in floodplain

J =Energy slope

L=Line normal to the bottom

$\hat{L}$=Angle between horizontal plane and line L

P =Total wetted perimeter

$P(b)$ =Wetted perimeter corresponding to bed

$P(w)$ =Wetted perimeter corresponding to the sidewalls

Pd =Dimensionless perimetric distance = s/P

Q=Discharge

r =Conduit radius

Rh =Hydraulic radius = S/P

S =Flow area

$u*$ =Shear velocity

y=Transverse span-wise distance measured from channel sidewall

$y1$ =Distance along the sidewall measured from the channel corner

β=Angle between sidewall and water surface

$\lambda1$ =Angle between radius OA and normal line OC

$\lambda2$ =Angle between radius drawn to the water surface OD and the normal line OC

v=Kinematic viscosity

ξ =Coefficient

ρ=Density of water

τ =Local boundary shear stress

$\tau*$ $=\tau/(\rho g Rh J)$ non-dimensionalized shear stress

$\tau(b)$ =Local bed shear stress

$\tau(w)$ =Local sidewall shear stress

$\bar{\tau}(b)$ =Mean bed shear stress

$\bar{\tau}(w)$ =Mean sidewall shear stress

ψ =Coefficient

%*SFw* =Percentage of total shear force acting on the walls

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