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# Loss of Energy in the Converging Compound Open Channels

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Abstract In overbank flow due to the interaction mechanism between the main channel and floodplain, the flow property of the compound sections gets affected. The complexity is more when the compound channels have non-prismatic floodplains. Additional complexity occurs during the interaction between the subsections as well as due to non-uniformity of flow through converging parts of the compound channel. For prediction of flow, calculation of energy loss parameters from section to section is an important task for river engineers. In this paper, an experimental investigation for the energy losses of converging compound channels for different flow depths along the converging path is performed. The loss of energy due to contraction and compound geometry for a compound channel is evaluated, and the dependency of energy loss for such channels is analyzed. A generalized multivariable regression model has been developed to predict the energy slope with high accuracy. Using the expression of the energy loss concept, the discharge capacity in the converging compound is found to provide good results as compared to other standard model exists in the literature.

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### List of symbols

- *B* Width of compound channel
- *b* Width of the main channel
- $E_1$  Total energy heads of section 1
- $E_2$  Total energy heads of section 2
- $h_1$  Energy loss
- *h* Height of the main channel
- H Bank full depth
- *L* Converging length
- R Hydraulic Radius
- $S_0$  Bed slope
- Se Energy slope
- S<sub>f</sub> Friction slope
- $V_1$  Mean flow velocities at section1
- *V*<sub>2</sub> Mean flow velocities at section2
- $Y_1$  Mean flow depths at section 1
- $Y_2$  Mean flow depths at section 2
- $Z_1$  Bed elevations above a given datum for section 1
- $Z_2$  Bed elevations above a given datum for section 2
- $\alpha$  Width ratio (*B*/*b*)
- $\alpha_1$  correction factors for velocity head for section 1
- $\alpha_2$  correction factors for velocity head for section 2
- $\delta$  Aspect ratio (b/h)
- *n* Composite Roughness
- $\beta$  Relative depth ((H h)/H)
- $\theta$  Angle of convergence
- $X_{\rm r}$  Relative distance (x/L)



#### **1** Introduction

In natural rivers, due to changes in the cross-sectional area, the state of the flow may be changed from uniform to nonuniform. Under such conditions, the hydraulic analysis will be more complicated compared to simple uniform flow. Calculation of energy loss in a compound channel is one of the key issues in river engineering studies, and it needs to be handled properly. In general, a compound channel consists of the main channel and two flood plains. Because of the exchange of momentum between the main channel and the floodplain, consumption of energy is more in a compound section as compared to a simple channel section. There is a continuous settlement of people near the bank of the river, causing a reduction in width of the compound channel, and as a result the converging of floodplains occurs. Wrong estimation of discharges in these regions will lead to much loss of life and properties. In converging compound channels, because of constant variation in the geometry of the floodplain along the direction of flow, the resultant interactions and exchange of momentum are increased further (Bousmar et al. [1]; Proust et al. [2]; Rezaei [3]). This increased momentum exchange is really an essential parameter and should be taken into consideration while doing flow modeling of such channel sections. The interaction of the flow between the main channel and a prismatic floodplain has been examined by many researchers such as Knight and Demetriou [4], Shiono and Knight [5], Khatua and Patra [6], and Khatua et al. [7]. Even in uniform flow conditions, there is a loss of energy occurs and the main sources of energy losses are bed friction, momentum flux due to both turbulent exchange and secondary current across the channel cross section. But in a non-prismatic compound channel (i.e., converging floodplain), the flow is non-uniform. Due to this non-uniformity, an additional contraction loss of energy is generated. Several two- and threedimensional approaches have been applied to the discharge prediction in compound channels (Knight and Shiono [8]; Cater and Williams [9]; Jazizadeh and Zarrati [10]; Marjang and Merkley [11]), but these models are complex and take too much time to reach satisfactory solutions. Therefore, onedimensional analytical methods are still widely used in engineering applications. Researchers have proposed a number of 1D methods to estimate the discharge capacity, which can be classified into six types: single channel method (SCM), divided channel method (DCM), area method(AM), Shiono and Knight method (SKM), coherence method (COHM), and apparent shear force method (ASFM). Although most of the modified discharge-estimating methods include the effect of momentum transfer mechanism and account for the interaction effect due to geometry and roughness conditions of a prismatic compound channel only, but there are no models exist in the literature which will account for the non-prismatic action of a compound channel. Several studies on contrac-



tions of the channels have been studied by many researchers. Head losses in different sections of the converging channel were being studied by Hager [12], and also he showed the effects of head losses on the computation of discharge. Molinas and Marcus [13] and Naik et al. [14] gave a model for discharge evaluation short, in abrupt channel contractions incorporating the energy losses due to contractions.

The present work investigates experimental findings on the effect of different converging angle and geometry on predicting the energy loss in such cases. Finally, an efficient approach is developed to compute the loss of energy in a compound channel with converging floodplains. Experimental work on non-prismatic compound channels has been done in the Hydraulics Laboratory of Civil Engineering Department at NIT, Rourkela, mainly to examine the water surface profile and the loss of energy caused by the convergence of the floodplains. Computation of the energy losses in different converging sections of a non-prismatic compound channel is carried out by considering different hydraulic and geometric conditions, and the functional relationships of energy loss with most influencing parameter are studied. An attempt is finally made to derive a generalized mathematical expression to predict the energy losses in a converging compound channel for different geometry and flow conditions. The results were finally tested with the observed experimental data and the data of other investigators. The work will be helpful for accurate flow estimation in different sections of a nonprismatic compound channel reaches.

## 2 Analyses of Energy Losses and Influencing Parameters

Under a uniform flow condition, the total energy loss gradient  $(S_e)$ , bed slope  $(S_0)$ , and friction slope  $(S_f)$  are equal. But in a symmetrically converging floodplain, the width of the compound channel reduces gradually in the flow direction due to which the flow depth varies significantly and the flow becomes non-uniform. Flow resistance increases with the presence of floodplain contractions. There are some methods exist for accounting the additional resistance occurred due to contraction, but they are valid for simple channels and meandering channels. So when those methods are applied to a converging compound straight channel, they failed to give a satisfactory result for calculation of discharge. In nonuniform flow conditions, the head loss gradient or energy slope  $(S_e)$  consists of a friction slope  $(S_f)$ , head loss due to shear stress related to interfacial turbulent exchanges  $(S_t)$  and head loss due to the mass exchange  $(S_m)$  (2009). So for computation of discharge, it is required to consider these losses so that we can get a better a model which can predict the discharge more accurately than any other existing method. An energy profile for a prismatic compound channel is shown in



Fig. 1 Sketch of energy profile for converging channels at NIT RKL

Fig. 1. The width of the compound and the main channel are considered as B and b, respectively. The width of the compound channel is being reduced gradually and becomes equal to the width of the main channel in the longitudinal direction (Fig. 1) (same as that of Naik and Khatua [15]). This non-prismatic section is being divided into five parts in the direction of the fluid flow. The mean flow depths at section 1 to section 5 are adopted as  $Y_1, Y_2, Y_3, Y_4$ , and  $Y_5$ , respectively. In a non-prismatic compound channel, along with frictional and viscosity loss, another two more energy loss factors exist such as the interaction loss and contraction loss. The interaction loss occurs due to the interaction between the main channel and floodplain, while the contraction loss takes place due to floodplain contraction. Head loss between two consecutive sections can be obtained by using the conservation of energy principle. For the energy calculation, the channel bottom at section 5 (i.e., extreme downstream end of the converging part of the channel) is considered as the datum of the channel. Applying the conservation of energy principle for the sections 1 and 2

$$E_1 = Z_1 + Y_1 + \frac{V_1^2 \alpha_1}{2g} \tag{1}$$

$$E_2 = Z_2 + Y_2 + \frac{V_2^2 \alpha_2}{2g},$$
(2)

where  $E_1$  and  $E_2$  are the total energy heads of section 1 and 2, respectively,  $Z_1$  and  $Z_2$  are the bed elevations above a given

datum for sections 1 and 2, respectively,  $V_1$  and  $V_2$  are the mean flow velocities at sections 1 and 2, respectively, and  $\alpha_1$  and  $\alpha_2$  are the correction factors for velocity head (Naik and Khatua [16]). Let  $h_1$  be the total head loss between sections 1 and 2. The conservation of energy principle can be written as:

$$Z_1 + Y_1 + \frac{V_1^2 \alpha_1}{2g} = Z_2 + Y_2 + \frac{V_2^2 \alpha_2}{2g} + h_1.$$
 (3)

The energy loss  $(h_1)$  in a converging compound channel does not linearly vary between sections to sections rather it changes with channel geometry, converging angle, and surface and flow conditions. So  $h_1$  can be written as:

$$h_1 = E_1 - E_2. (4)$$

After estimating the energy loss, the energy slope  $S_e$  for the given channel can be obtained by using the Eq. 5a,

$$S_{\rm e} = h_1/l = (E_1 - E_2)/l \quad (\text{Neglecting the } S_{\rm t} \text{ and } S_{\rm m}).$$
(5a)

Alternately the energy slope can also be computed from Manning's equations

$$Q = \frac{1}{n} R_{\rm e}^{\frac{2}{3}} S_{\rm e}^{\frac{1}{2}}$$
(5b)

$$R = \frac{A}{P},\tag{5c}$$



where Q = total discharge, n = composite roughness (Lotters Method), R = hydraulic radius i.e., (A/P).

So by putting the computed value of the energy slope in Manning's, Chezys or Darcy-Weisbach equations, the flow or average velocity calculation can be done in any part of the non-prismatic channel. In this paper, using Eq. 5, the energy slope between sections is calculated and applied in the present experimental channels and channels of Rezaei [3] for different cross sections of the channel and flow conditions. Here, section 1 is adopted as a reference for computation of the length of the reach. The energy slope calculation is based on a prediction of the total energy at any section. The computation of energy slope is an important issue as it has a significant effect on predicting the important variables like stage-discharge relationship, distribution of shear stress, flow distribution, etc., so by looking at these aspects, an attempt is made here to develop a model to predict the energy slope of a converging compound channel using the multi-linear regression approach.

#### 2.1 Selection of Hydraulic Parameters

Flow fields in a non-prismatic compound channel are significantly influenced by both geometrical and hydraulic parameters. The computation of the energy slope becomes complicated when the width of the floodplain is contracted and becomes zero. The factors which are considered to be responsible for the estimation of the energy slope are given below:

- (i) Angle of convergence  $(\theta)$
- (ii) Relative flow depth [ $\beta$ = bank full depth (H) depth of main channel (h/H)]
- (iii) Width ratio ( $\alpha$ ) [i.e., ratio of the width of the floodplain (*B*) to the width of the main channel (*b*)]
- (iv) Aspect ratio ( $\delta$ ) (i.e., ratio of the width of the main channel to depth of the main channel)
- (v) Longitudinal relative distance  $(X_r)$  (i.e., of point velocity in the lengthwise direction of the channel /total length of the non-prismatic channel).

#### **3 Experimental Setup and Procedure**

Experimentation had been carried out in three different sets of non-prismatic compound channels by varying the cross section built over a cement concrete flume. The dimension of the flume was  $15 \text{ m} \log \times 0.90 \text{ m}$  width  $\times 0.55 \text{ m}$  depth. Perspex sheets of same dimensions are used inside the concrete flume. The width ratio ( $\alpha$ ) and the aspect ratio ( $\delta$ ) of the channel were 1.8 and 5, respectively. Three different angles of convergence for the channels were adopted such as 5°, 9°, and 12.38°, respectively. The converging lengths



of the channels for three different angles were found to be 2.28, 1.26, and 0.84 m, respectively. Centrifugal pumps of 11.2 kW capacities were used for water supply into a large overhead tank. There is a measuring tank built at the downstream end of the channel. A sump is also built nearer to the measuring tank. The water was supplied to the overhead tank from this sump through pumping. Due to this arrangement, there is a complete recirculation of water is possible within experimental channels. Figure 2a shows the plan view of experimental sections while Fig. 2b shows the experimental setup of the channel. At the downstream, a regulating tailgate was provided to maintain the flow depth and to keep the flow uniform in the channel. A movable bridge was provided across the flume for taking measurements of each location on the plan of the compound converging channel. Velocity readings were taken by the Pitot tube having an outer diameter of 4.77 mm

The Pitot tube was rotated manually with respect to the flow direction till it records the maximum manometer reading. The steady uniform discharge was preserved in every run of the experiments, and several runs were performed for overbank flow with different relative depths varying from 0.15 to 0.51. Table 1 shows the hydraulic parameters of the experimental channels of NITRKL as well as Rezaei [3] channel. The energy losses in the flow due to the convergence of floodplain at different sections of the converging lengths were being evaluated.

#### **4** Experimental Results

The stage–discharge relationship of prismatic and nonprismatic sections for the converging compound channels of angles  $5^{\circ}$ ,  $9^{\circ}$  and 12.38° from in bank to overbank flow conditions are shown in Fig. 3a–c. A total 13 stage–discharge runs are observed at each test reach. The stage–discharge relationship for all cases is found to be power function given by  $H = AQ^n$  where A, n are coefficients with  $R^2$  value more than 0.97.

#### 5 Development of the Energy Slope Model

Generally, for the prismatic compound channel, the energy slope  $S_e$  depends importantly upon the geometrical parameter like width ratio, depth ratio, aspect ratio i.e  $S_e = F$  $(\alpha, \beta, \delta)$  where F is the functional symbol. But for converging compound channel which has a non-prismatic cross section the energy slope depends on two more influencing geometrical parameters such as converging angle  $\theta$  and relative distance  $X_r$  so  $S_e = F(\alpha, \beta, \delta, \theta, X_r)$  for a converging compound channel cases. From the experimental results, different plots are drawn between the energy slope  $S_e$  and these



Fig. 2 a Plan view of experimental setup. b Experimental setup of the channel with velocity readings using Pitot tube and pointer gauge

five non-dimensional independent parameters. The Se has been calculated from a wide range of experimental datasets of NIT, Rourkela, India, and three more series of converging compound channels data of Rezaei [3] (details of experimentation are provided in Table 1). All these converging compound channels have homogeneous roughness both in the main channel and floodplain subsections with Manning's n values 0.01 bearing smooth surface. This will help us to investigate the effect of all these influencing parameters of prediction of flow variables without considering the roughness effects. The variation of  $S_e$  has been found out for all the six converging compound channels. The variation of  $S_e$  with relative depth  $\beta$ , converging angle  $\theta$ , relative distance  $X_r$  and width ratio  $\alpha$  are plotted for different converging angles  $\theta$ , respectively, and shown in Figs. 4, 5, 6 and 7. Figure 4 shows that the  $S_e$  increase with increase in relative depth and this happen for lower aspect ratio of NIT Rourkela channel, but for higher aspect ratio channel of Rezaei [3] the  $S_e$  found to decrease exponentially this may be because of more energy consumed in higher aspect ratio channels as compared to that in low aspect ratio channels. However, the best fit for both the cases (higher and lower aspect ratio channel) are found to exponential nature shown in Fig. 4. Again, by fitting the

curve between  $S_e$  and the converging angle  $\theta$ , a relationship can be established (Fig. 5).

It increases with converging angle  $\theta$ , and the best trend follows a power function with  $R^2$  value 0.97. Similarly,  $S_e$ has been plotted with  $X_r$  for all the relative flow depths for all the channels shown in Fig. 6, and the best fits are found to be exponential functions with a very high  $R^2$  value of 0.98. Finally, the variations of  $S_e$  have been plotted with a different width ratios keeping the same angle and same relative flow depths, i.e., plotted to keep relative flow depth constant, i.e., 0.2, 0.25, 0.3, and 0.4 shown in Fig.7. Figure shows that the  $S_e$  decreases with width ratio  $\alpha$  for different relative depths and the best trend found for  $S_e$  with width ratio  $\alpha$  found to be logarithmic in nature for all the cases. Finally, the best functional relationships between  $S_e$  with five nondimensional parameters are listed below. All the equations bear the  $R^2$  value varying between 0.91 and 0.99.

 $S_e = F_1(\beta) = A(e)^{-3.38\beta}$  For higher aspect ratio, i.e.,  $\delta$  up to 5 (6)

$$S_{e} = F_{2} (\beta) = B(e)^{1.98\beta} \text{ For lower aspect ratio, i.e., } \delta > 5 (7)$$
  

$$S_{e} = F_{3} (\theta) = C(\theta)^{0.57} (8)$$



Verified test channel	Types of channel	Angle of convergence $(\theta)$	Longitudinal slope (S)	Cross-sectional geometry	Total channel width $(B)$ in m	Main channel width $(b)$ in m	Main channel depth $(h)$ in m	Width ratio $(s^{-1})$ $B/b(\alpha)$	Longitudinal relative distance $(X_{\rm r})$ in m	Aspect Rati $b/h(\delta)$
Rezaei [3]	Convergent (CV2)	$11.31^{\circ}$	0.002	Rectangular	1.2	0.398	0.05	3	2	7.96
Rezaei [3]	Convergent (CV6)	$3.81^{\circ}$	0.002	Rectangular	1.2	0.398	0.05	3	6	7.96
Rezaei [3]	Convergent (CV6)	$1.91^{\circ}$	0.002	Rectangular	1.2	0.398	0.05	3	6	7.96
N.I.T. Rkl	Convergent	$5^{\circ}$	0.0011	Rectangular	0.9	0.5	0.1	1.8	2.28	5
N.I.T. Rkl	Convergent	9°	0.0011	Rectangular	0.9	0.5	0.1	1.8	1.26	5
N.I.T. Rkl	Convergent	$12.38^{\circ}$	0.0011	Rectangular	0.9	0.5	0.1	1.8	0.84	5

$$S_{\rm e} = F_4 \left( x_{\rm r} \right) = D(e)^{0.56x_{\rm r}} \tag{9}$$

$$S_{\rm e} = F_5(\alpha) = E \log(\alpha) + 0.0023$$
 (10)

All these relationships of the non-dimensional parameters with  $S_e$  are attempted to compiled to get a generalize expression for  $S_e$ . To achieve this, we have applied all these Eqs. (6–10) to get multi-linear regression model and finally Eq. 11 is obtained. Table 2 represents the Unstandardized Coefficient and Regression Statistics of the regression analysis. Equations 11 and 12 represent the final form of the multiple linear regression results after compiling Eq. 6 to 10.

$$S_{e} = 0.0073 + 0.00024 (F_{1}) - 0.00013 (F_{2}) - 0.0022 (F_{3}) - 0.0036 (F_{4})$$
(11)

After simplifying the above equation for higher aspect ratio, i.e.,  $\delta$  up to 5

$$S_{\rm e} = -0.0016 + 0.00024e^{-3.38} - 0.0009\theta^{0.57} + 0.0092e^{0.56X_{\rm r}}$$
(12a)

For channels with Lower aspect ratio, i.e.,  $\delta > 5$ 

$$S_{\rm e} = -0.0016 + 0.00013e^{1.98} - 0.0009\theta^{0.57} + 0.0092e^{0.56X_{\rm r}}$$
(12b)

The modified energy slope of a converging compound channel can now be calculated using the results of  $S_e$  from Eq. 12. Here, the effect of momentum transfer, as well as the effect of converging angle and other geometrical effects of such channels, has been incorporated in terms of the improved energy slope expressions. Knowing the energy slope, the stage–discharge relationships of a converging compound channel from section to sections can now be calculated using Manning's equation (Eq. 5).

#### **6** Results and Discussion

#### **6.1 Error Analysis**

The variation between the calculated values of  $(S_e)$  using Eq. (12) and the corresponding observed values for all the six types of channels is shown in Fig. 8. The percentage error in estimated  $(S_e)$  is less when compared to observed values for both present experimental channel and Rezaei [3] channel.

#### 6.2 Stage–Discharge Models

The discharge results obtaining from the Eq. (12) are compared with six standard 1D approaches. The datasets used for validation purpose are different from the calibrated datasets.



Fig. 4 Variation of  $S_e$  of non-prismatic compound channel at typical sections

Fig. 5 Variation of  $S_{\rm e}$  with converging angles for different relative depths

This means dataset considered for contrasting the new model with other models are not from the dataset, which are used for the regression and calibration of the model. Total dataset was divided in the ratio of 7:3 where approximately 70% data were used in the regression meanwhile remaining datasets were used for the validation purpose. Among the other approaches, the following methods are considered, i.e., single channel methods (SCM), vertical divi-

sion methods (VDM), horizontal division methods (HDM), diagonal division method (DDM), area method (AM), apparent shear force method (ASFM). The standard errors for calculating discharge by different methods are shown in Fig. 9. It indicates that the proposed approach appears to be better method among all the discussed methods. The method is finally tested with the results obtained from the software package developed in the Conveyance Estimation System (CES). The free software has been developed by joint





Fig. 6 Variation of Se with section to section along the converging angle for different relative depths



Fig. 7 Variation of  $S_e$  with width ratio for different relative depths

 Table 2
 Unstandardized coefficient and regression statistics

	Coefficients	Regression statistics	
Intercept	0.0073	Multiple <i>R</i>	0.911
β	0.00024	R square	0.861
θ	- 0.00013	Adjusted R square	0.846
X <sub>r</sub>	-0.0022	-	-
α	-0.0036	-	-



Fig. 8 Scatter plot for observed and modeled value of Energy slope

Agency/DEFRA research program on flood defense, with contributions from the Scottish Executive and the Northern Ireland Rivers Agency, HR Wallingford, This proves the ade-



Fig. 9 Absolute error of discharge calculation for present experimental channel and channel of Rezaei [3]

quacy of the proposed energy slope models for a converging compound channel.

#### 7 Conclusions

The following conclusions can be derived from the above research work on converging compound channels presented in this work.

- 1. The energy slope is found to decrease with increase in width ratio of compound channels and increase with increase in converging angle  $\theta$  of the floodplains. Further the energy slope is  $S_e$  found to increase exponentially with overbank flow depth for higher aspect ratio channels and decrease exponentially for lower aspect ratio channels; this is due to consumption of more energy in higher aspect ratio channels as compared to that in low aspect ratio channels.
- 2. The dependency of energy slope with five most influencing non-dimensional geometric and hydraulic parameters of a converging compound channel is analyzed. For all the parameters, it is found to bear the nonlinear functions. A multiple linear regression analysis has been done to model a generalized expression to predict the energy slope of a converging compound channel.
- 3. Different standard models to predict the energy slope are applied to the present channel and the converging compound channels of other investigators. CES is found to provide good discharge prediction results as compared to other approaches; however, the present approach is providing the least discharge error as compared to all other approaches.

The overall model developed for the converging compound channel using non-prismatic characterized parameters is giving good result though converging angle ranges from only  $1.91^{\circ}$  to  $12.38^{\circ}$ . In case of higher angles or any case where angle falls out of this bracket are not investigated. Beside this, the overall roughness criteria for two-stage channel are normalized considering same roughness throughout the channel, which is not the case in real field scenario. However, roughness effect can also be considered in the further investigation and can give this model a comprehensive investigation in the near future.

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